CSCE 355
2/9/2026

(ε-)NFA ⟶ regex

conclude: regex $\subseteq$ ε-NFA $\Leftrightarrow$ NFA $\Leftrightarrow$ DFA
all describe the regular langs

① (top right)

State elimination method
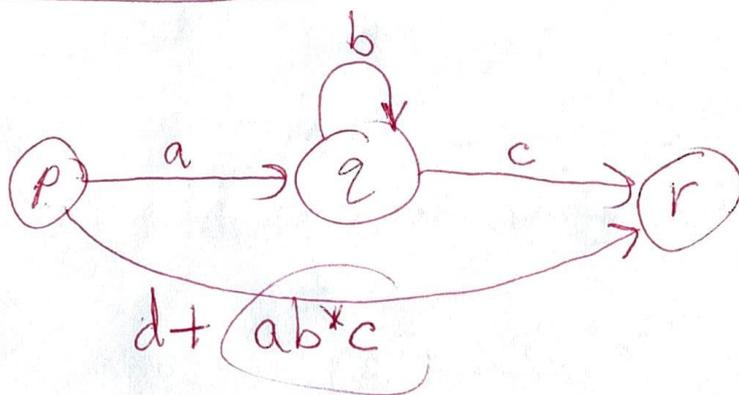
Can now eliminate q

Edge consolidation:
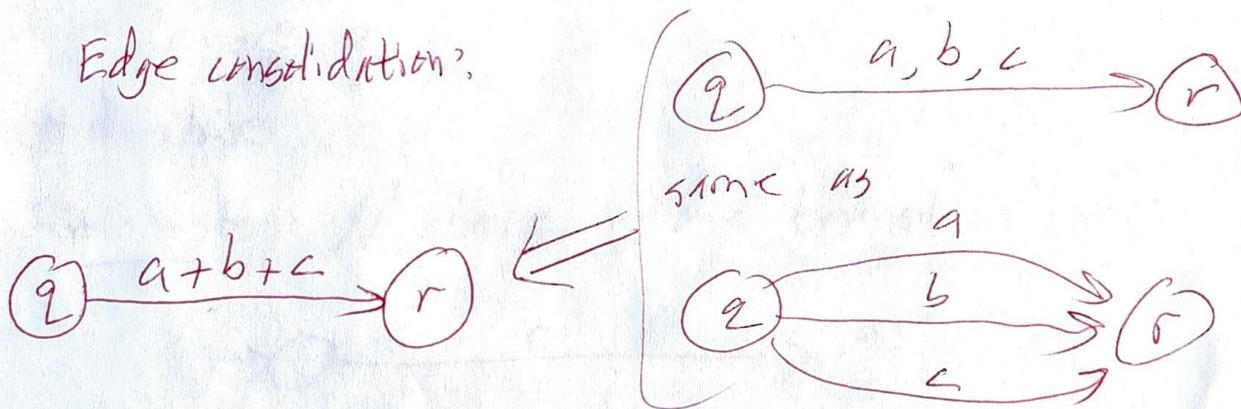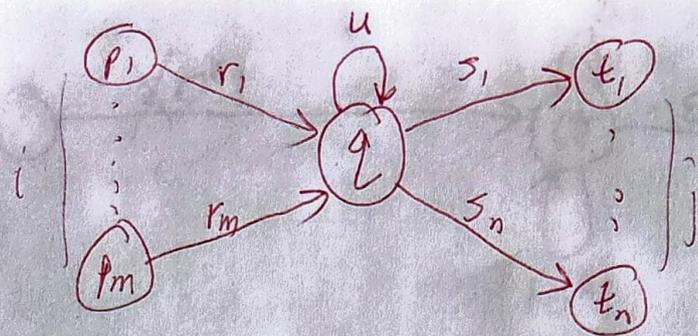

Method: Input: a clean ε-NFA ~~DFA~~
Output: An equivalent regex

Step 0: Do edge consolidation, $\xrightarrow{\varepsilon} \Rightarrow \xrightarrow{\emptyset^*}$

while there is an intermediate state q, do
not the start state
"   " accept   "
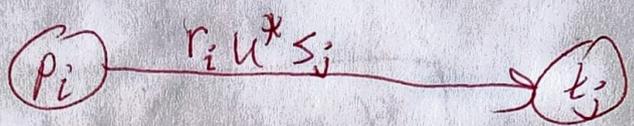
For every $1 \le i \le m$ and $1 \le j \le n$

    a) create a bypass;
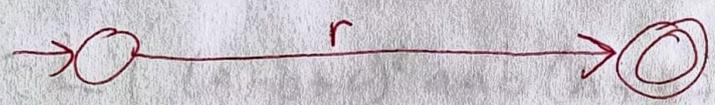   add an edge ~~with~~

mn
many
bypasses

    b) edge consolidation
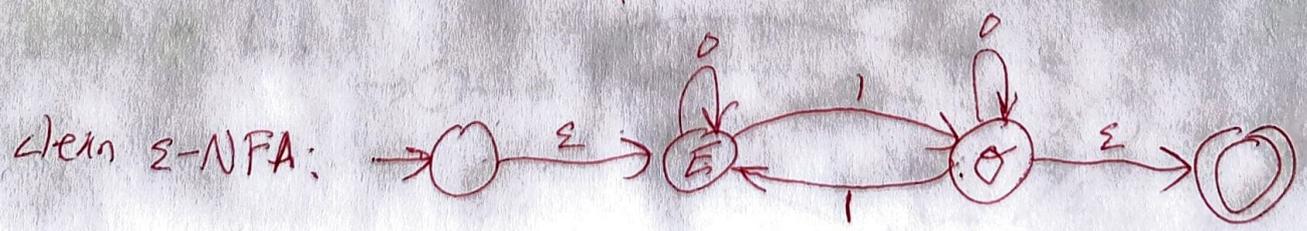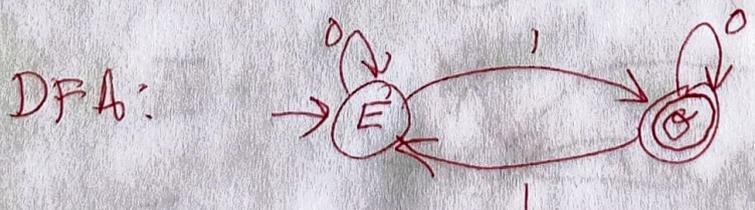
Remove q

end-while

Finish step; // there is one transition left

Return r

Examples: $L = \{ w \in \{0,1\}^* : w \text{ has an odd } \# \text{ of } 1\text{'s} \}$
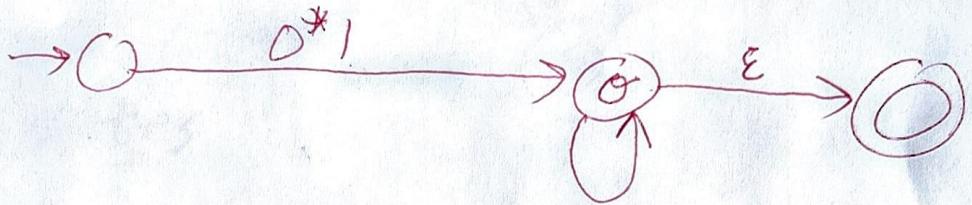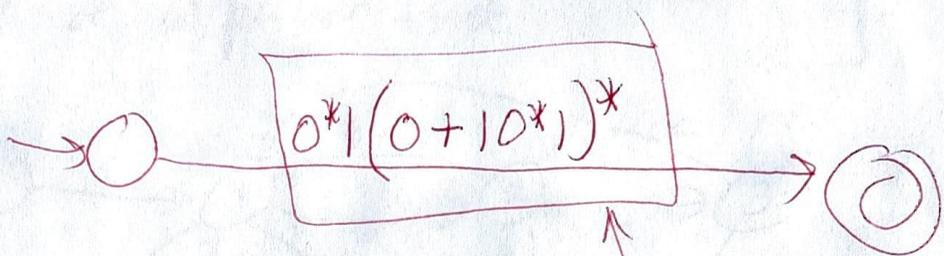
DFA:

clean ε-NFA:

Eliminate E:

$$\rightarrow O \quad \cancel{\cancel{0^*1}} \quad \xrightarrow{} \quad O \xrightarrow{\varepsilon} \odot$$

(with self-loops 0 and 10^*1)

Edge consolidate:

$$\rightarrow O \quad \xrightarrow{0^*1} \quad O \xrightarrow{\varepsilon} \odot$$

(self-loop $0 + 10^*1$)

Eliminate O:

$$\rightarrow O \quad \boxed{0^*1\left(0 + 10^*1\right)^*} \quad \rightarrow \odot$$
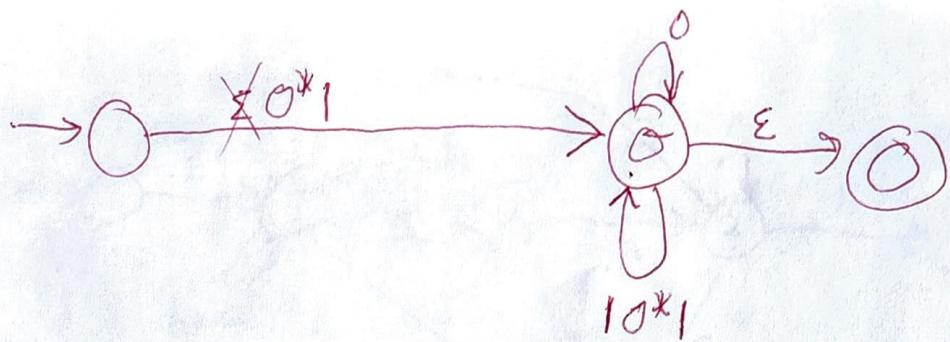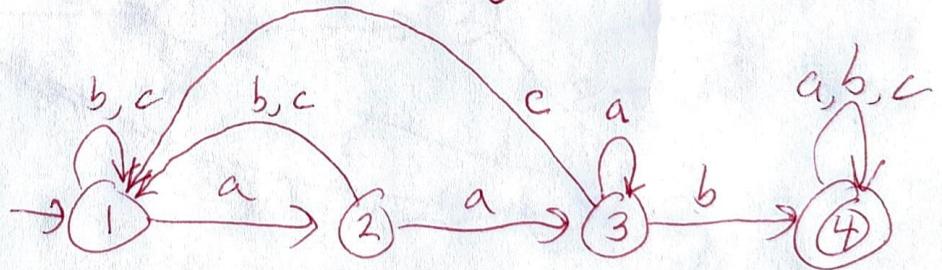
↑
Answer

Ex: $L = \{w \in \{a, b, c\} : w \text{ has } aab \text{ as a substring}\}$

$$L = L\left((a+b+c)^* \, aab \, (a+b+c)^*\right)$$

what about $\bar{L}$?
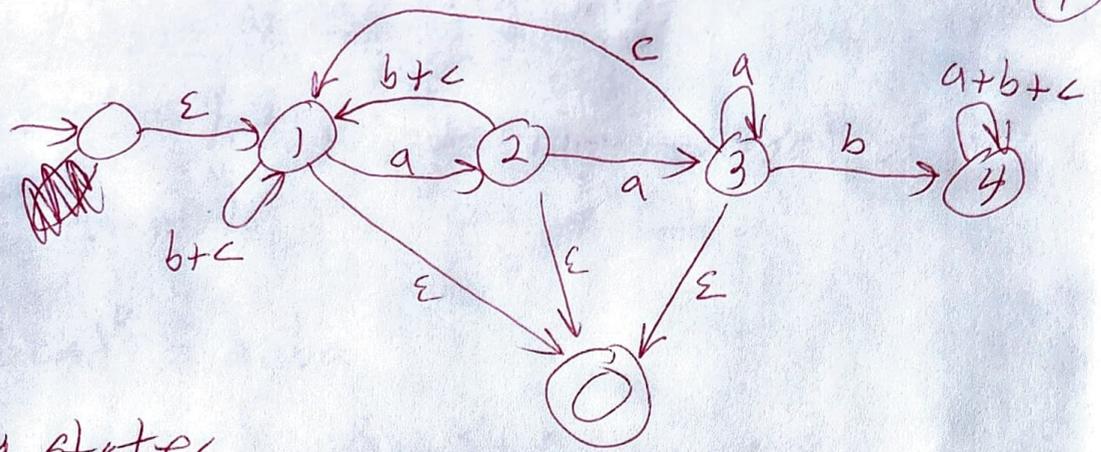
DFA for L:

DFA for $\bar{L}$:
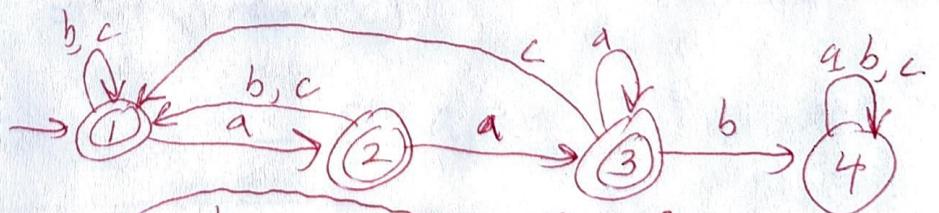
clean ε-NFA for $\bar{L}$:
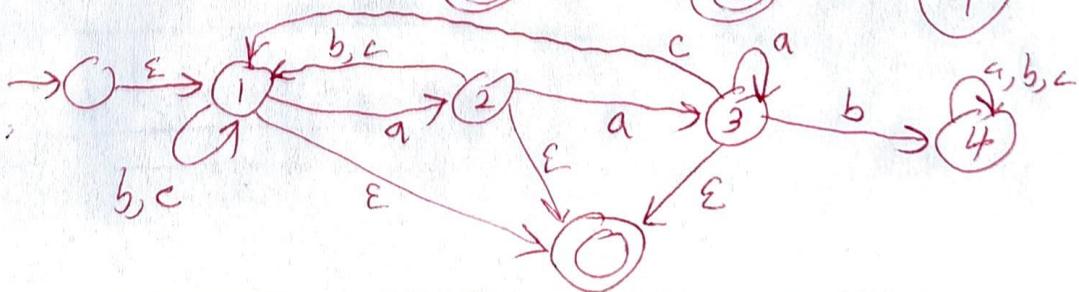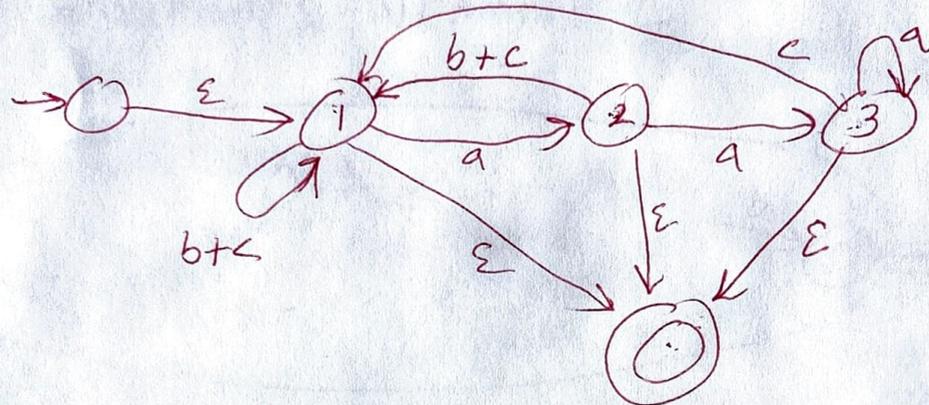
consolidate
edges:

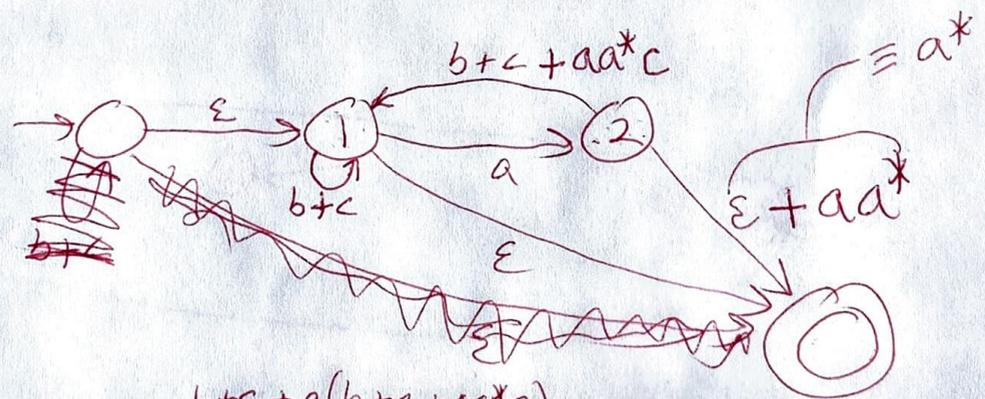
Start eliminating states

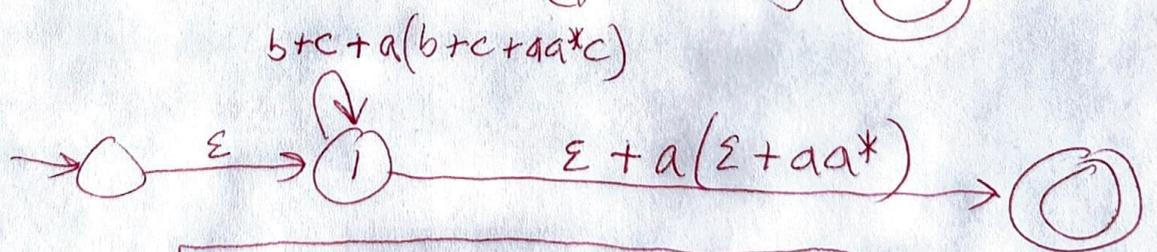Rule: Eliminate the state that triggers the least # of bypasses first

Elim 4
(no bypasses!)

Elim 3
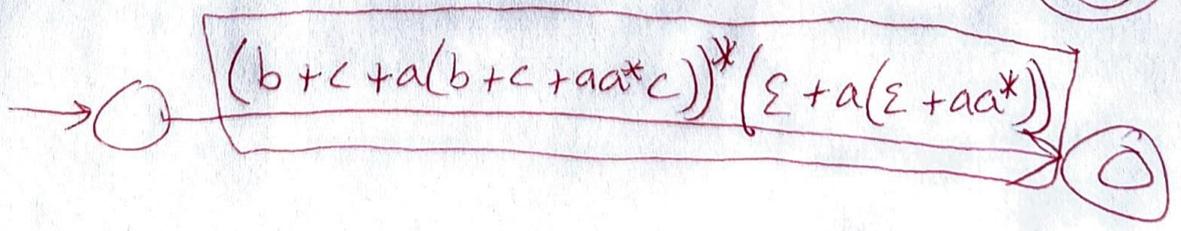(2 bypasses)

$\equiv a^*$

$b+c+aa^*c$

$\varepsilon + aa^*$

Elim 2
(2 bypasses)

$b+c+a(b+c+aa^*c)$

$\varepsilon + a(\varepsilon + aa^*)$

Elim 1
(1 bypass)

$$\left(b+c+a(b+c+aa^*c)\right)^*\left(\varepsilon+a(\varepsilon+aa^*)\right)$$
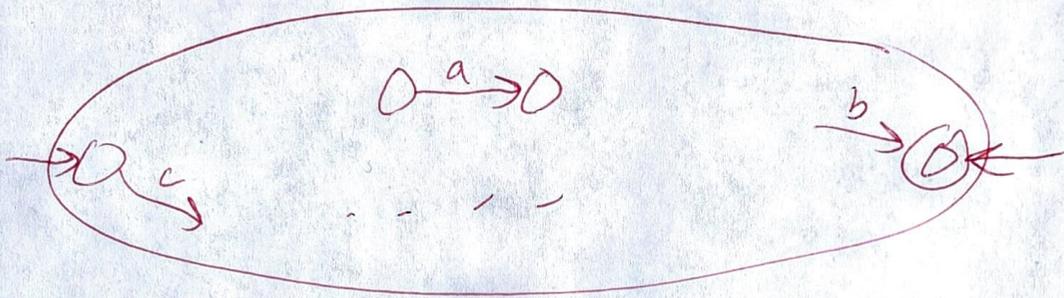
More closure properties of the reg. langs.

Def: Given $w \in \Sigma^*$, let w^R be the reversal of w
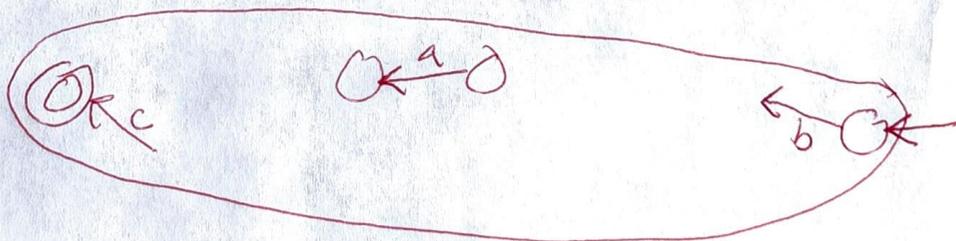(symbols in reverse)

Ex. $(abca)^R = acba$

Given $L \subseteq \Sigma^*$, define $L^R := \{w^R : w \in L\}$

Prop: If L is regular then so is L^R.

Proof: Given a clean ε-NFA for L,

make this ε-NFA:

- swap start & accept states
- reverse all arrows.   //