Basic Concepts in Digital Image Processing

Announcement

Homework #1 was posted in Blackboard and on class website.

Due time: 2:20pm Monday, Jan 29

Submit it via Blackboard

Today's Agenda

- Interpolation
- Basic Relationships between Pixels
- Mathematical tools for image processing

Representing Digital Images

(a)*: f*(*x*,*y*), *x*=0, 1, …, *M*-1, *y*=0,1, …, *N*-1

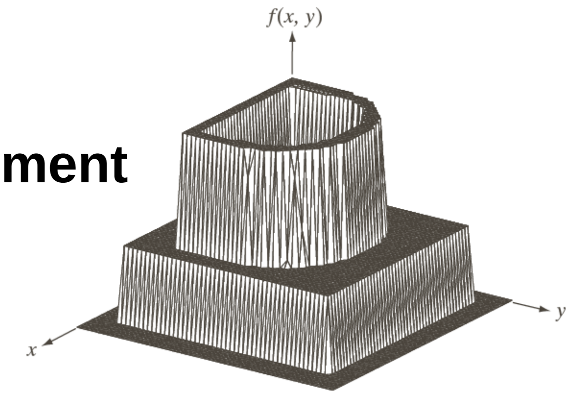
***x*, *y*: spatial coordinates → spatial domain**

(b): suitable for visualization

(c): processing and algorithm development

***x*: extend downward (rows)**

***y*: extend to the right (columns)**



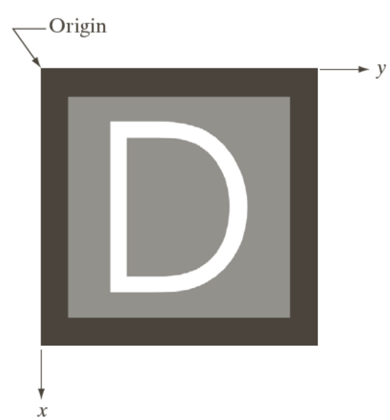
Number of bits storing the image

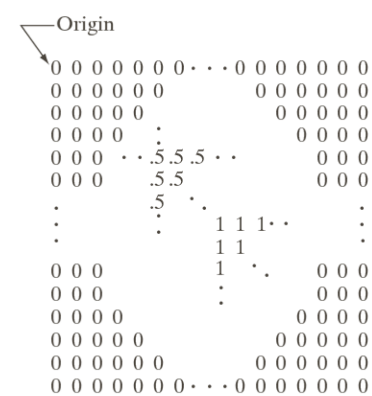
↑

$$b = M \times N \times k$$

a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).





Store an Image

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial Resolution

Spatial resolution: **smallest discernible details**

- # of line pairs per unit distance
- # of dots (pixels) per unit distance
  - Printing and publishing
  - In US, dots per inch (dpi)

Newspaper ⟶ magazines ⟶ book

Large image size itself does not mean high spatial resolution!

⟶ **Scene/object size in the image**

1280*960

http://www.shimanodealer.com/fishing_reports.htm

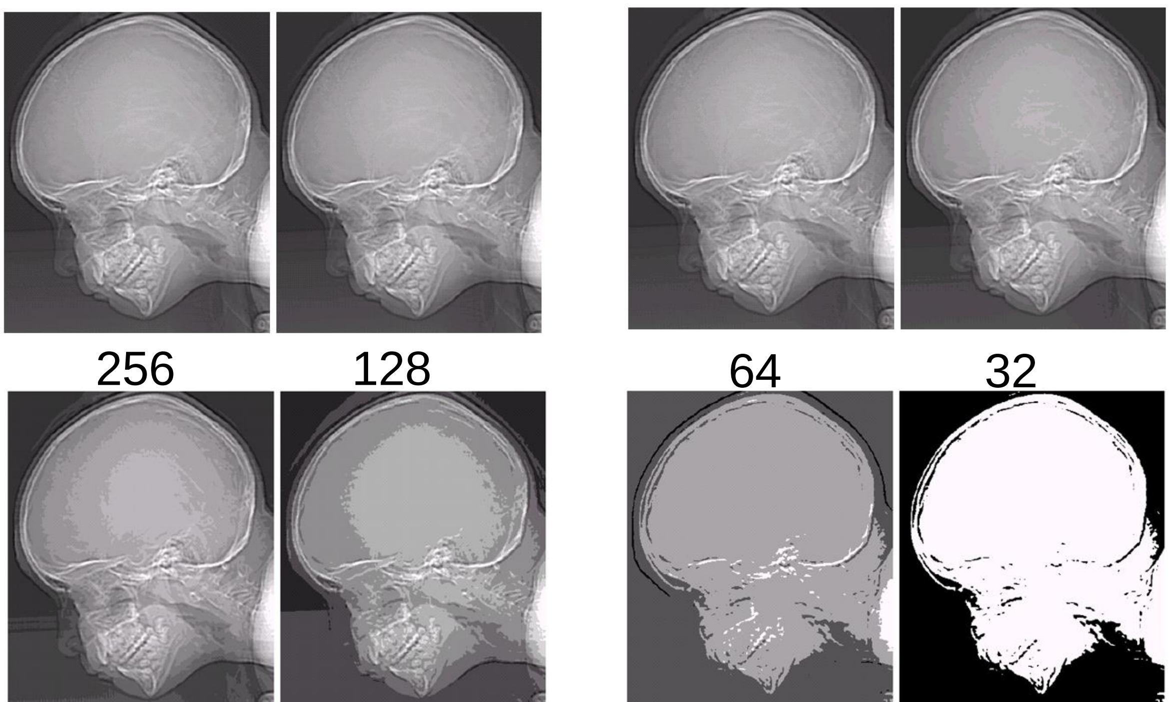
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Intensity Resolution

Intensity resolution

- Smallest discernible change in intensity levels
- Using the number of levels of intensities
- False contouring (banding) when *k* is small - undersampling



256 128 64 32

16 8 4 2

Isopreference Curves

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Vary the spatial and intensity sampling simultaneously:

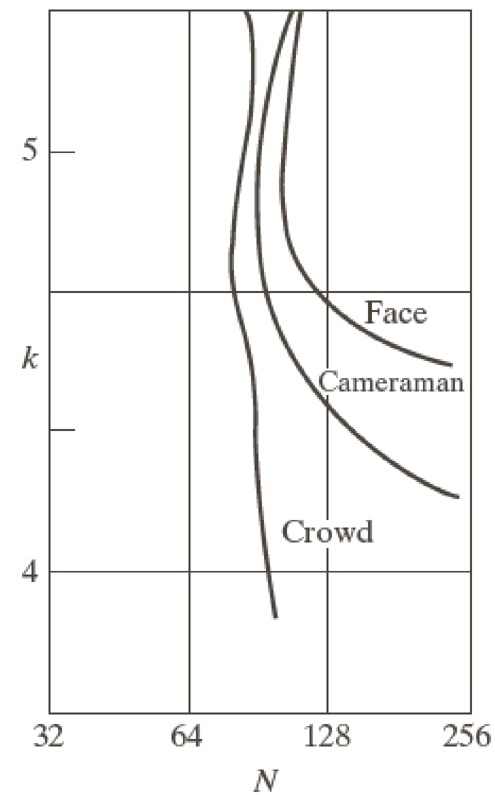


FIGURE 2.23 Typical isopreference curves for the three types of images in Fig. 2.22.

Image Resampling & Interpolation

Need to resample the image when

- Rescaling
- Geometrical transformation
- The output image coordinates are not discrete

Interpolation methods:

- Nearest neighbor
  - Fast and simple
  - Loss of sharpness
  - Artifacts (checkerboard)
- Bilinear
- Bicubic
  - Images are sharpest
  - Fine details are preserved
  - Slow

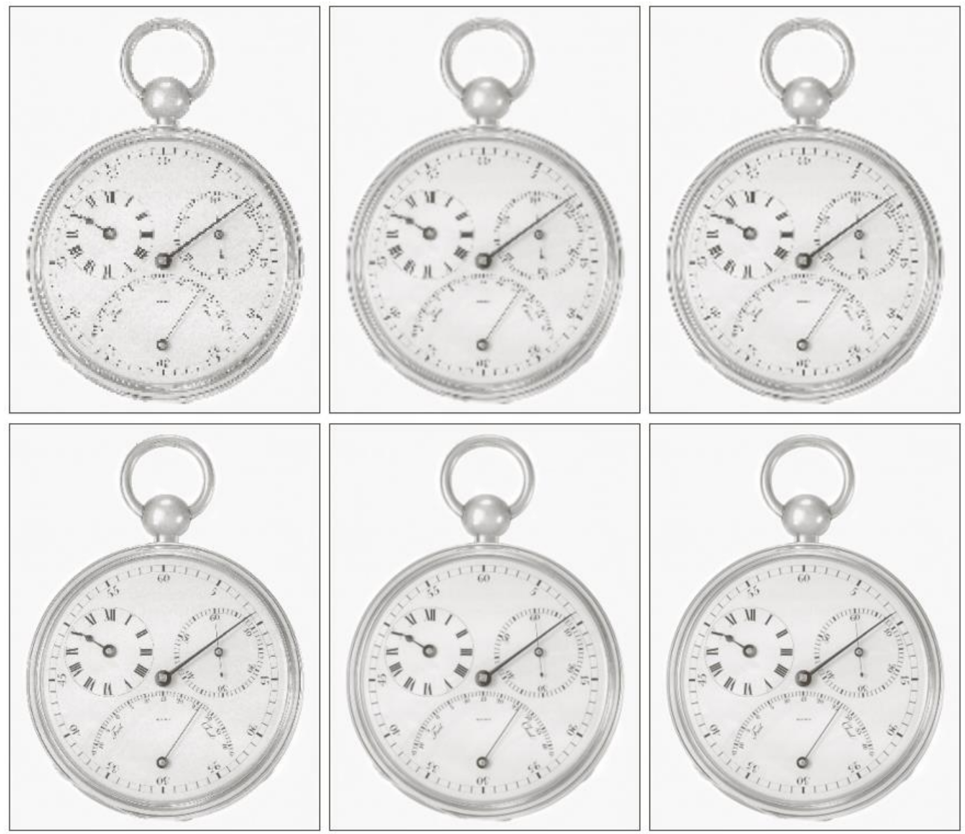
Image Resampling & Interpolation

Original
1250dpi

72dpi

150dpi

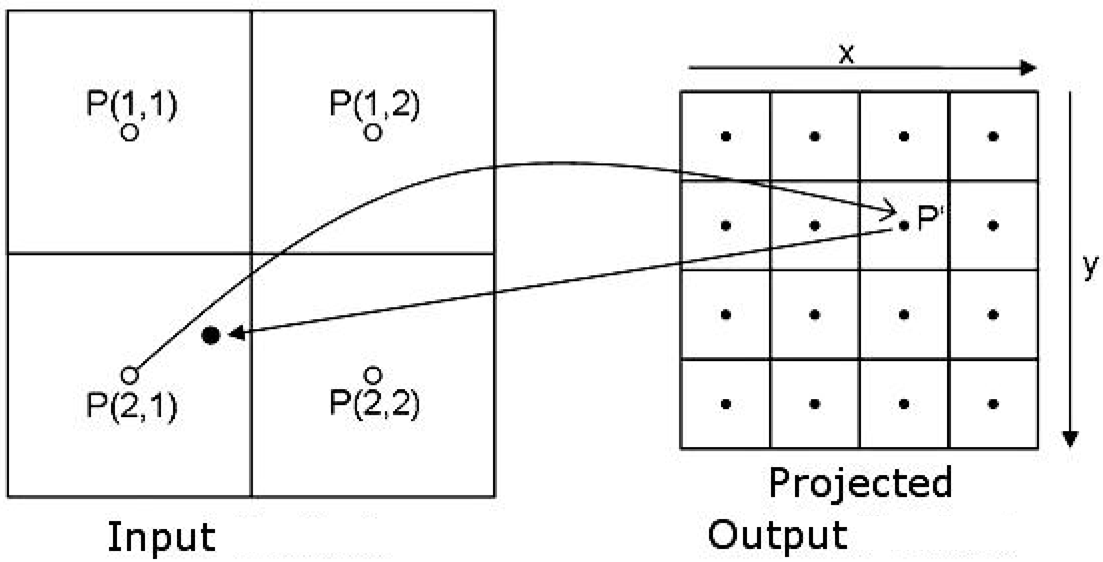
NN | Bilinear | Bicubic



a b c
d e f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

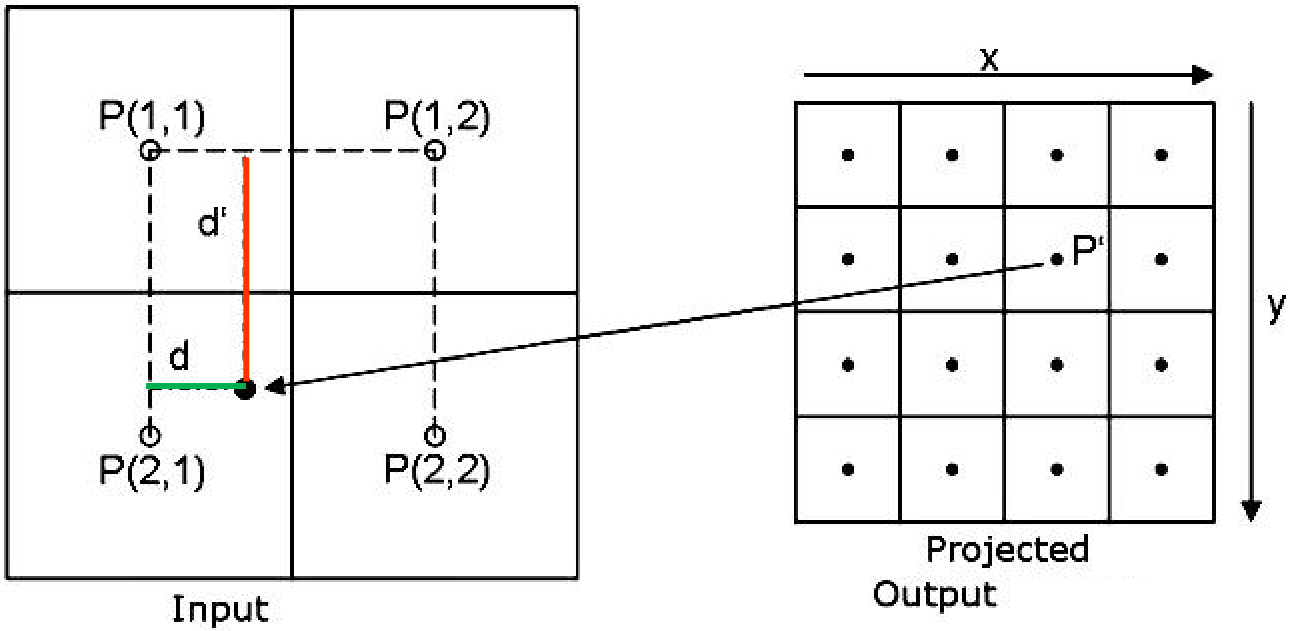
Image Interpolation – Nearest Neighbor



http://www.brockmann-consult.de/beam/doc/help/general/ResamplingMethods.html

Assign each pixel in the output image with the nearest neighbor in the input image.

Image Interpolation – Bilinear

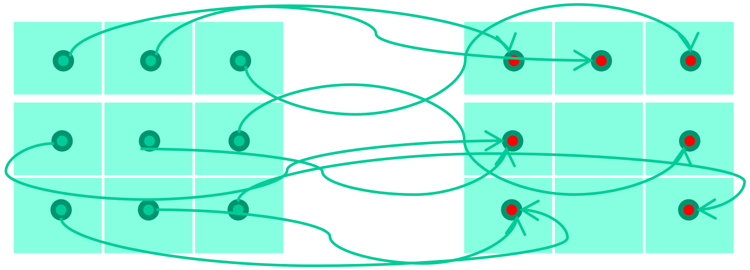


http://www.brockmann-consult.de/beam/doc/help/general/ResamplingMethods.html

$$\boldsymbol{P' = P(1,1)(1-d)(1-d') + P(1,2)d(1-d') + P(2,1) * d' * (1-d) + P(2,2)dd'}$$

Image Resampling & Interpolation

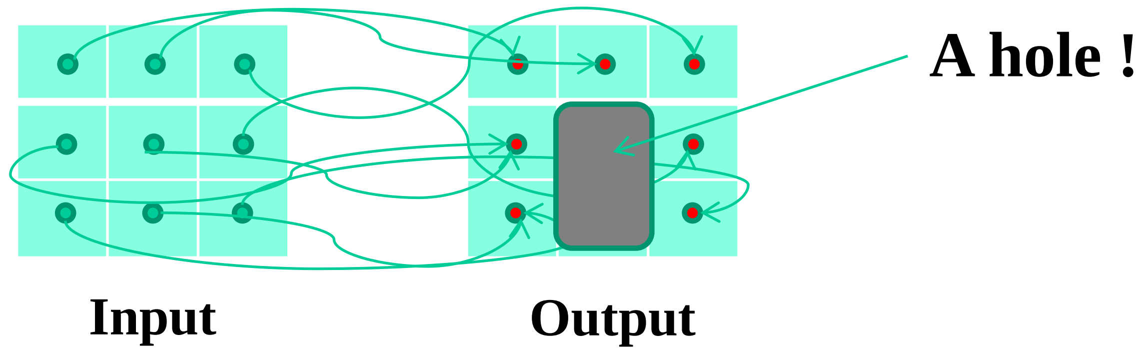
Forward mapping



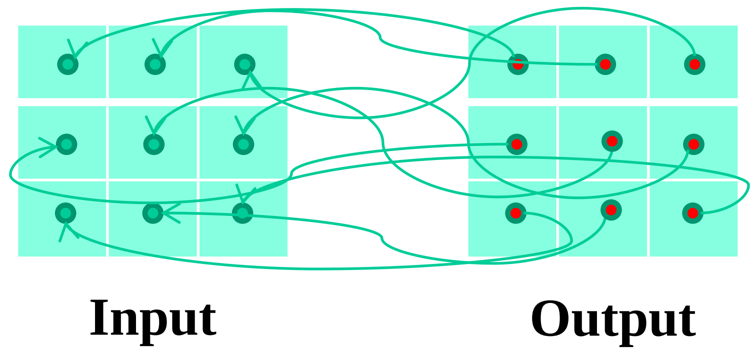
Input Output

Issues on Image Resampling & Interpolation

Missing points in forward mapping



Solution: perform a backward mapping



Basic Set and Logical Operations

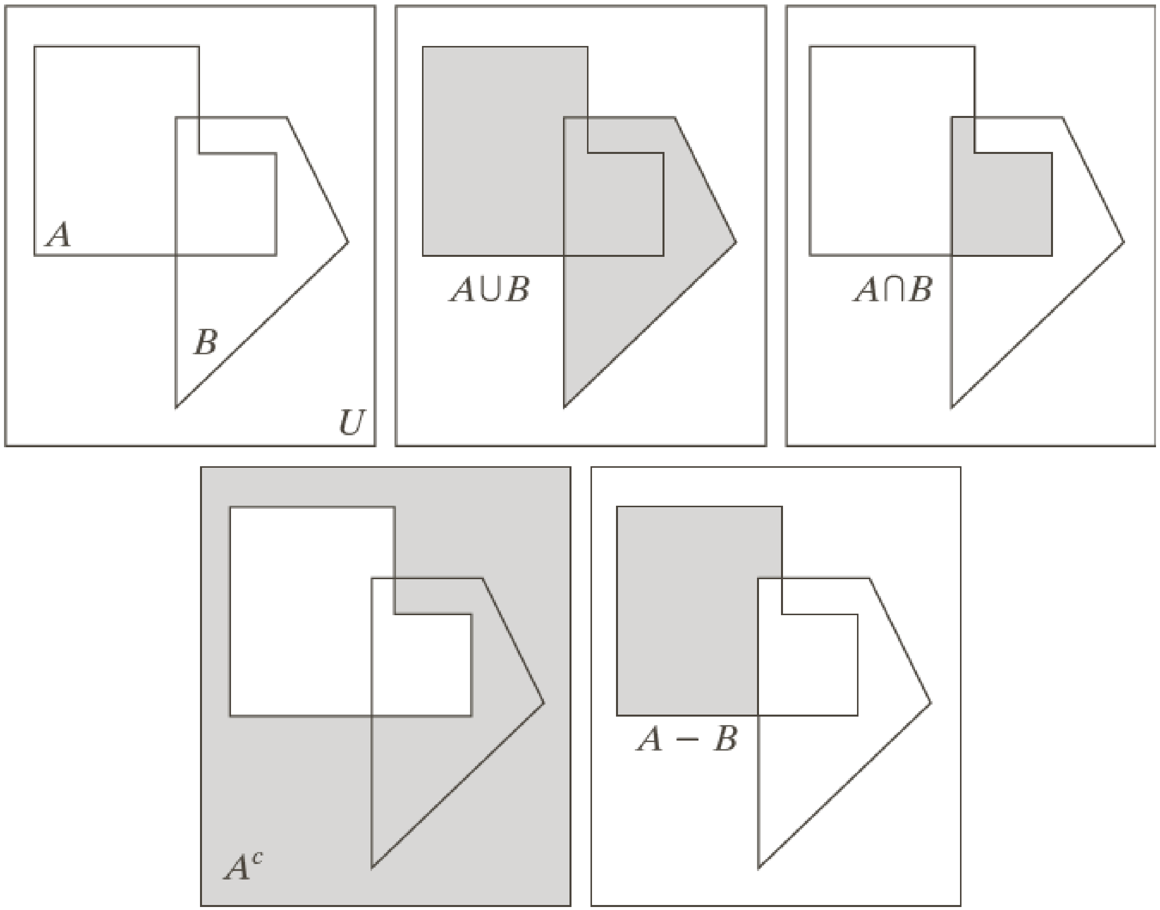
- A is a set: A={.}   e.g. A={1,…,255} or $A = \{w | w = 1, \ldots, 255\}$

$A = \emptyset$ for empty set

- a is an element of A$(a \in A)$ or a isn't an element of A$(a \notin A)$
- A is a *subset* of B if every element in A also is in B $(A \sqsubseteq B)$
- C is the *union* of two sets A and B $(C = A \cup B)$
- C is the *intersection* of A and B $(C = A \cap B)$
- Disjoint or mutual exclusive sets $(A \cap B = \emptyset)$
- *Set universe* is the set of all elements in an application
- *Set difference* $(A - B = \{w | w \in A, w \notin B\})$

Set Operations Based on Coordinates

A region in an image is represented by a set of coordinates within the region



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Some Basic Relationships between Pixels

Neighbors of a pixel

$N_4(\mathbf{p})$

	(x-1,y)	
(x,y-1)	(x,y)	(x,y+1)
	(x+1,y)	

$N_D(\mathbf{p})$

(x-1,y-1)		(x-1,y+1)
	(x,y)	
(x+1,y-1)		(x+1,y+1)

$N_8(\mathbf{p})$

(x-1,y-1)	(x-1,y)	(x-1,y+1)
(x,y-1)	(x,y)	(x,y+1)
(x+1,y-1)	(x+1,y)	(x+1,y+1)

Adjacency

Adjacency is the relationship between two pixels *p* and *q*

***V* is a set of intensity values used to define adjacency**

- **Binary image: V={1} or V={0}**
- **Gray level image:** $\boldsymbol{V \sqsubseteq \{0, 1, \dots, 255\}}$

$\boldsymbol{f(p) \in V}$ and $\boldsymbol{f(q) \in V}$ ⇒ **Intensity constraints**

Three types of adjacency:

4-adjacency	**8-adjacency**	**m-adjacency**
$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$
p		X
$\boldsymbol{q \in N_4(p)}$	$\boldsymbol{q \in N_8(p)}$	$\boldsymbol{q \in N_D(p)}$ **and** $N_4(q) \cap N_4(p) = \emptyset$ *or* $q \in N_4(p)$

Connectivity

- **Path from p to q: a sequence of <u>distinct</u> and <u>adjacent</u> pixels with coordinates**

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Starting point p **adjacent** **ending point q**

- ***Closed* path: if the starting point is the same as the ending point**
- **p and q are *connected*: if there is a path from p to q in S**
- ***Connected component*: all the pixels in S connected to p**
- ***Connected set*: S has only one connected component**

Are they connected sets?

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Regions

- R is a region if R is a connected set
- R_i and R_j are adjacent if $R_i \cup R_j$ is a connected set

$$\begin{array}{l}\left.\begin{matrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{matrix}\right\} R_i\\ \left.\begin{matrix}0 & 0 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{matrix}\right\} R_j\end{array}$$

Boundaries

- Inner boundary (boundary) -- the set of pixels each of which has at least one background neighbor

- Outer boundary – the boundary pixels in the background