Announcement

Homework #3 has been posted in Blackboard and class website.

Due 2:20pm, Wednesday, Feb. 14.

On the Midterm Exam

- Monday, March 11th in class
- Closed book and closed notes
- A single page letter-size cheat sheet is allowed
- A calculator is allowed for +-*/
- Covers the topics until the class on Wednesday, Feb. 28th

Proposal of Final Project

Due: 11:59 pm, Feb. 21.

Late submission penalty applies.

Include

- Title and names of the team members
- Topic: a research project or a survey
- Brief introduction on the background
- Timeline and project management for a teamwork
- An initial list of papers being reviewed (Survey project only)

At most one page

Each team only needs one abstract

On the Paper Reading

Each student has 10 minutes (9 minutes for presentation and 1 minute for questions) to present the paper chosen by yourself.

Presentation days:

- Wednesday, March 13
- Monday, March 18
- Wednesday, March 20

On the Paper Reading

Send me an email (tongy@cse.sc.edu) by 11:59pm of Feb. 21, which includes:

- The paper you are going to present
  - Title, authors, where and when it was published, pages
  - Example: Sing Bing Kang, Ashish Kapoor, Dani Lischinski , "Personalization of Image Enhancement ", in *Proceedings of IEEE Conference on computer vision and Pattern Recognition (CVPR)*, 2010
- Your name and preference of these three days in a decreasing order. Earlier email has higher priority in choosing the day

I will provide feedback (approve/suggest to change) to your selected paper

Where to Find the Paper

The paper you choose must be published in an official journal or conference!

A journal paper is preferred!

You can find papers from journals

IEEE Transactions on Pattern Analysis and Machine Intelligence

http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?reload=true&punumber=34

IEEE Transactions on Image Processing

http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=83

Other premier conferences or journals, CVPR, ICCV, ECCV, IEEE Trans. on Medical Imaging …

Deadline for your email: 11:59pm, Feb. 21

Today’s Agenda

- **Fourier Transform**
  - FT of simple functions

Concept of Fourier Series And Transforms

Fourier series: any periodic function can be represented by a discrete weighted sum of sines and cosines

Fourier transform: an arbitrary function with finite duration (non-periodic function) can be expressed by a weighted integrals of sines and cosines

Fourier transform is more general!

Fourier Series

$f(t)$ **is a continuous function with period** T**, we have**

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{j2\pi nt}{T}} \Longrightarrow \cos\frac{2\pi n}{T}t + j\sin\frac{2\pi n}{T}t$$

Coefficient

Discrete frequency

Where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-\frac{j2\pi nt}{T}}dt, \quad n = 0, \pm 1, \pm 2, \ldots$$

Video demo https://en.wikipedia.org/wiki/Fourier_transform#/media/File:Fourier_transform_time_and_frequency_domains_(small).gif

Fourier Transform in 1D

$f(t)$ **is an arbitrary non-periodic function and can be represented by**

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Coefficient

Continuous frequency

where

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Fourier Transform in 1D

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$$f(t)=\int_{-\infty}^{\infty}F(\mu)e^{j2\pi\mu t}d\mu$$

Coefficient

Continuous frequency

where

$$F(\mu)=\int_{-\infty}^{\infty}f(t)e^{-j2\pi\mu t}dt$$

Fourier series

Discrete frequency

$$f(t)=\sum_{n=-\infty}^{+\infty}c_n e^{\frac{j2\pi nt}{T}}$$

$$c_n=\frac{1}{T}\int_{-T/2}^{T/2}f(t)e^{-\frac{j2\pi nt}{T}}dt$$

Fourier Transform in 1D

Spatial domain → Frequency domain

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt \qquad \text{Forward transform}$$

Frequency domain → Spatial domain

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu \qquad \text{Inverse transform}$$

Fourier transform pair

Basic Properties of FT

Linearity $h(t) = af(t) + bg(t) \leftrightarrow H(\mu) = aF(\mu) + bG(\mu)$

Translation $h(t) = f(t - t_0) \leftrightarrow H(\mu) = e^{-j2\pi t_0 \mu} F(\mu)$

Translation in spatial domain → Rotation in frequency domain

Modulation $h(t) = e^{j2\pi \mu_0 t} f(t) \leftrightarrow H(\mu) = F(\mu - \mu_0)$

Rotation in spatial domain → Translation in frequency domain

Basic Properties of FT

Scaling

$$h(t) = f(at) \leftrightarrow H(\mu) = \frac{1}{|a|} F(\frac{\mu}{a})$$

Conjugation

$$h(t) = f^*(t) \leftrightarrow H(\mu) = F^*(-\mu)$$

Symmetry

$$f(t) \leftrightarrow F(\mu) \Rightarrow F(t) \leftrightarrow f(-\mu)$$

FT of Simple Functions

$$f(t) = \begin{cases} A & -\frac{w}{2} \leq t \leq \frac{w}{2} \\ 0 & otherwise \end{cases}$$

$$F(\mu) = \frac{A}{\pi\mu} \sin \pi w\mu = Aw \frac{\sin \pi w\mu}{\pi w\mu} = Aw\ sinc(\pi w\mu)$$

FT of a Rectangle Function

Rectangle function → Sinc function

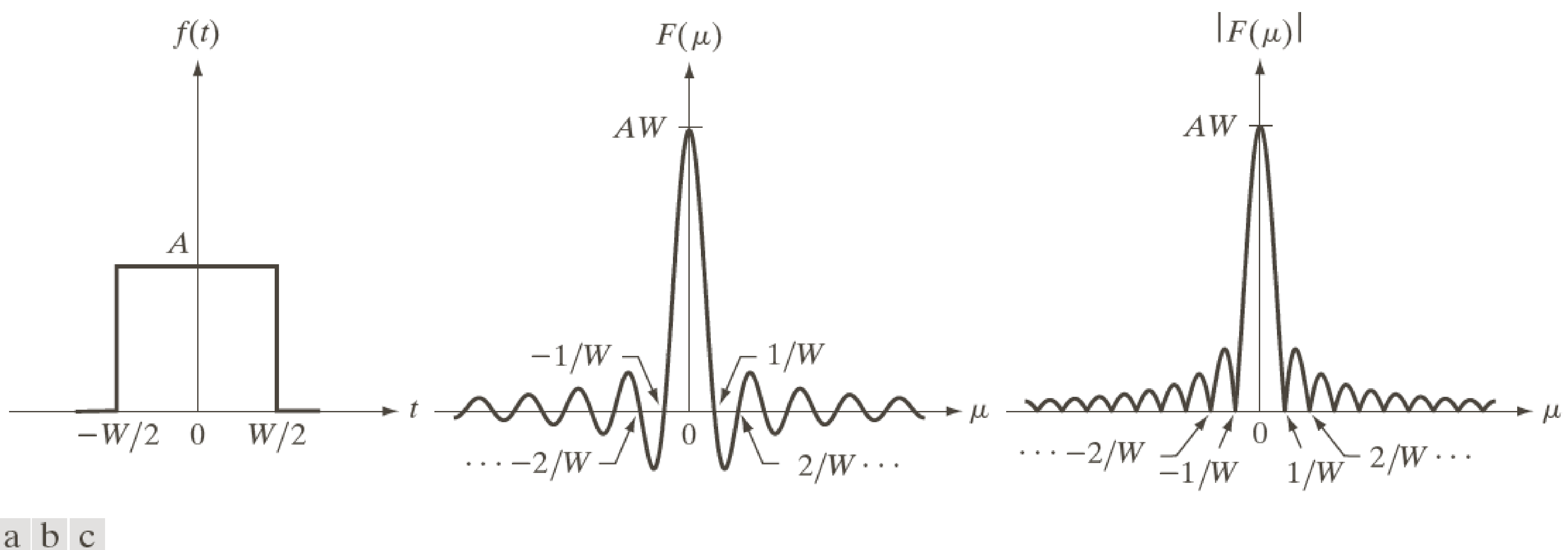


FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Continuous Impulses and Sifting Property

Unit impulse

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Sifting property

$$\int_{-\infty}^{\infty} \delta(t) g(t)\,dt = g(0)$$

$$\int_{-\infty}^{\infty} \delta(t - t_0) g(t)\,dt = g(t_0)$$

The value of function at
the impulse location

FT of an Impulse

$$\delta(t) \leftrightarrow ?$$

$$\delta(t - t_0) \leftrightarrow ?$$

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Proof with

- sifting property

$$\int_{-\infty}^{\infty} \delta(t - t_0) g(t) dt = g(t_0)$$

- translation property

$$h(t) = f(t - t_0) \leftrightarrow H(\mu) = e^{-j2\pi t_0 \mu} F(\mu)$$

FT of an Impulse

$$\delta(t) \leftrightarrow F(\mu) = 1$$

$$\delta(t - t_0) \leftrightarrow F(\mu) = e^{-j2\pi\mu t_0}$$

FT of an Impulse

$$e^{j2\pi t_0 t} \leftrightarrow ?$$

$$F\left(e^{j2\pi t_0 t}\right) = \delta(\mu - t_0)$$

Symmetry property

$$f(t) \leftrightarrow F(\mu) \Rightarrow F(t) \leftrightarrow f(-\mu)$$

$$\delta(t - t_0) \leftrightarrow F(\mu) = e^{-j2\pi u t_0}$$

$$F\left(e^{-j2\pi t_0 t}\right) = \delta(-\mu - t_0)$$
$$= \delta(\mu + t_0)$$

Scaling property

$$h(t) = f(at) \leftrightarrow H(\mu) = \frac{1}{|a|} F\left(\frac{\mu}{a}\right)$$

$$F\left(e^{j2\pi t_0 t}\right) = \delta(\mu - t_0)$$

Discrete Impulses and Sifting Property

Unit impulse

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \quad \text{and} \quad \sum_{x=-\infty}^{+\infty} \delta(x) = 1$$

Sifting property

$$\sum_{x=-\infty}^{\infty} \delta(x) g(x) = g(0)$$

$$\sum_{x=-\infty}^{\infty} \delta(x - x_0) g(x) = g(x_0)$$

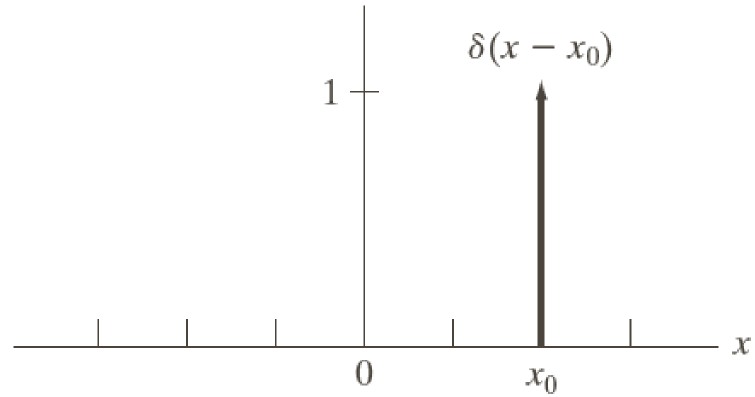


FIGURE 4.2
A unit discrete impulse located at $x = x_0$. Variable x is discrete, and δ is 0 everywhere except at $x = x_0$.

Impulse Train

$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

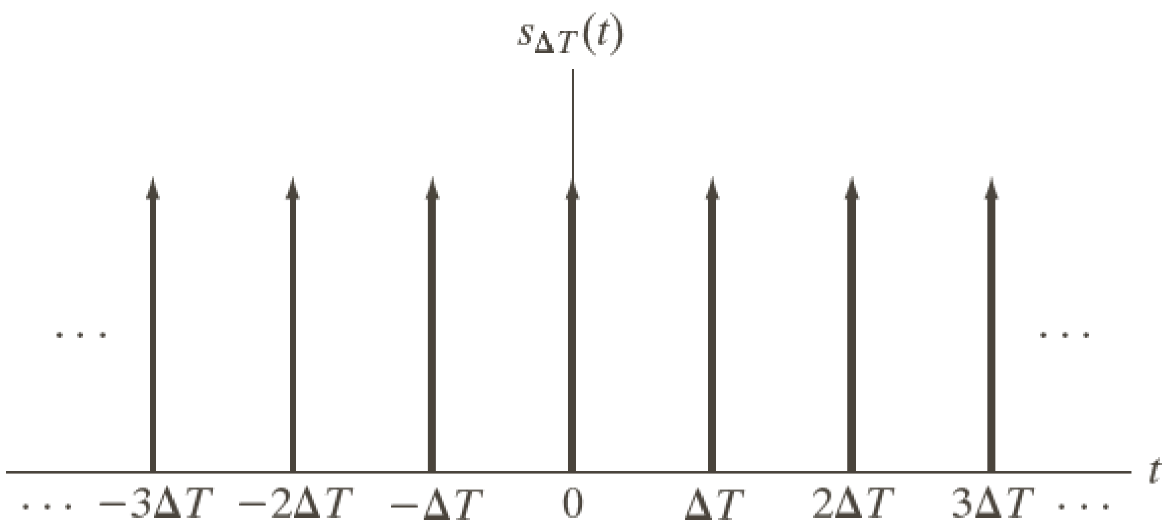
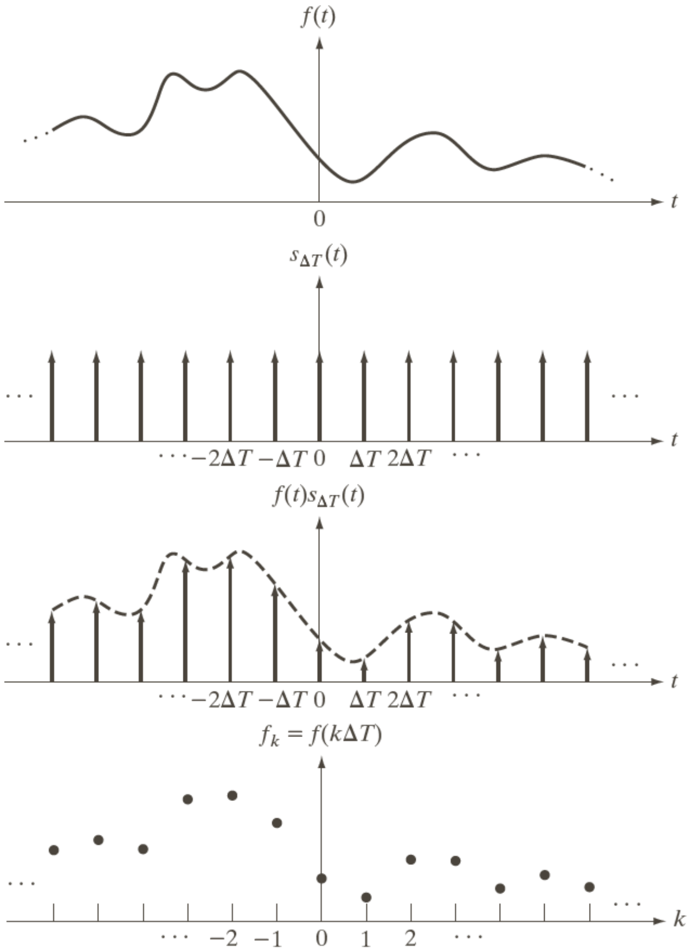


FIGURE 4.3 An impulse train.

Sampling in Spatial Domain



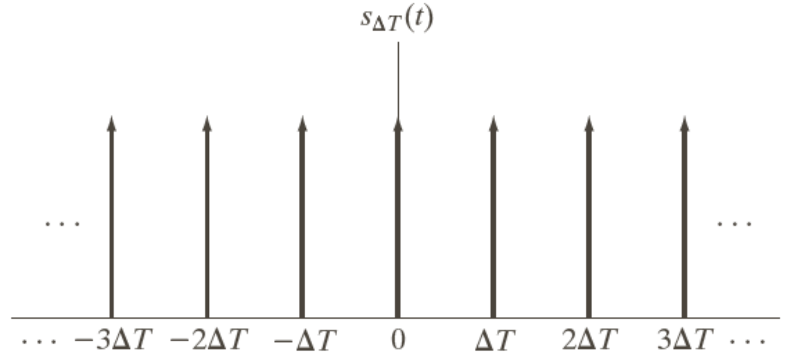
$$\tilde{f}(t) = f(t)s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

FIGURE 4.5 (a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

$$\tilde{f}(t) = \sum_{k=-\infty}^{\infty} f(k\Delta T)$$

$\frac{1}{\Delta T}$ is the sample rate

FT of an Impulse Train



$$s_{\Delta T}(t) \leftrightarrow ?$$

Fourier series of an impulse train $$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{\Delta T}t}$$

where $$c_n = \frac{1}{\Delta T}\int_{-\frac{\Delta T}{2}}^{\frac{\Delta T}{2}} S_{\Delta T}(t) e^{-\frac{j2\pi nt}{\Delta T}} dt \Rightarrow c_n = \frac{1}{\Delta T}$$

$$\Rightarrow \quad s_{\Delta T}(t) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}$$

FT of an Impulse and Impulse Train

$$e^{j2\pi t_0 t} \leftrightarrow \delta(\mu - t_0)$$

$$s_{\Delta T}(t) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j2\pi\frac{n}{\Delta T}t}$$

Let $t_0 = \frac{n}{\Delta T}$

$$\Downarrow$$

$$S(\mu) = F(s_{\Delta T}(t)) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\Delta T})$$

$\Rightarrow$ FT of an impulse train is an impulse train in frequency domain