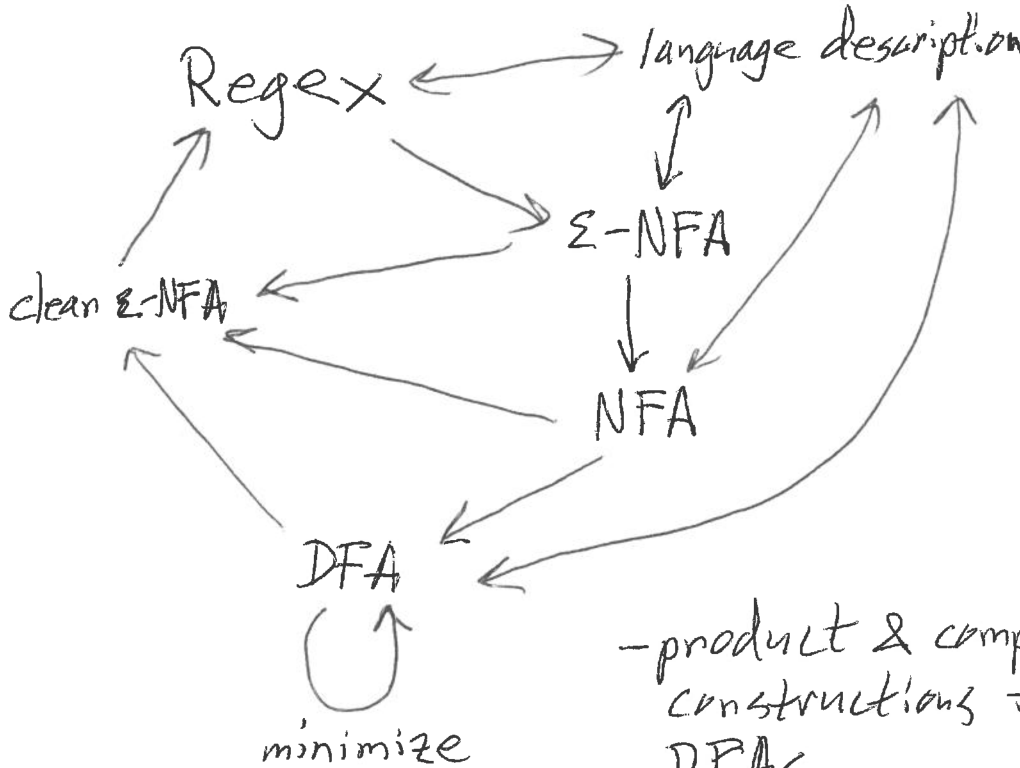
CSCE 355
4/28/2025

Review Day

Course Topics

Regular Languages

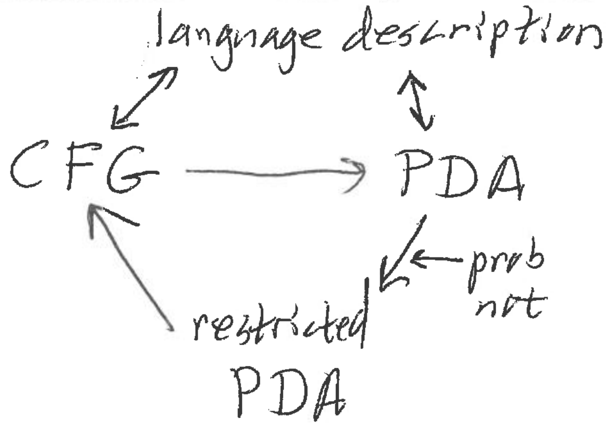


- product & complement constructions for DFAs
- closure properties of the reg. langs.

regex(es) FA(s)

- Pumping lemma for reg langs

Context-free Languages (CFLs)



- CFGs, derivations, parse trees, ambiguity
- closure properties of CFLs (less likely (?))
- Pumping Lemma for CFLs

Turing Machines (TMs)

lang description $\longleftrightarrow$ TM (state diag or tabular form)

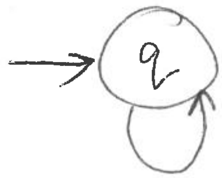
- deciding vs recognizing a language
- Editing problem

Restricted PDA to CFG example

$\Gamma = \{Z, *\}$

0 = (

1 =)



0, $\overset{* \text{ or } Z}{\text{anything}}$ / push $*$ ~~0/push *~~

~~1, anything~~ ~~1/pop~~

$1, *$ / pop ~~Z/pop~~

ε, Z / pop

CFG:

Variables:

$S, [qZq], [q*q]$

$$S \longrightarrow [qZq]$$

$$[qZq] \longrightarrow \varepsilon \mid 0[q*q][qZq]$$

$$[q*q] \longrightarrow 1 \mid 0[q*q][q*q]$$

$[qXr] \Longrightarrow \ldots\ldots \Longrightarrow$ those string that ~~are~~ can be read starting in state q with X on the stack and ending in state r with X popped.

Pumping Lemma:

Reg lang example:

$$L = \{ ww : w \in \{0,1\}^* \}$$

To show L not pumpable:

Given $p > 0$, let $s :=$ (some string that has length $\geq p$ and is in L).

$0^p 1^p 0^p 1^p$ works

$0^p 1 0^p 1$ works

0^p does not work

Given $x, y, z \in \{0,1\}^*$ such that

- $s = xyz$
- $|xy| \leq p$
- $|y| > 0$,

let $i :=$ some value (never 1) such that $xy^iz \notin L$.

0

Then $xy^iz \notin L$ because (brief justification).

Let $k = |y| > 0$. So $y = 0^k$

$xy^0z = xz = 0^{p-k}1^k 0^p 1^k \notin L$

Example:

$$L = \{ a^m b^n : 2m \leq n \leq 3m \}$$

Given $p > 0$, let $s :=$ $a^p b^{2p}$

Given $x, y, z \in \{a, b\}^*$ such that . . . .

Let $i := 2$? yes.

$xyyz \notin L$ because $y = a^k$ some $k > 0$,
so $xyyz = a^{p+k} b^{2p}$. But $2p < 2(p+k)$,
so $xy^2z \notin L$.

Ex: $L = \{a^m b^n : n < 2m \text{ or } n > 3m\}$

~~$L = \overline{\{a^m b^n : 2m \le n \le 3m\}} \cap L(a^* b^*)$~~

$\underbrace{\{a^m b^n : 2m \le n \le 3m\}}_{\text{showed not pumpable}} = \overline{L} \cap L(a^* b^*)$

So if L is regular, then so is $\overline{L}$ and
$\underbrace{\overline{L} \cap L(a^* b^*)}_{\text{not pumpable, hence not regular.}}$ ↯

Try to show that L is not pumpable.

Pumping Lemma for CFLs example:

$L = \{www : w \in \{0,1\}^*\}$

L is not CFL pumpable.

Proof: $(s \in L \;\&\; |s| \geq p)$

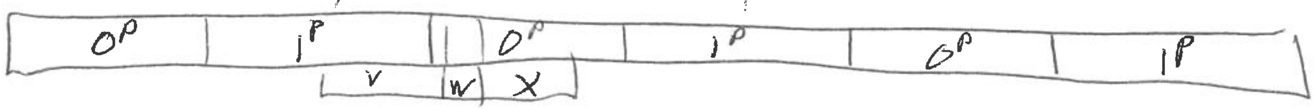
Given $p > 0$, let $s := 0^p 1^p 0^p 1^p 0^p 1^p$

Given $u, v, w, x, y \in \{0,1\}^*$ such that

- $s = uvwxy$
- $|vwx| \leq p$
- $|vx| > 0$,

let $i := 0$ (or anything $\neq 1$)

then $uv^iwx^iy \notin L$ because



Does $s := 0^p 1 \, 0^p 1 \, 0^p 1$ work? yes