Top-down Ground Proof Procedure

Idea: search backward from a query to determine if it is a logical consequence of KB.

An answer clause is of the form:

$$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$$

The SLD Resolution of this answer clause on atom a_i with the clause:

$$a_i \leftarrow b_1 \wedge \ldots \wedge b_p$$

is the answer clause

$$yes \leftarrow a_1 \wedge \cdots \wedge a_{i-1} \wedge b_1 \wedge \cdots \wedge b_p \wedge a_{i+1} \wedge \cdots \wedge a_m.$$



Derivations

- An answer is an answer clause with m=0. That is, it is the answer clause $yes \leftarrow$.
- A derivation of query " $?q_1 \wedge ... \wedge q_k$ " from KB is a sequence of answer clauses $\gamma_0, \gamma_1, ..., \gamma_n$ such that
 - γ_0 is the answer clause $yes \leftarrow q_1 \wedge \ldots \wedge q_k$,
 - \triangleright γ_i is obtained by resolving γ_{i-1} with a clause in KB, and
 - $ightharpoonup \gamma_n$ is an answer.

Top-down definite clause interpreter

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To solve the query ?q_1 \wedge \ldots \wedge q_k:
ac := "yes \leftarrow q_1 \wedge \ldots \wedge q_k"
\textbf{repeat}
\textbf{select} \ \text{atom} \ a_i \ \text{from the body of} \ ac;
\textbf{choose} \ \text{clause} \ C \ \text{from} \ KB \ \text{with} \ a_i \ \text{as head};
\text{replace} \ a_i \ \text{in the body of} \ ac \ \text{by the body of} \ C
\textbf{until} \ ac \ \text{is an answer}.
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Nondeterministic Choice

- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives.
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

Example: successful derivation

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a \leftarrow b \land c. a \leftarrow e \land f. b \leftarrow f \land k. c \leftarrow e. d \leftarrow k. e. f \leftarrow j \land e. f \leftarrow c. j \leftarrow c.
```

Query: ?a

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\gamma_0: yes \leftarrow a \gamma_4: yes \leftarrow e

\gamma_1: yes \leftarrow e \land f \gamma_5: yes \leftarrow

\gamma_2: yes \leftarrow f

\gamma_3: yes \leftarrow c
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Example: failing derivation

$$a \leftarrow b \wedge c$$
. $a \leftarrow e \wedge f$. $b \leftarrow f \wedge k$. $c \leftarrow e$. $d \leftarrow k$. e . $f \leftarrow j \wedge e$. $f \leftarrow c$. $j \leftarrow c$.

Query: ?a

```
\begin{array}{lll} \gamma_0: & \textit{yes} \leftarrow \textit{a} & \gamma_4: & \textit{yes} \leftarrow \textit{e} \land \textit{k} \land \textit{c} \\ \gamma_1: & \textit{yes} \leftarrow \textit{b} \land \textit{c} & \gamma_5: & \textit{yes} \leftarrow \textit{k} \land \textit{c} \\ \gamma_2: & \textit{yes} \leftarrow \textit{f} \land \textit{k} \land \textit{c} \\ \gamma_3: & \textit{yes} \leftarrow \textit{c} \land \textit{k} \land \textit{c} \end{array}
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Search Graph for SLD Resolution



