781 2011-02-22

Note Title 2011-01-20

Yasuhara Ch. 9 (Propositional Calculus)

Lemma 9.1 (1) - alternate proof (11-01-20)

1. $(A \supset B), (B \supset C), A \vdash A$ hypothesis

2. " " " $\vdash (A \supset B)$ "

3. " " " $\vdash B$ m.p. on 1 and 2

4. " " " $\nvdash (B \supset C)$ m.p. on 3 and 4

5. " " " $\vdash C$

Exercise 9.14

$L(Y) = L(P_0)$ language

$((A \supset B) \supset (A \supset A))$ axiom

$\{A, (A \supset B)\} \to B$ rule of inference (mp)

(a) The axiom is a tautology, as one can check by TT!

A	B	$A \supset B$	$A \supset A$	$((A \supset B) \supset (A \supset A))$
f	f	t	t	t
f	t	t	t	t
t	f	f	t	t
t	t	t	t	t

mp preserves tautologies as shown in Exercise 9.9 (d).

So, yes

(b) 1. $\vdash ((A \supset B) \supset (A \supset A))$ axiom

2. $\vdash ((A \supset B) \supset (A \supset A)) \supset ((A \supset B) \supset (A \supset B))$ axiom with $A \leftarrow A \supset B$, $B \leftarrow A \supset A$

3. $\vdash ((A \supset B) \supset (A \supset B))$ mp on 1, 2

Case 1 Let $\bar{A}$ be an axiom and $(\bar{A} \supset \bar{B})$ be also an axiom

$$\{\bar{A}, (\bar{A} \supset \bar{B})\} \to \bar{B} \qquad ((A \supset B) \supset (A \supset B)) \quad (mp)$$

$$((A \supset B) \supset (A \supset A)) \supset (A \supset B) \supset (A \supset B)$$

Or:

1 $\vdash_{y} ((A \supset B) \supset (A \supset A))$ axiom

2 $\vdash_{y} (((A \supset B) \supset (A \supset A)) \supset ((A \supset A) \supset (A \supset A)))$ ax. [illegible]

3 $\vdash_{y} ((A \supset A) \supset (A \supset A))$ m p on 1, 2 then.

Case 2

1. $\vdash_Y ((A \supset B) \supset (A \supset A))$ axiom

$\vdash_Y ((A \supset B) \supset (A \supset A)) \supset ($ non-axiom thm

impossible

Case 3 $\vdash_Y ((A \supset A) \supset (A \supset A))$ non-ax. thm.

$\vdash_Y (((A \supset A) \supset (A \supset A)) \supset ((A \supset A) \supset (A \supset A)))$ non ax. thm

$\vdash_Y ((A \supset A \supset (A \supset A))$ non ax. thm

Case 4 non~~-~~ax thm, ax. $\Rightarrow$ non ax thm (like case 3

QED