

CSCE 355
2/10/2025

Today: regex \rightarrow ϵ -NFA (cont.)
Intro to GNFA's

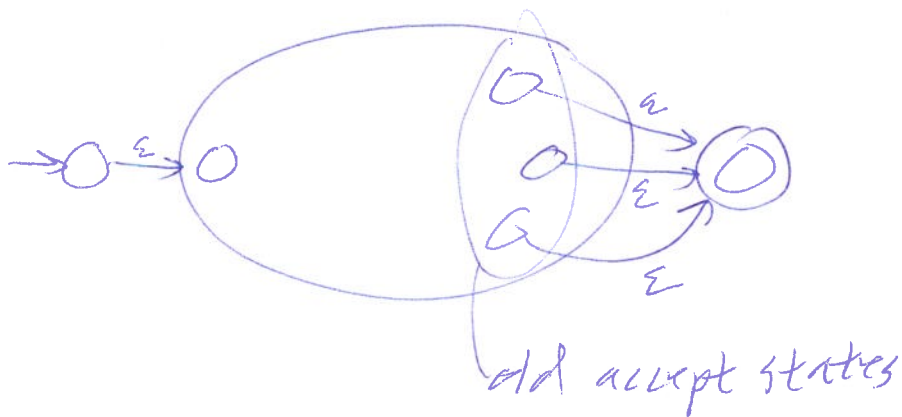
1

Recall from last time: rules for converting a regex into an ϵ -NFA. Recursion on regex syntax.

An ~~ATP~~ ϵ -NFA is clean if

- one accept state & it is \neq start state,
- no transitions into start state
- " " out of accept state

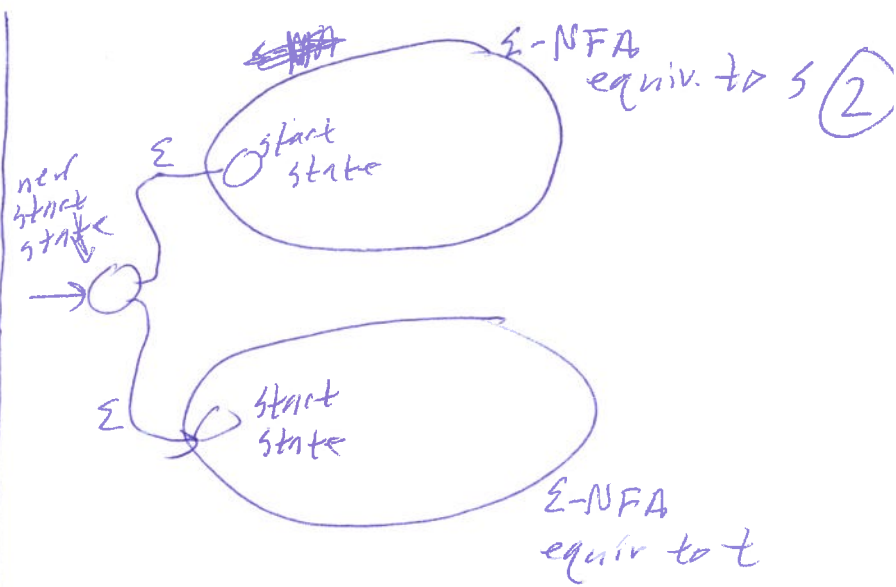
Showed: Every ϵ -NFA has an equivalent clean ϵ -NFA:



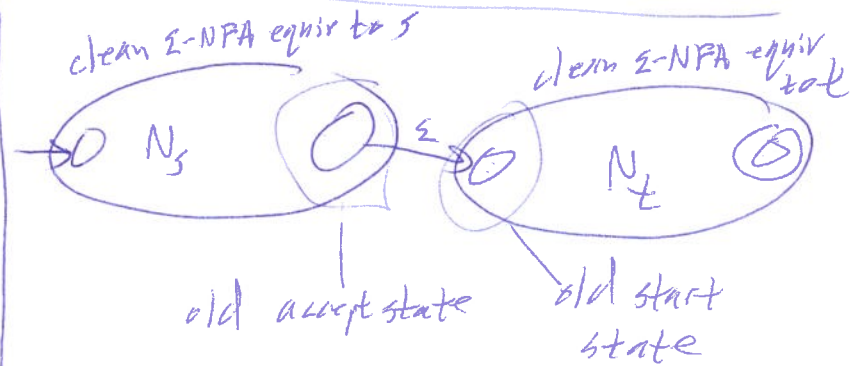
Rules for regex \rightarrow ϵ -NFA (fix alphabet Σ)

regex r	equiv ϵ -NFA
\emptyset	$\rightarrow \bigcirc$
$(a \in \Sigma)$ a	$\rightarrow \bigcirc \xrightarrow{a} \bigcirc$

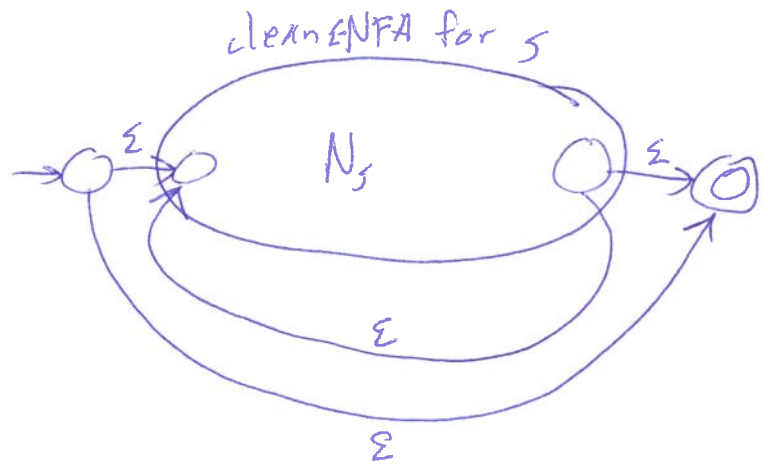
$s + t$
 (regexes s, t)



st



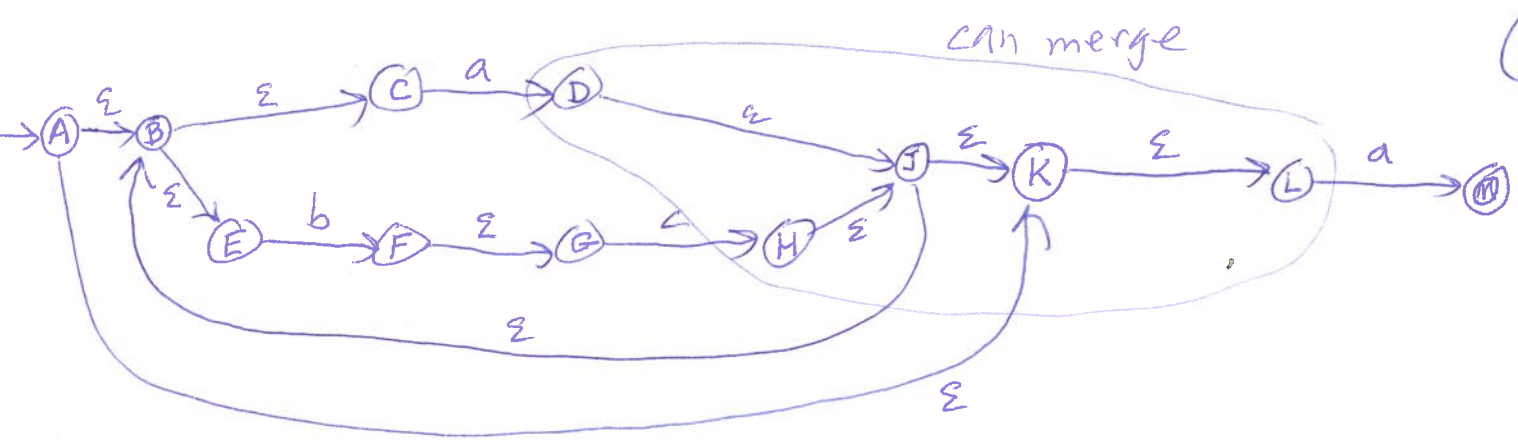
s^*



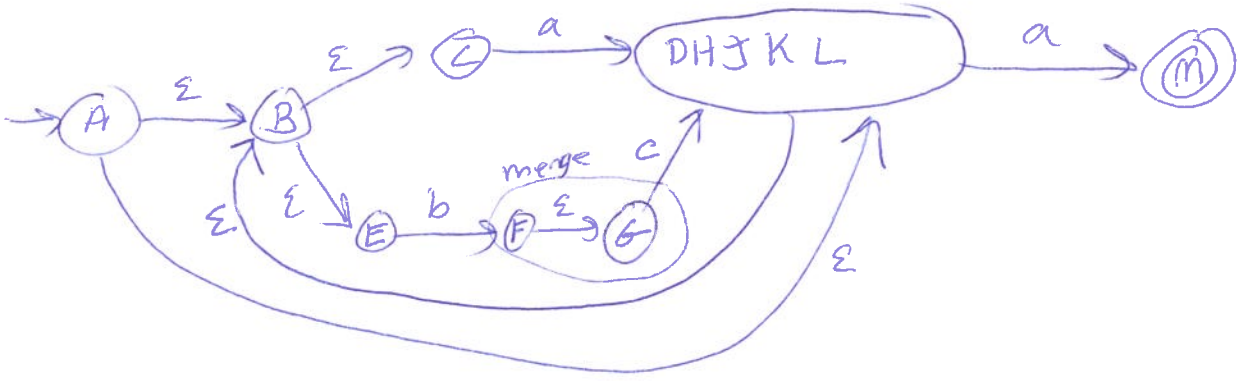
Example: $(a + bc)^* a$

Work from inside out — start with the atomic regexes, then combine the resulting ϵ -NFAs.

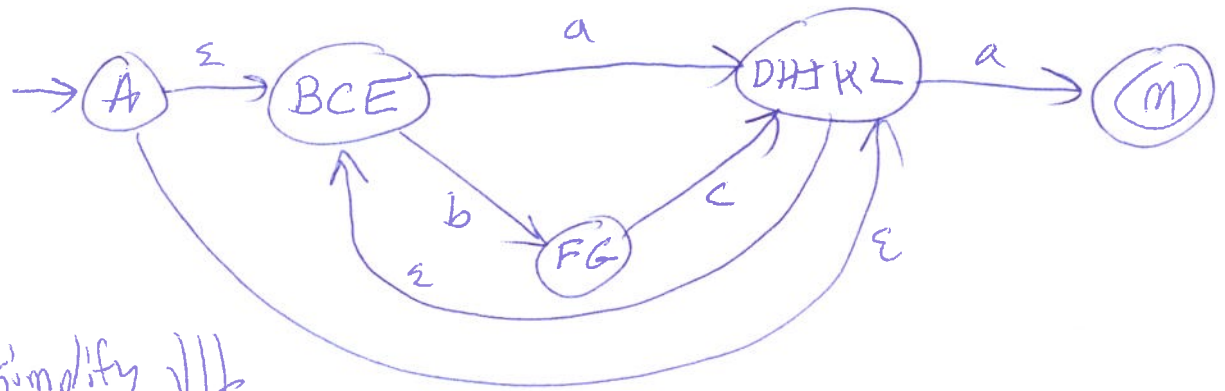
3



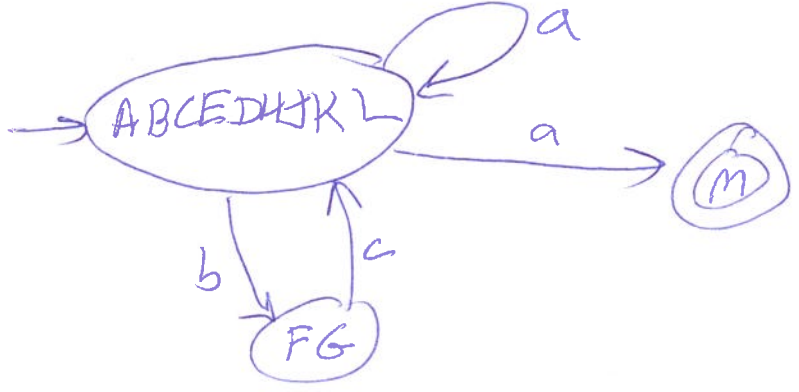
simplify



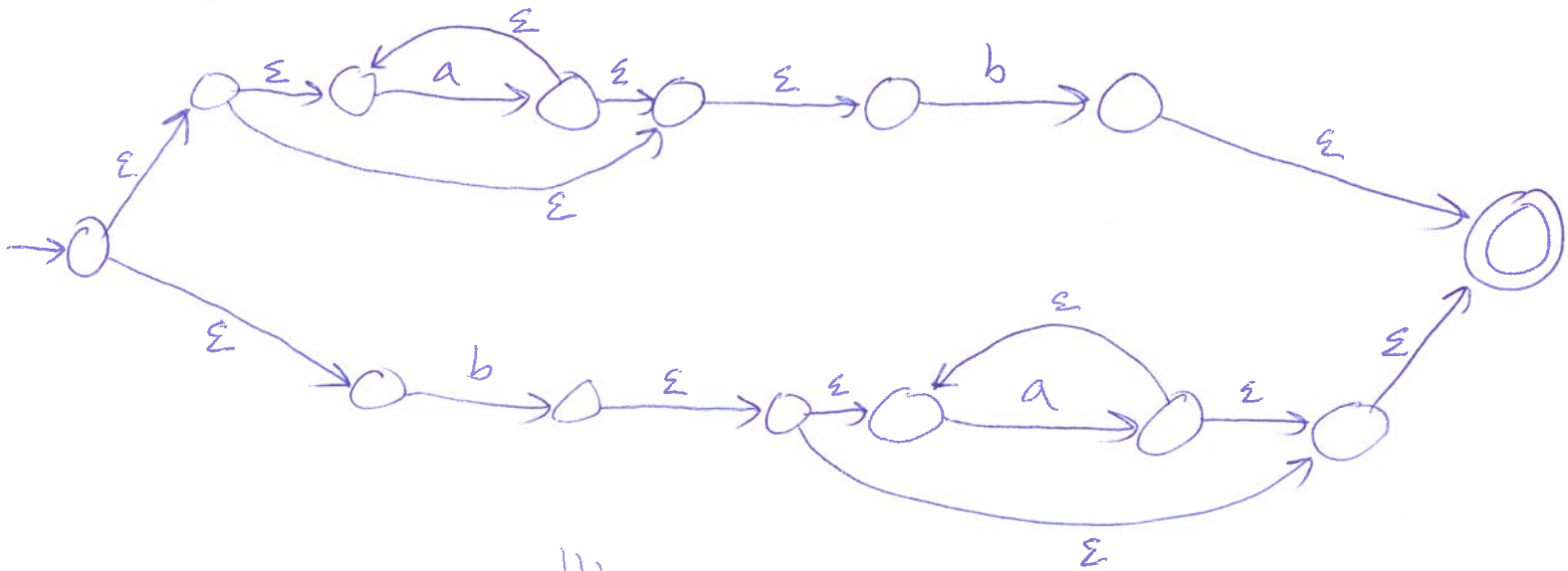
simplify



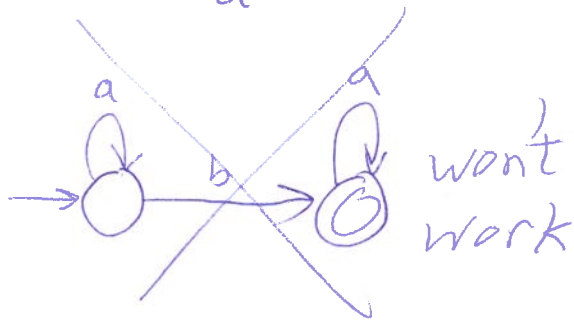
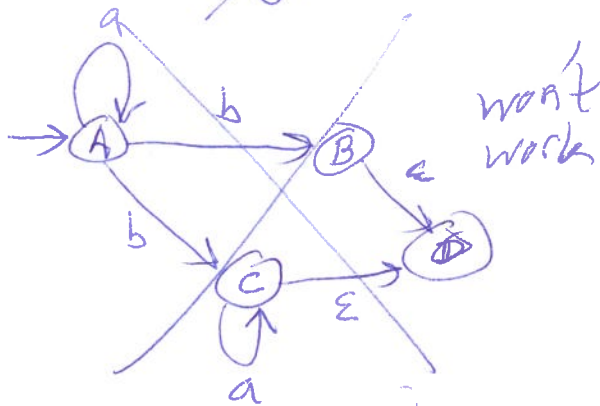
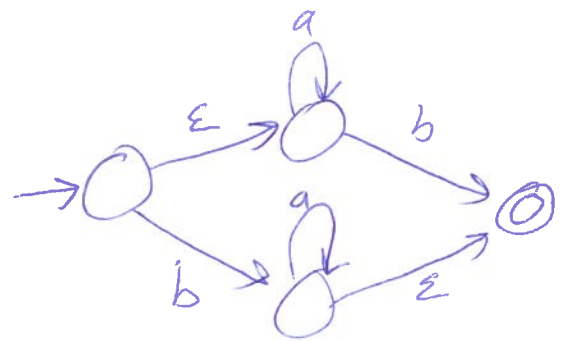
simplify



Example $a^*b + ba^*$



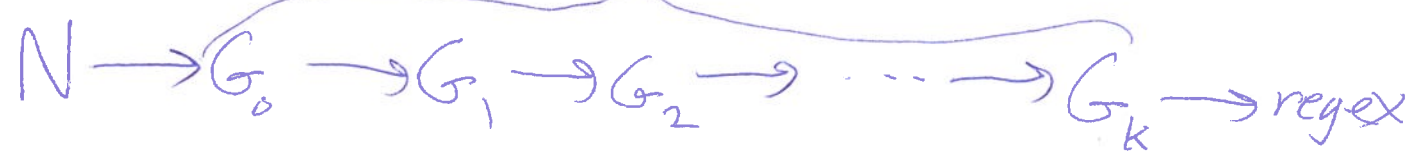
⋮



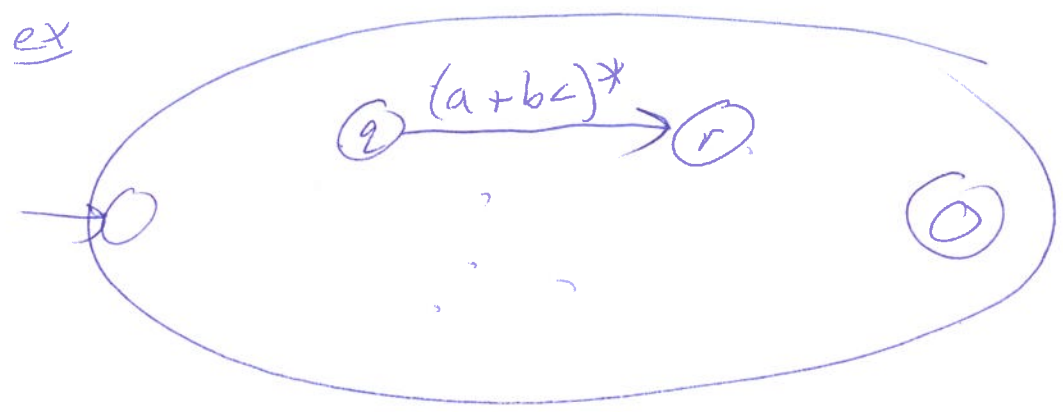
Σ -NFA $\xrightarrow{\text{state elimination method}}$ equivalent regex

clean Σ -NFA

GNFAs



* In a GNFA (Generalized NFA), edge labels can be arbitrary regexes over the input alphabet.



Def: A generalized NFA (GNFA) is a tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are as with an NFA, and

$$\delta : Q \times Q \rightarrow \text{REX}_{\Sigma}$$

REX_{Σ} = set of all regexes over Σ .

⑥

If q, r are states and R is a regex,
then in the transition diagram,



means

$$\delta(q, r) = R$$



$$\delta(q, r) = \emptyset$$



Def: Let $G = \langle Q, \Sigma, \delta, q_0, F \rangle$ be a GNFA
and let $w \in \Sigma^*$ be any string.

A computation path of G on input w is
a sequence of states

$$s_0, s_1, \dots, s_k \in Q$$

such that there exist $w_1, \dots, w_k \in \Sigma^*$
such that

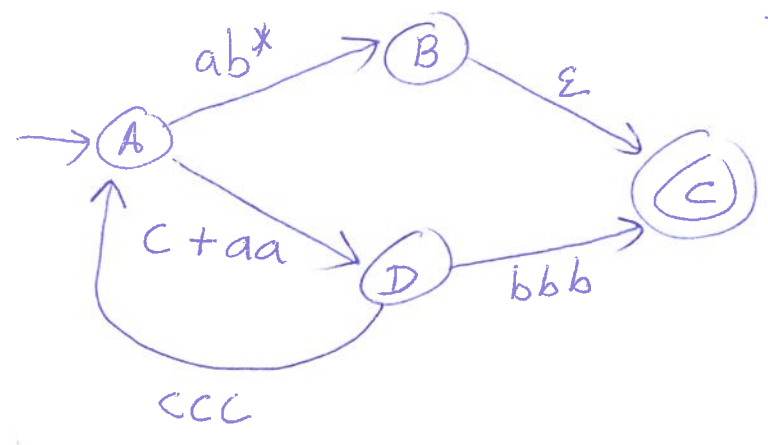
- 1) $s_0 = q_0$
- 2) $w = w_1 \dots w_k$
- 3) $\forall i, 1 \leq i \leq k, w_i \in L(\delta(s_{i-1}, s_i))$

[w_i matches the regex R_i



~~Def~~ G accepts w if there exists a computation path s_0, \dots, s_k with $s_k \in F$.

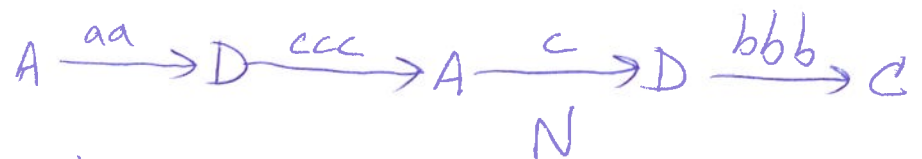
Example



Tabular form:

	A	B	C	D
$\rightarrow A$	\emptyset	ab^*	\emptyset	$c+aa$
B	\emptyset	\emptyset	ϵ	\emptyset
*C	\emptyset	\emptyset	\emptyset	\emptyset
D	ccc	\emptyset	bbb	\emptyset

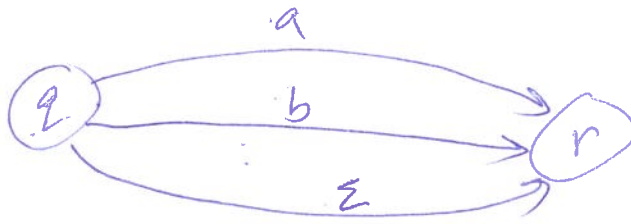
$aaccccbbb$



Note that an ϵ -NFA_N is "essentially" a GNFA_N:
 The transition diagrams for N and G look the same by interpreting the edge labels of N as regexes [$\epsilon := \emptyset^*$]

One exception:

In N :



edge consolidation

In G :

$$\delta(q, r) = a + b + \epsilon$$

