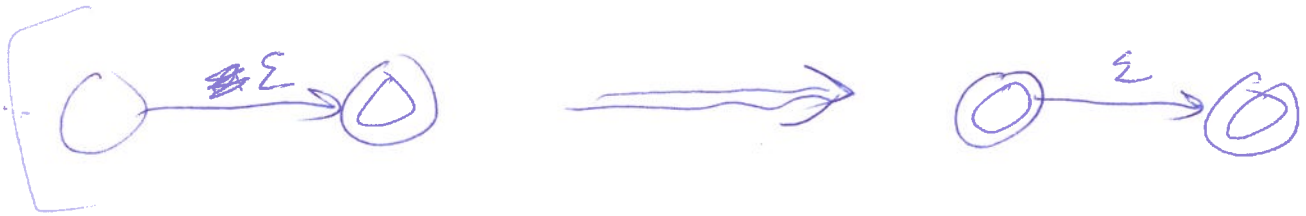
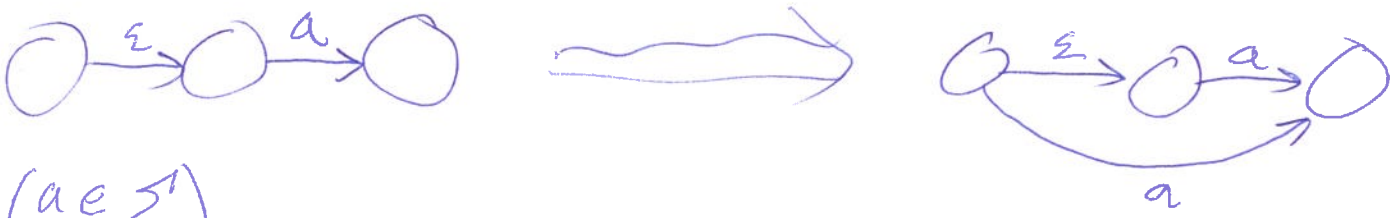
CSCE 355
2/3/2025

ε-NFA ⟶ NFA example

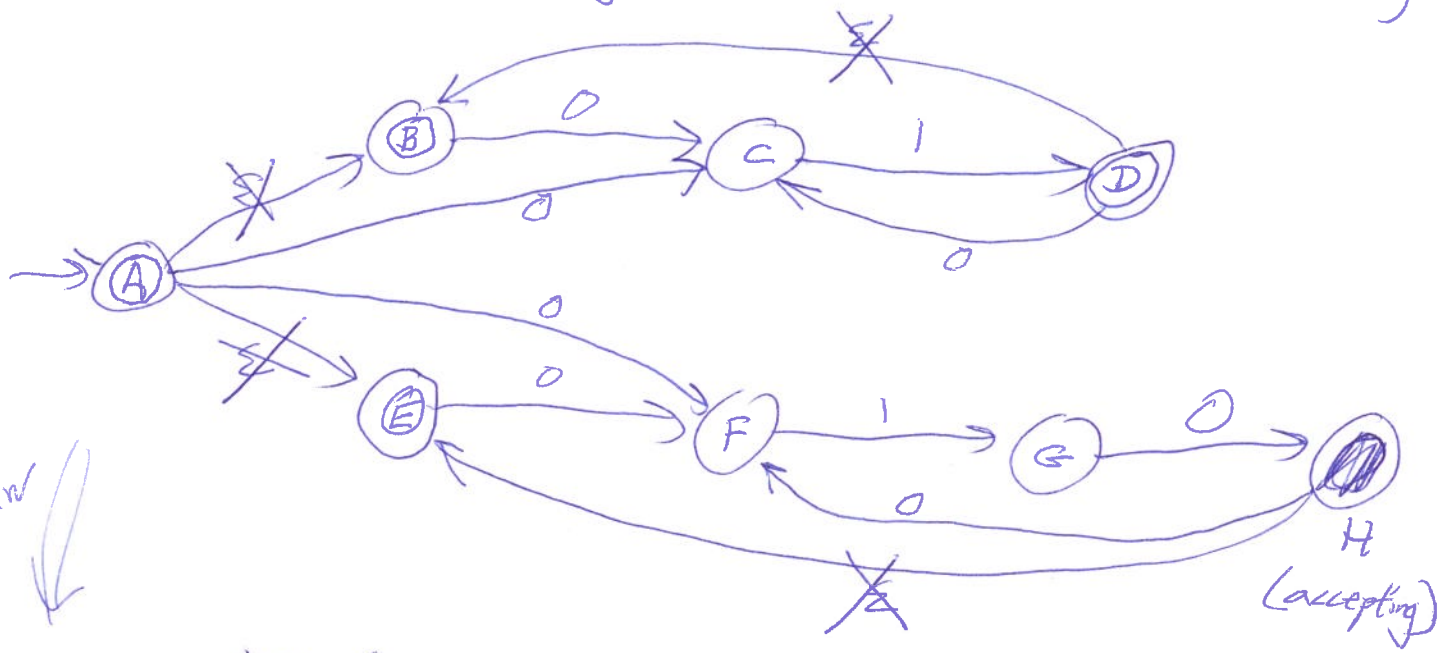
2 kinds of transformations:



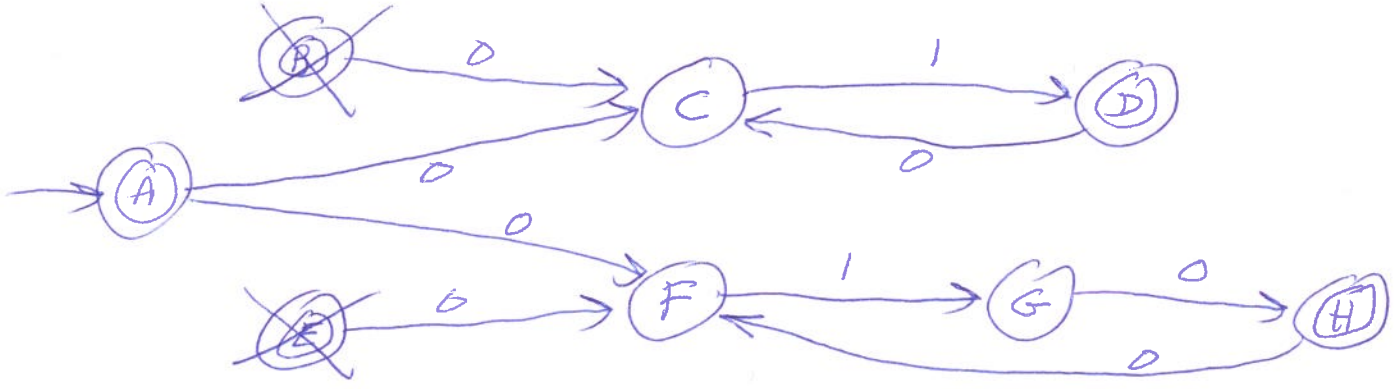


$(a \in \Sigma)$

Ex: ε-NFA recognizing $\{(01)^n : n \geq 0\} \cup \{(010)^n : n \geq 0\}$



redraw



DFA minimization: Given a DFA, find an equivalent DFA with the fewest possible states. [The minimum equivalent DFA is unique.]

Def: A DFA is sane if there are no states unreachable from the start state:

For DFA $A := \langle Q, \Sigma, \delta, q_0, F \rangle$,

A is sane means

$$\{ \hat{\delta}(q_0, w) : w \in \Sigma^* \} = Q.$$

Observe: Removing unreachable states yields an equivalent DFA.

Def: Let $A = \langle Q, \Sigma, \delta, q_0, F \rangle$ be a DFA. Two states $q, r \in Q$ are distinguishable if there exists a string $w \in \Sigma^*$ such that one of $\hat{\delta}(q, w)$ and $\hat{\delta}(r, w)$ is accepting and the other is rejecting. q, r are distinguished by w.

[If not, q and r are indistinguishable.]

↳ Also say that r distinguishes q from r

DFA minimization of DFA A:

1. Remove unreachable states (so A is sane)
2. Merge states indistinguishable from each other into single states.

To do (1): Do BFS from q_0 to find all reachable states, then remove the rest.

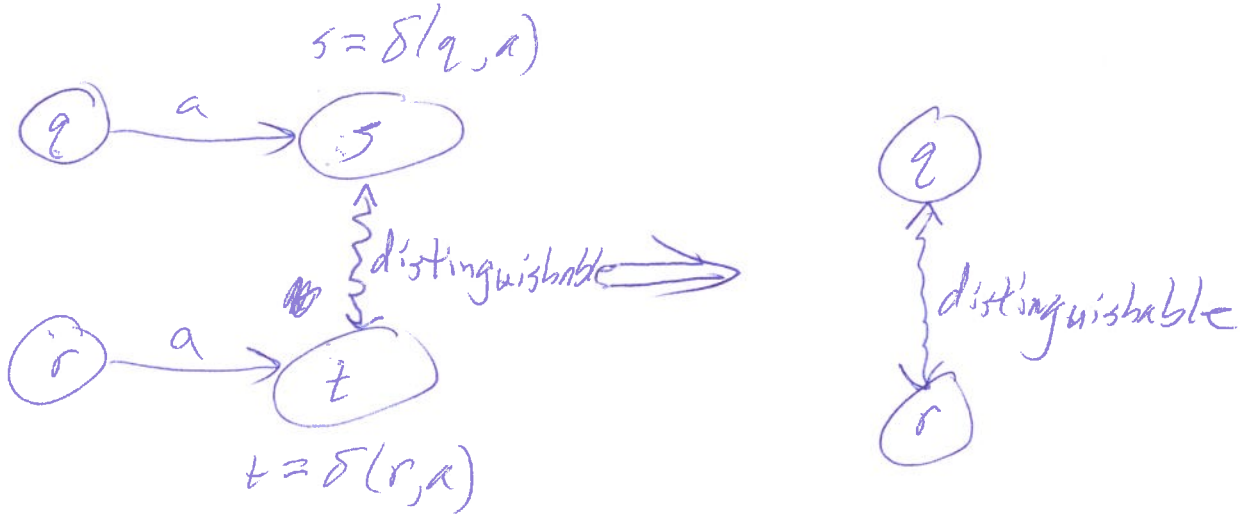
For (2): Systematically find all pairs of distinguishable states. Then merge states not found by this.

Two rules that determine distinguishability:

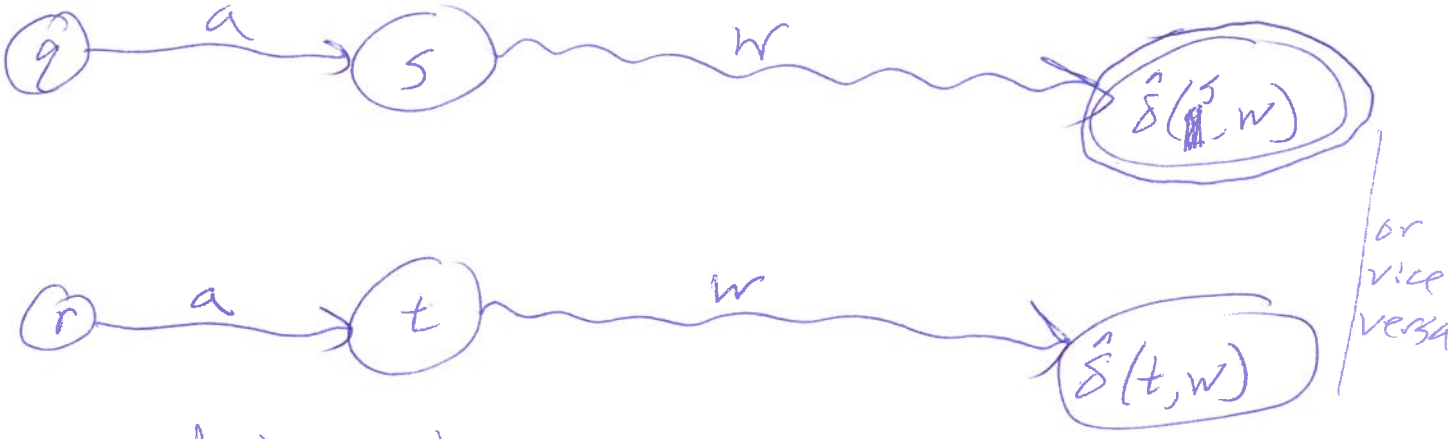
Rule 1: If q is accepting and r is rejecting, or " " rejecting " " " accepting, then q, r are distinguishable (distinguished by ε)

or

Rule 2: If there exists $a \in \Sigma$ such that $\delta(q, a)$ and $\delta(r, a)$ are distinguishable, then q and r are distinguishable.



Why? Let $w \in \Sigma^*$ distinguish s from t.

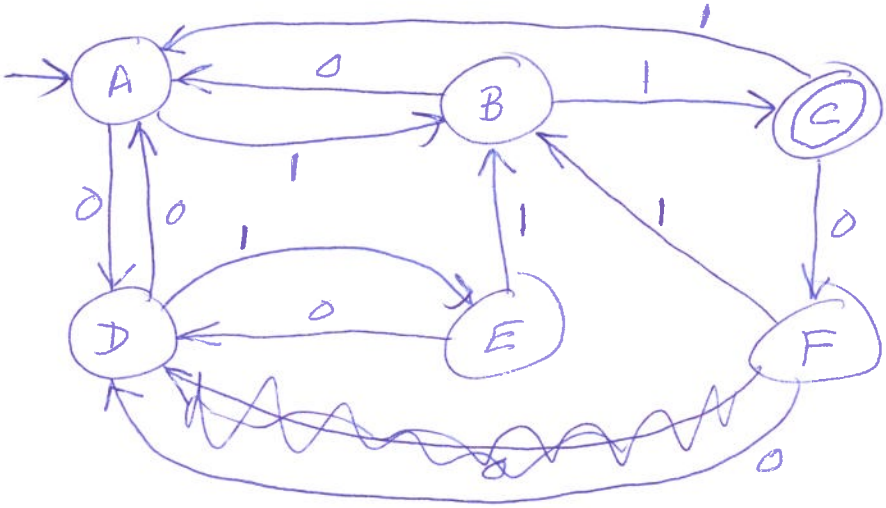


aw distinguishes q from r.

These two rules are exhaustive. Applying them repeatedly finds all distinguishable pairs.

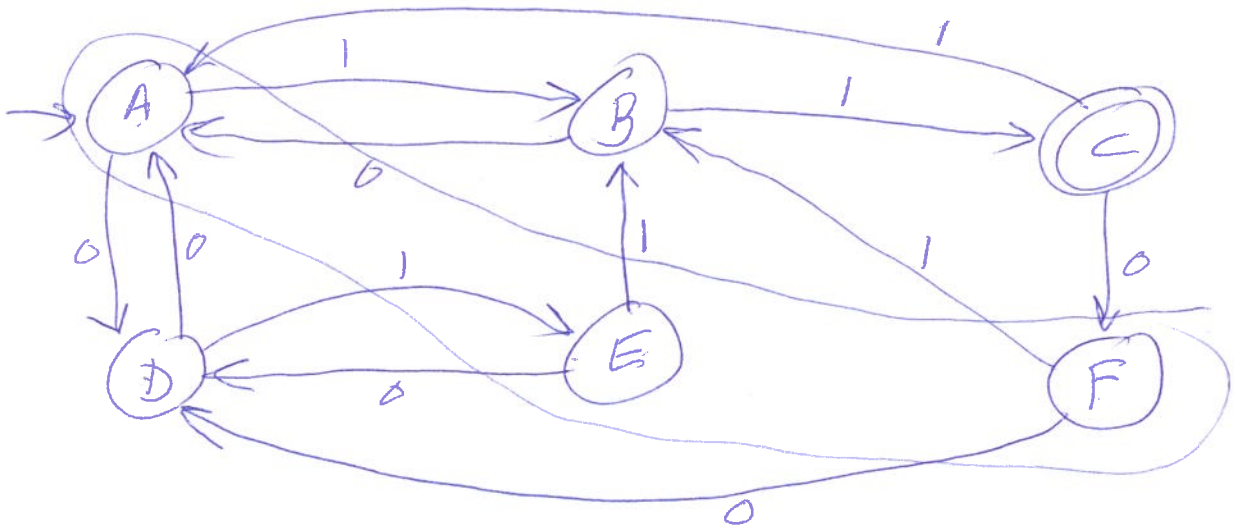
Ex:

$\Sigma = \{0, 1\}$



T[p,q] initially unmarked.

Each time I find a dist. pair p,q, will mark T[p,q] with a string distinguishing p from q (not needed, could just use "X")

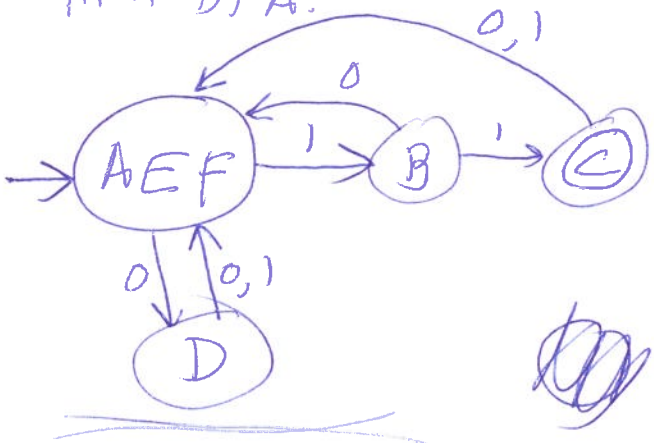


T:

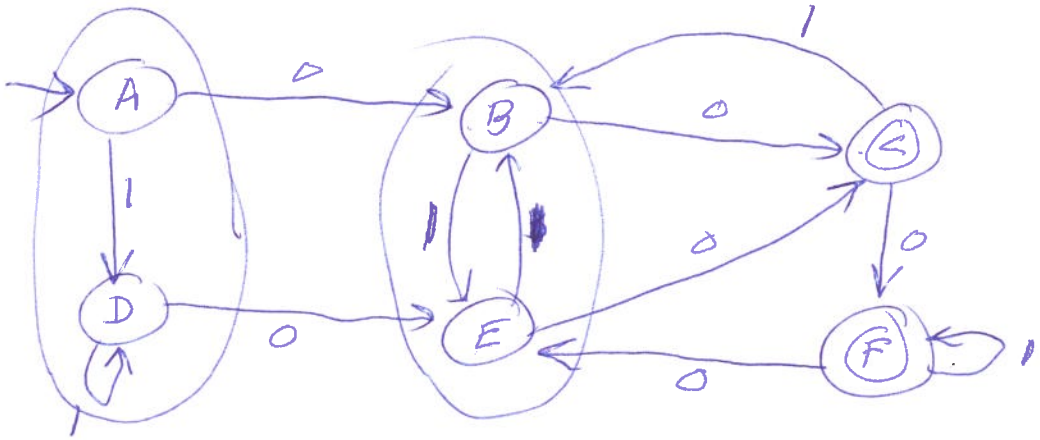
	A	B	C	D	E
B	1				
C	ε	ε			
D	11	1	ε		
E		1	ε	11	
F		1	ε	11	

Merge A, E, F into a single state

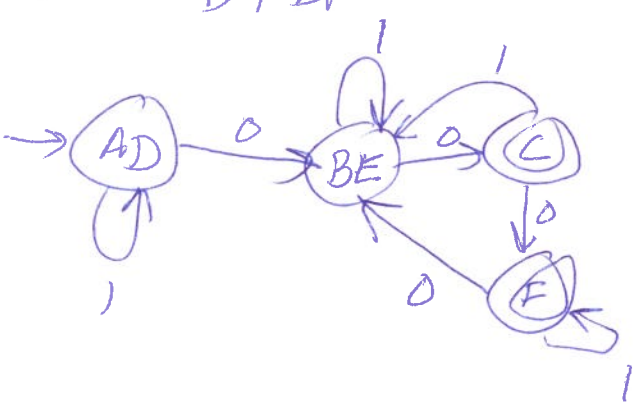
Min DFA:



Ex:



Min DFA



	A	B	C	D	E
B	X				
C	X	X			
D		X	X		
E	X		X	X	
F	X	X	X	X	X