781 _ 2011-03-29

Note Title 2011-03-29

HW3

Exercises 76, 85, 86 from

Schöning's Ch. 2 handout

Herbrand universe $D(F)$ for a closed formula

F in Skolem form

Ex. $F = \forall x \forall y \forall z \; P(x, f(y), g(z, x))$

$D(F) = \{ a, f(a), g(a, a), f(g(a, a)),$
$g(a, g(a, a)), g(g(a, a), g(a, a)),$
$f(g(a, g(a, a))), \ldots \}$

Ex. $G = \forall x \forall y \; Q(c, f(x), h(y, b))$

$D(G) = \{ b, c, f(b), f(c), h(b, b), h(c, b),$

$f(f(b)), f(f(c)), f(h(b,b)), \ldots \}$

The Herbrand structure $\mathfrak{Q}$

$D(\mathfrak{Q}) = \{a, f(a), g(a,a), \ldots\}$

$f^{\mathfrak{Q}} \longrightarrow f$

$g^{\mathfrak{Q}} \to g$

$P^{\mathfrak{Q}}$ defined as $(t_1, t_2, t_3) \in P^{\mathfrak{Q}}$ iff $g(t_1, t_2) =$

$= g(t_1, f(t_1))$

is not a model of $F = \forall x \forall y \forall z\, P(x, f(y), g(z,x))$

b/c $\mathcal{Q}(F) = 0$ for the assignment

$x \to a$, $y \to a$, $z \to a$ and,

$\mathcal{Q}(F) = 1$ iff $\mathcal{Q}(P(x), f(y), g(z,x)) = 1$

for every assignment to x, y, z.

~~Exercise~~ 72. Call this structure $\mathcal{B}$

The universe and interpretation of f and g

are set — they are the same for all Herbrand structures

For P, let $P^{\mathcal{B}} = \{(\alpha, \beta, \gamma) \mid \alpha, \beta, \gamma \in D(F)\}$

Königs lemma