Topics

Hierarchical modeling

- Examine the limitations of linear modeling
  - Symbols and instances

Model Complicated Objects

So far, we have discussed modeling simple geometrical objects.

How can we generate a complicated object, e.g., a robot, which is made up from several parts?

Construct complex objects from a collection of basic objects.

Instance Transformation

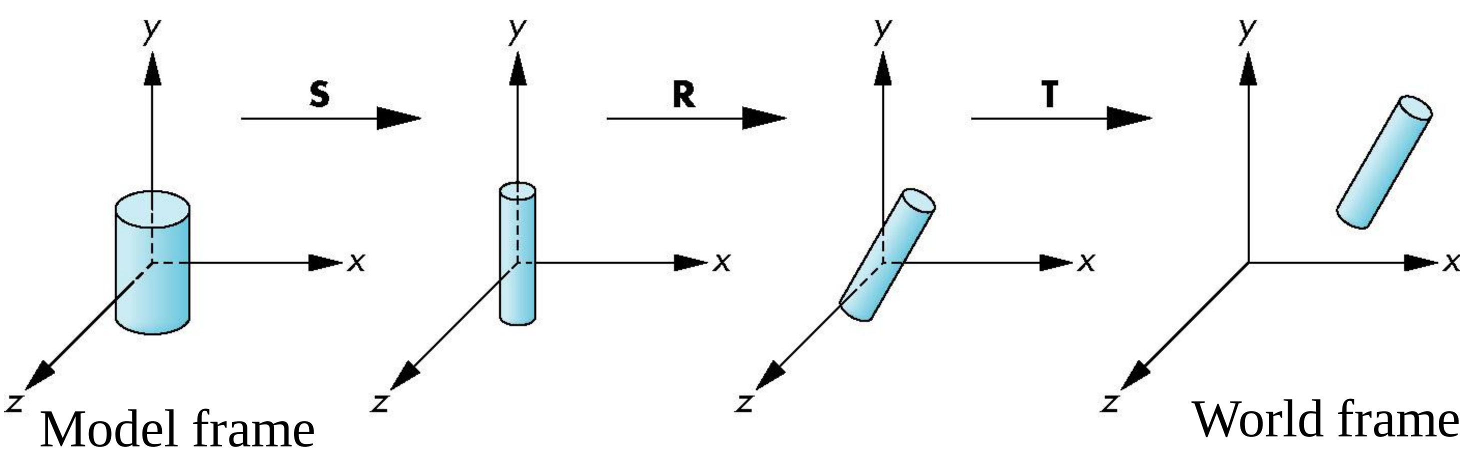
Start with a prototype object (a *symbol*), e.g.,

- Geometric objects
- Fonts

Each appearance of the object in the model is an *instance*

- A instance transformation from model frame to world frame by scaling, rotation, and translation

$$\mathbf{M} = \mathbf{TRS}$$



Instance Transformation

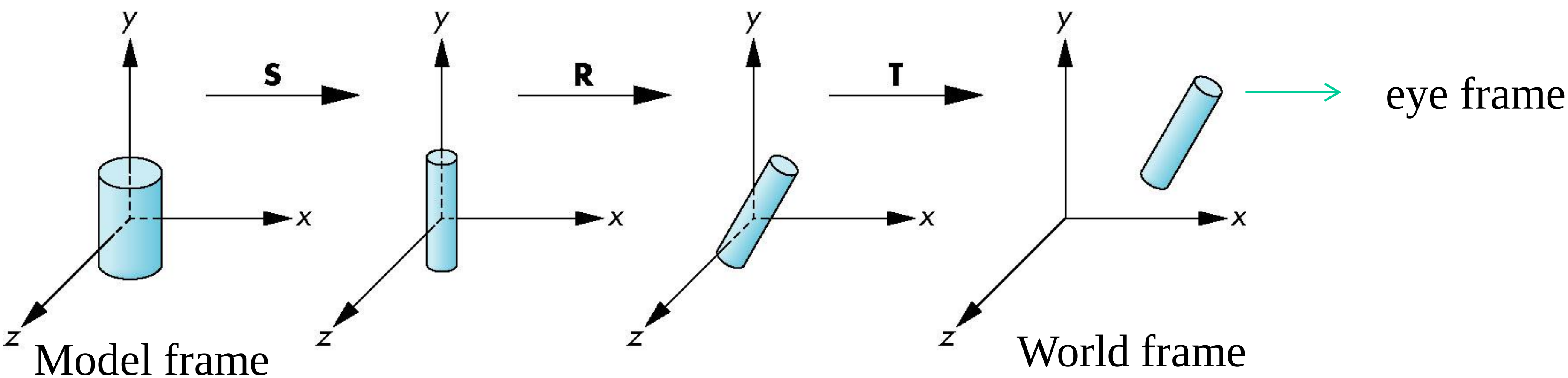
A model view matrix consists of

- instance transformation and
- a transformation from the world frame to the eye frame

```
mat4 instance; mat4 model_view;

instance = Translate(dx, dy, dz)*RotateZ(rz)*RotateY(ry)*RotateX(rx)*Scale(sx, sy, sz);

model_view = model_view*instance;
```



Symbol-Instance Table

Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation

What's the problem with the table?

A flat structure - each symbol is processed independently.

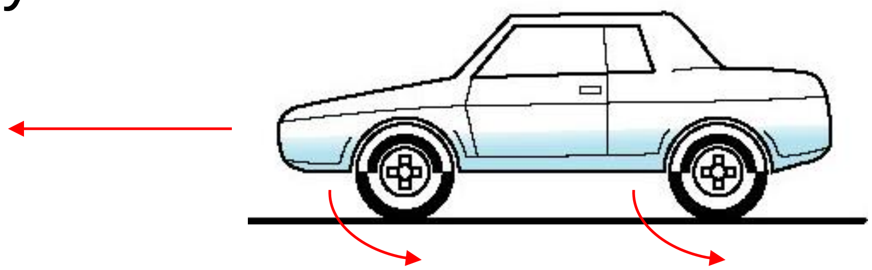
Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	$\theta_x, \theta_y, \theta_z$	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			

Relationships in Car Model

Symbol-instance table does not show relationships between parts of model

Consider model of car

- Chassis + 4 identical wheels
- Two symbols



Rate of forward motion determined by rotational speed of wheels

Structure Through Function Calls

```
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```

Fails to show relationships well

Represent the relationships among different parts using a graph

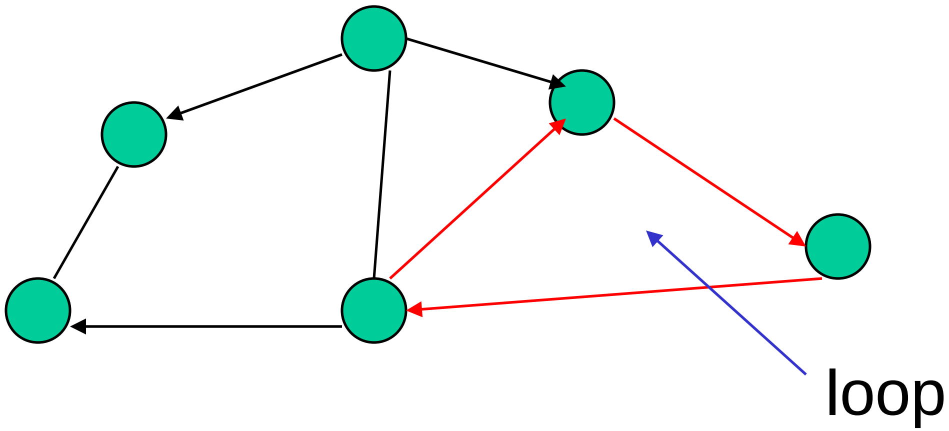
Graphs

Set of ***nodes*** and ***edges*** *(links)*

Edge connects a pair of nodes

- Directed or undirected

Cycle: directed path that is a loop

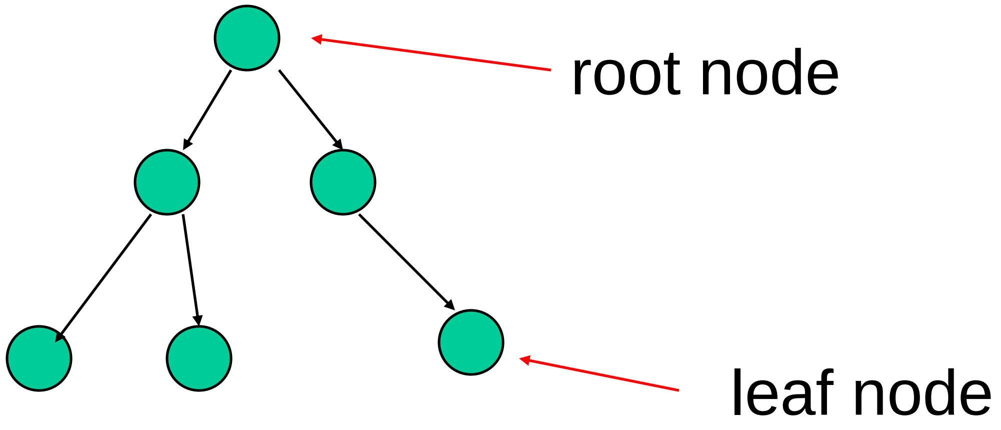


Graphs: Tree

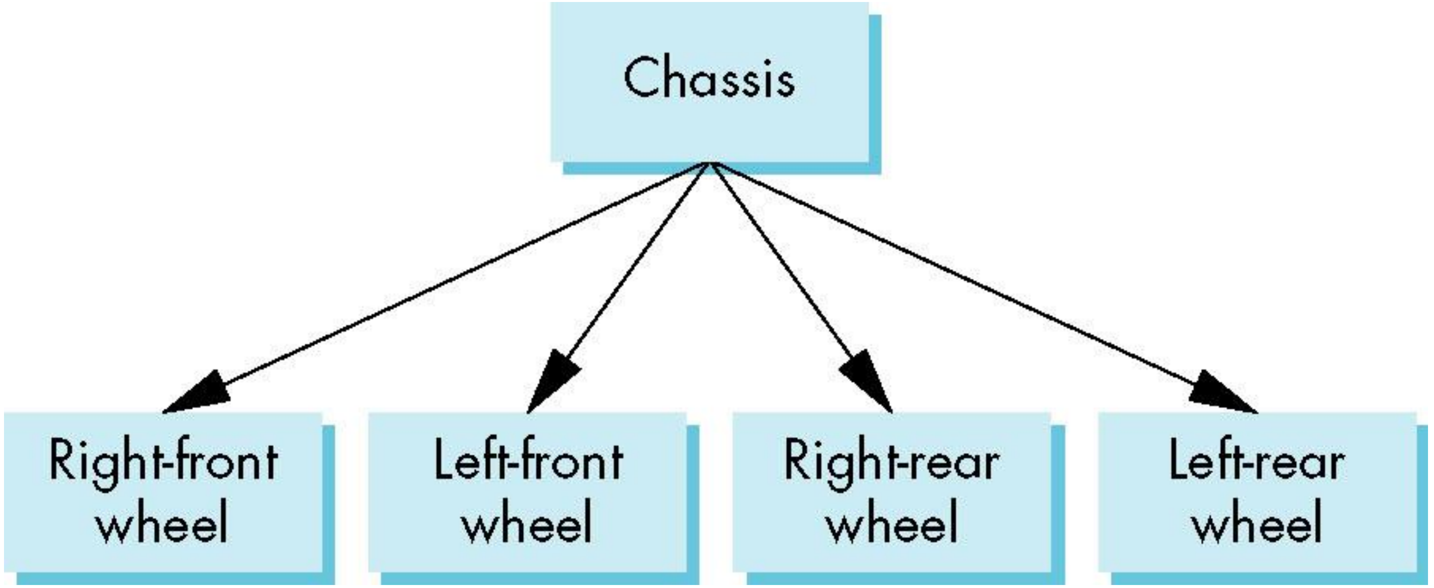
Tree is a directed acyclic graph (DAG)

A directed graph in which each node (except the root) has exactly one parent node

- No loops
- May have multiple children
- Leaf or terminal node: no children



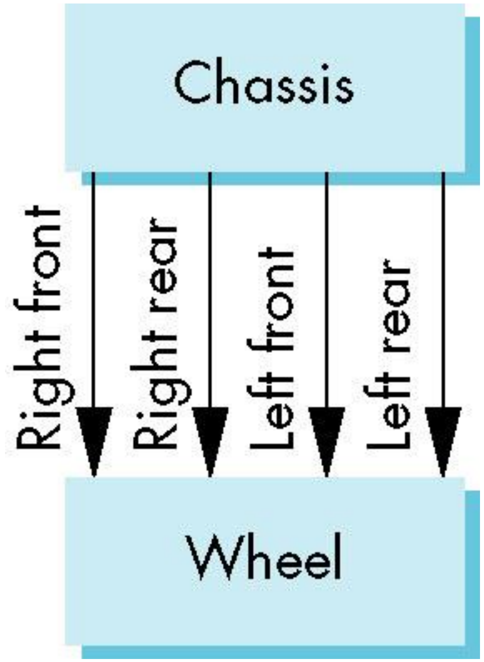
Tree Model of Car



DAG Model

If we use the fact that all the wheels are identical, we get a *directed acyclic graph*

- Not much different than dealing with a tree



Modeling with Trees

Must decide what information to place in nodes and what to put in edges

Nodes

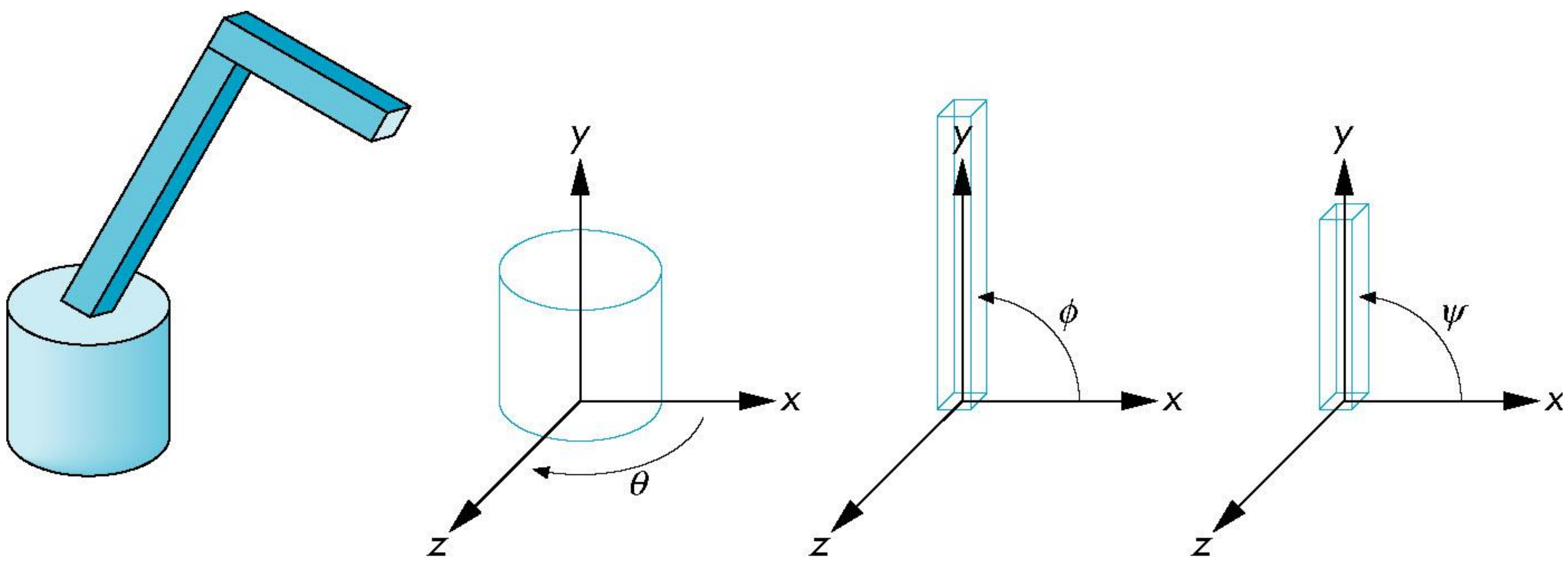
- What to draw
- Pointers to children

Edges

- May have information on incremental changes to transformation matrices (can also store in nodes)

A Robot Arm

A robot arm consists of two parallelepipeds and a cylinder



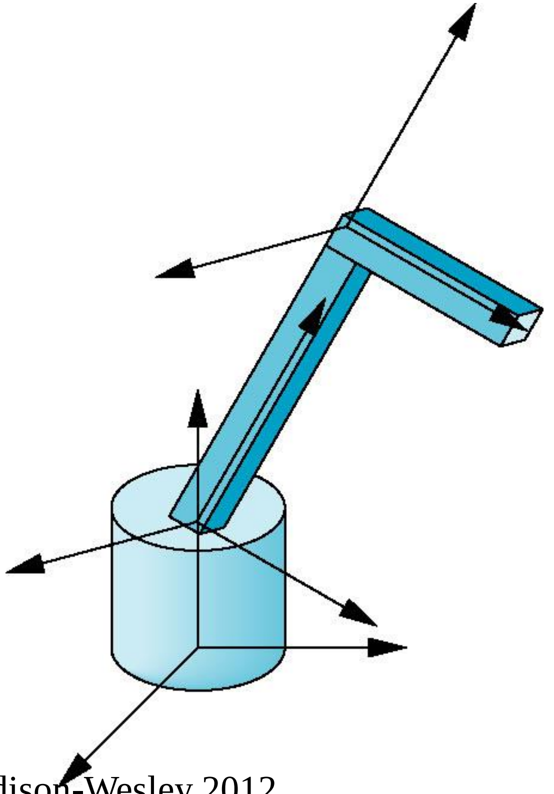
robot arm

parts in their own
coodinate systems

Articulated Models

Robot arm is an example of an *articulated model*

- Parts connected at joints
- Three degrees of freedom described by
  - –joint angles measured in its local frame
  - –Angle between the base and the ground



Relationships in Robot Arm

Base rotates independently

- Single angle determines position

Lower arm attached to base

- Its position depends on rotation of base
- Must also translate relative to base and rotate about connecting joint

Upper arm attached to lower arm

- Its position depends on both base and lower arm
- Must translate relative to lower arm and rotate about joint connecting to lower arm

Required Matrices

Base:

- Rotation of base: R_b
  - Apply $\mathbf{M} = \mathbf{R}_b$ to base

Lower arm:

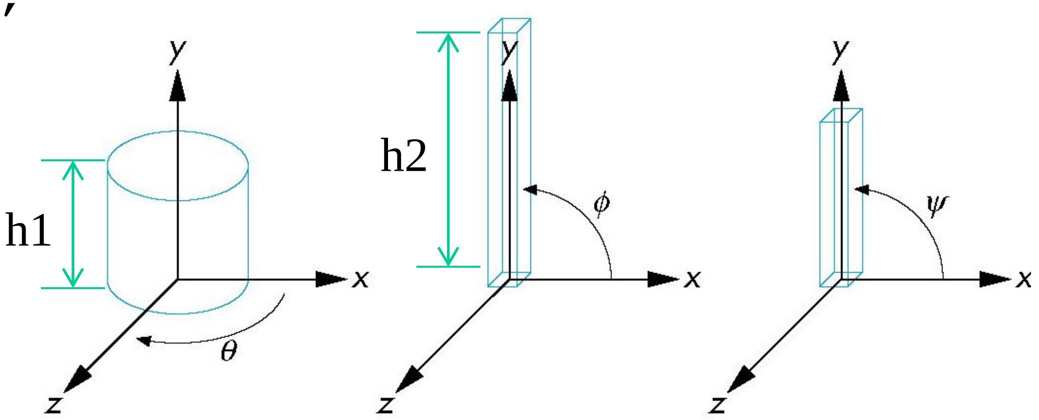
- Translate lower arm <u>relative</u> to base: T_{lu}
- Rotate lower arm around joint: R_{lu}
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu}$ to lower arm

Upper arm:

- Translate upper arm <u>relative</u> to lower arm: T_{uu}
- Rotate upper arm around joint: R_{uu}
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \mathbf{T}_{uu} \mathbf{R}_{uu}$ to upper arm

OpenGL Code for Robot

```
mat4 ctm;
robot_arm()
{
    ctm = RotateY(theta);
    base();
    ctm *= Translate(0.0, h1, 0.0);
    ctm *= RotateZ(phi);
    lower_arm();
    ctm *= Translate(0.0, h2, 0.0);
    ctm *= RotateZ(psi);
    upper_arm();
}
```



OpenGL Code for Robot Base

```
void base()

{

   mat4 instance = ( Translate( 0.0, 0.5 * BASE_HEIGHT,
0.0 ) *Scale( BASE_WIDTH, BASE_HEIGHT,BASE_WIDTH ) );

    glUniformMatrix4fv( ModelView, 1, GL_TRUE,
model_view * instance );

    glDrawArrays( GL_TRIANGLES, 0, NumVertices );

}
```

OpenGL Code for Robot

The lower arm and the upper arm are modeled similar to the base.

All the three parts are modeled based on cubes – the same symbol.

Only one set of vertices are needed to send to the buffer!

Tree Model of Robot

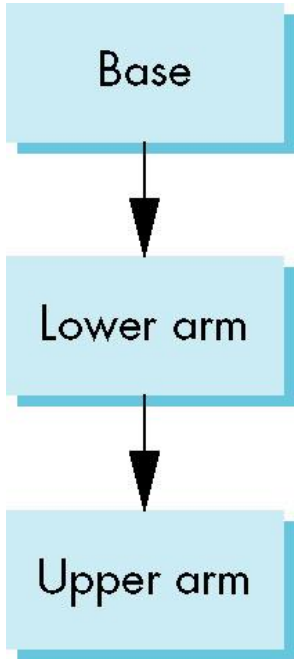
Note code shows relationships between parts of model

- Can change "look" of parts easily without altering relationships

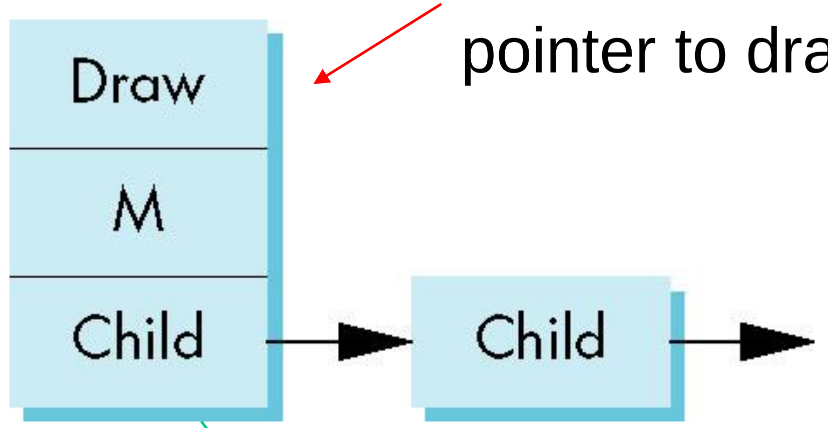
Simple example of tree model

Want a general node structure for nodes

-- storing all information in nodes



Possible Node Structure



Code for drawing part or pointer to drawing function

linked list of pointers to children

A matrix relating node to parent

Generalizations

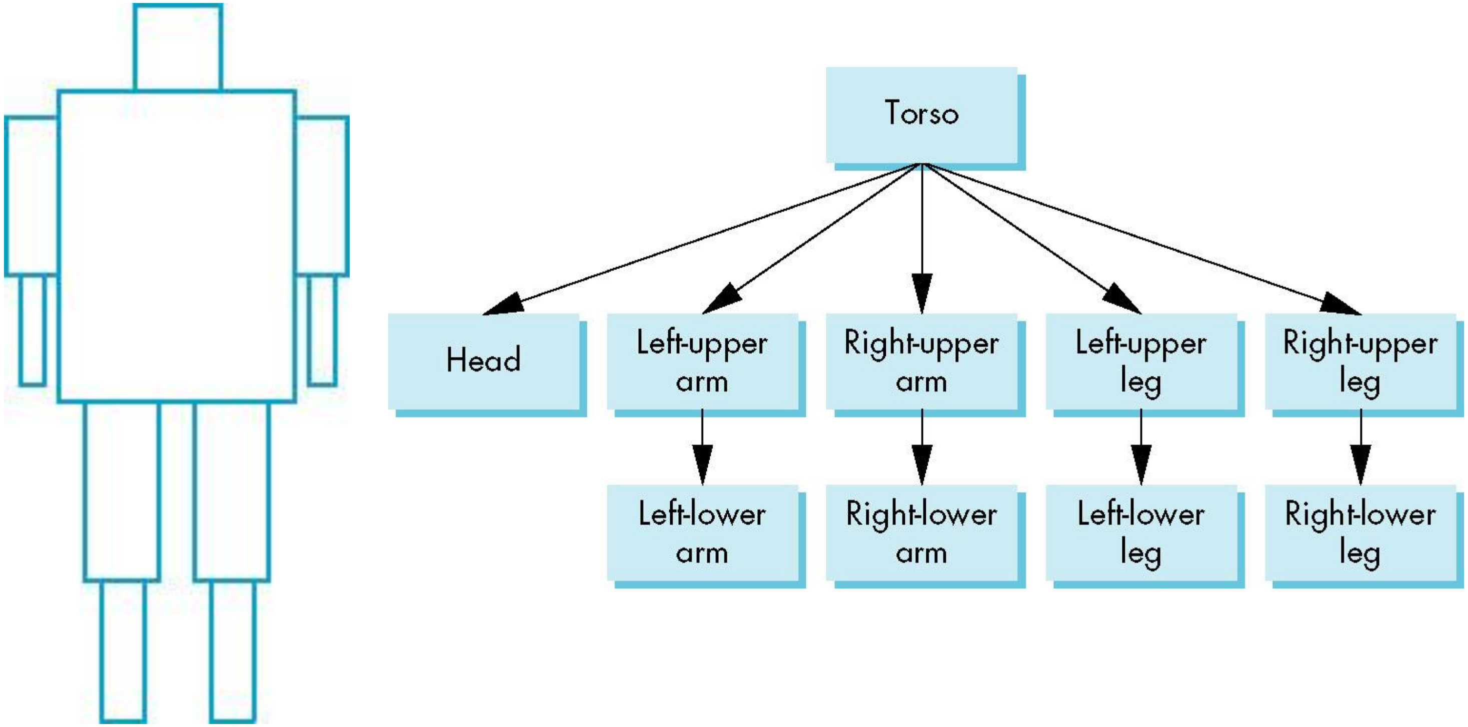
Need to deal with multiple children

- How do we represent a more general tree?
- How do we traverse such a data structure?

Animation

- How to use dynamically?
- Can we create and delete nodes during execution?

Humanoid Figure



Building the Model

Can build a simple implementation using quadrics:

- ellipsoids and cylinders

Access parts through functions drawing individual parts in their own frames

- `torso()`
- `left_upper_arm()`

Matrices describe position of node with respect to its parent

- $\mathbf{M}_{lla}$ positions left lower arm with respect to left upper arm

Tree with Matrices