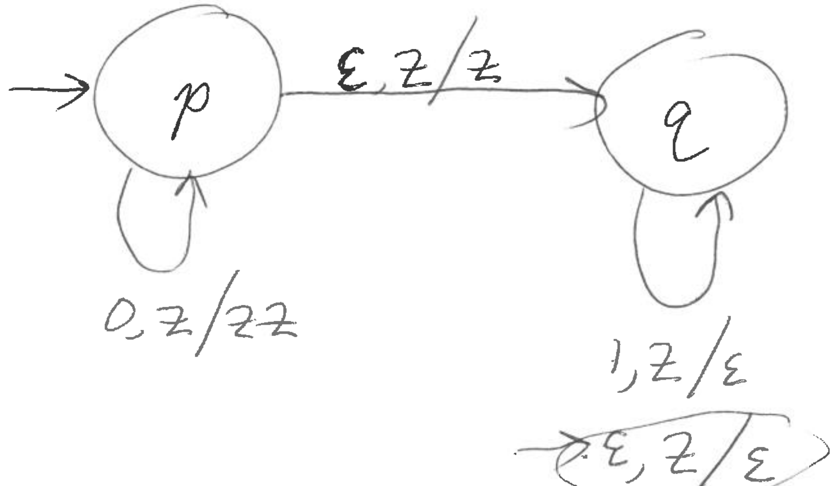
CSCE 355
3/19/2025

Computation examples
Two acceptance criteria are equivalent

Example: $L = \{ 0^n 1^n : n \geq 0 \}$

PDA for L:

accept by empty stack



Input
$w = 000111$

$\underbrace{\langle p, 000111, Z \rangle}_{\text{initial config}} \vdash \langle p, 00111, ZZ \rangle \vdash \langle p, 0111, ZZZ \rangle$

$\vdash \langle p, 111, ZZZZ \rangle \vdash \langle q, 111, ZZZZ \rangle$

$\vdash \langle q, 11, ZZZ \rangle \vdash \langle q, 1, ZZ \rangle$

$\vdash \langle q, \varepsilon, Z \rangle \vdash \underbrace{\langle q, \varepsilon, \varepsilon \rangle}_{\text{empty-stack accepting config}}$

accept!

Not the only accepting path!

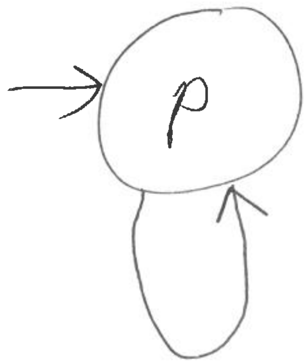
$\langle p, 000111, Z \rangle \vdash \langle q, 000111, Z \rangle$

$\vdash \langle q, 000111, \varepsilon \rangle$ no successor rejecting

Ex: $L = \{ w \in \{0,1\}^* : w \text{ has} = \# \text{ of } 0\text{'s} \ \& \ 1\text{'s} \}$

Stack alphabet is $\{Z, +, -\}$

empty stack acceptance



$0, Z / -Z$

$1, Z / +Z$

~~$0, + / ++$~~

$0, - / --$

$1, - / \varepsilon$

$0, + / \varepsilon$

$1, + / ++$

$\varepsilon, Z / \varepsilon$

Input is 1001

Accepting path:

$\langle p, 1001, Z \rangle$

$\vdash \langle p, 001, +Z \rangle$

$\vdash \langle p, 01, Z \rangle$

$\vdash \langle p, 1, -Z \rangle$

$\vdash \langle p, \varepsilon, Z \rangle$

$\vdash \langle p, \varepsilon, \varepsilon \rangle$

accept

Theorem:

1) For every PDA P there exists a PDA P' such that $N(P') = L(P)$

2) " " " " " P'' such that $L(P'') = N(P)$

Recall:

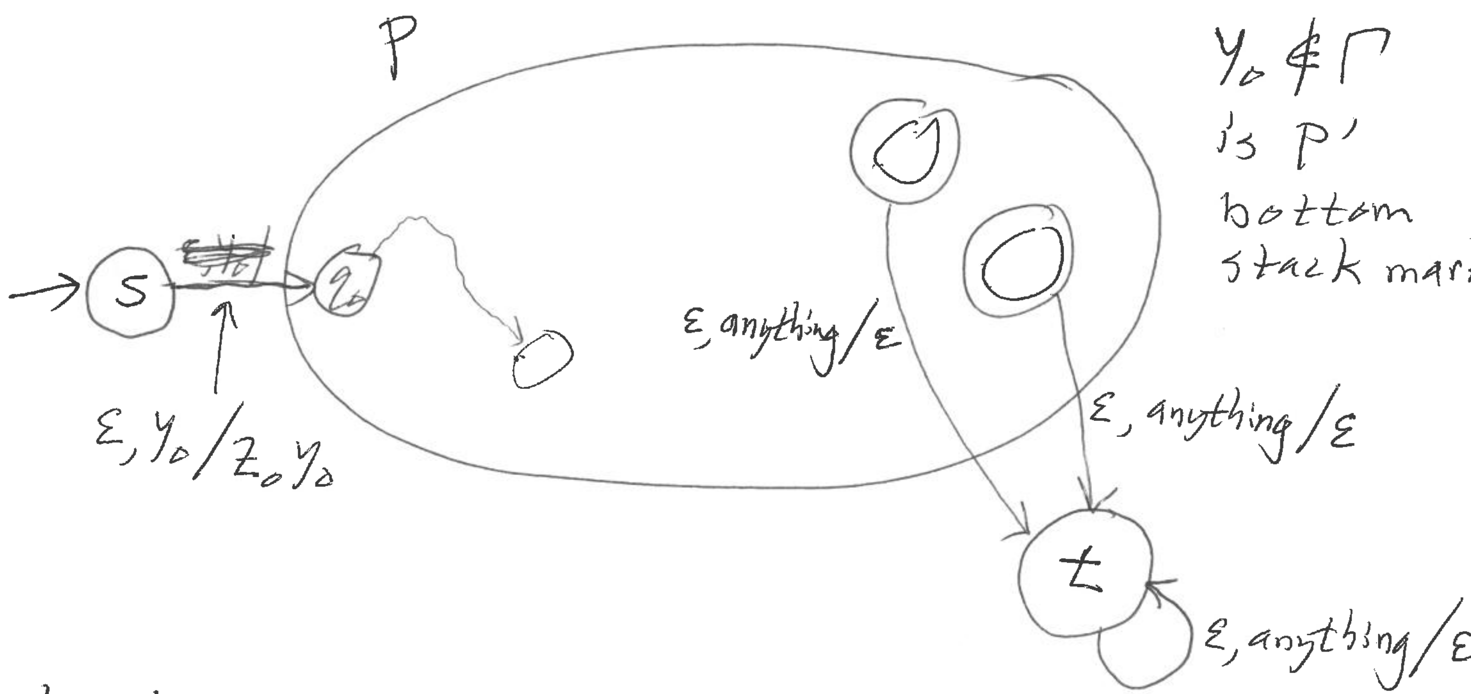
$L(P) = \{w : P \text{ accepts } w \text{ via final state}\}$

$N(P) = \{w : P \text{ accepts } w \text{ via empty stack}\}$

<u>Proof</u>: Let $P := \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle$ be any PDA.

For (1) we define P' as follows:

$Y_0 \notin \Gamma$ is P' bottom stack marker



$P' = \langle Q \cup \{s, t\}, \Sigma, \Gamma \cup \{Y_0\}, \delta', s, Y_0, \underline{\text{anything}} \rangle$

where $s, t \notin Q$, $s \neq t$, $Y_0 \notin \Gamma$, and

$\delta(s, \varepsilon, Y_0) = \{(q_0, Z_0 Y_0)\}$

For every $q \in F$, $\delta(q, \varepsilon, \text{anything})$ includes (t, ε)

$\delta(t, \varepsilon, \text{anything})$ includes (t, ε)

otherwise, δ' agrees with δ on all other arguments.

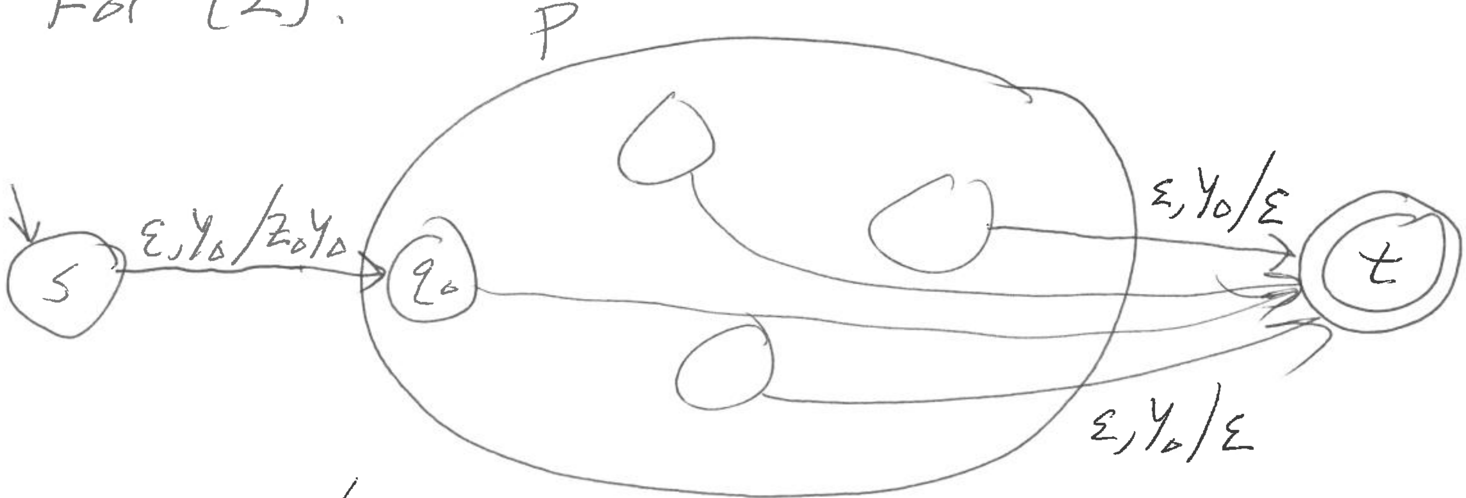
For all w, if P accepts w via final state ($w \in L(P)$),

then P' accepts w via ~~∅~~ empty stack ($w \in N(P')$).
(Fairly clear)

Conversely, if P′ accepts w via empty stack, then P must accept w via final state.

[New Y_0 is needed, otherwise P could empty the stack but end in a rejecting state after reading the whole input. But then P' accepts via empty stack but P reject via final state.]

For (2):



(make all states in P rejecting)

P'' uses its own bottom stack marker Y_0
pushes Z_0 onto stack and transfers control to P.

P empties its stack means Y_0 is the top of P'''s stack.

then can move to accepting state t
if and only if this happens.

$\blacksquare$

Next up:

$$\begin{tikzcd} \text{CFGs} \arrow[r, bend left, "\text{next}"] & \text{PDAs} \binom{\text{empty}}{\text{stack}} \arrow[l, bend left] \arrow[d, shift right] \\ & \text{PDAs} \binom{\text{final}}{\text{state}} \arrow[u, shift right] \end{tikzcd}$$

← already done

Grammar to PDA:

Given a grammar $G = \langle V, \Sigma, S, P \rangle$

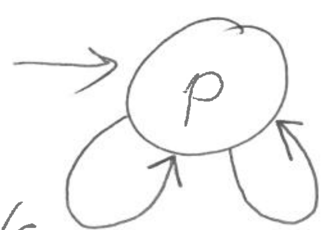
vars, input terminals, start symbol, productions

Ex: G: $S \to 0S1 \mid \varepsilon$ $\qquad L(G) = \{0^n 1^n : n \geq 0\}$

PDA ~~$P = \langle \{p\}, \Sigma, \Sigma \cup V, \delta, p$~~

$P = \langle \{p\}, \Sigma, \Sigma \cup V, \delta, p, S, \emptyset \rangle$

where δ is this:



matching transitions $\left[\begin{array}{l} 0,0/\varepsilon \\ 1,1/\varepsilon \end{array}\right.$

$\left.\begin{array}{l} \varepsilon, S/0S1 \\ \varepsilon, S/\varepsilon \end{array}\right]$ expansion transitions

Example: Input $w = 000111 \in L(G)$

$\langle p, 000111, S\rangle \vdash \langle p, 000111, 0S1\rangle$

match 0

$\vdash \langle p, 00111, S1\rangle \vdash \langle p, 00111, 0S11\rangle$

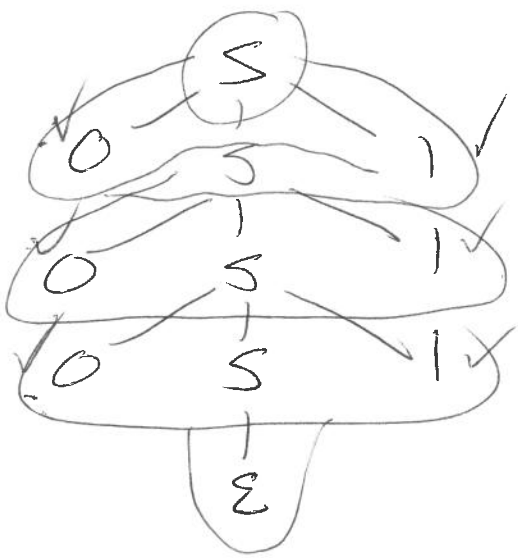
$\vdash \langle p, 0111, S11\rangle \vdash \langle p, 0111, 0S111\rangle$

$\vdash \langle p, 111, S111\rangle \vdash \langle p, 111, 111\rangle$

match 1

$\vdash \langle p, 11, 11\rangle \vdash \langle p, 1, 1\rangle \vdash \langle p, \varepsilon, \varepsilon\rangle$

accept!



$w = 001$

$\langle p, 001, S\rangle \vdash \langle p, 001, 0S1\rangle$

$\vdash \langle p, 01, S1\rangle$

$\vdash \langle p, 01, 0S11\rangle$

$\vdash \langle p, 1, S11\rangle$

$\vdash \langle p, 1, 11\rangle$

$\vdash \langle p, \varepsilon, 1\rangle$ reject