2011-02-10

Note Title 2011-02-10

Watch Watson in action on TV next week. Root for the machine!

Binary resolution, as defined in Ch. 1 of Schöning's book is refutation complete for the propositional calculus

Non binary resolution (in which

all pairs of complementary literals are removed from the resolvent during a resolution step) is also refutation-complete.

Factoring (i.e., the removal of complementary literals in one clause) may also be used.

Preview: binary resolution is sound but not complete for First-Order Logic

(FOL) ; either non-binary resolution
or binary resolution and factoring
are refutation complete

Example: Show that $F = (P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee Q) \wedge (\sim P \vee \sim Q)$ is unsatisfiable by resolution

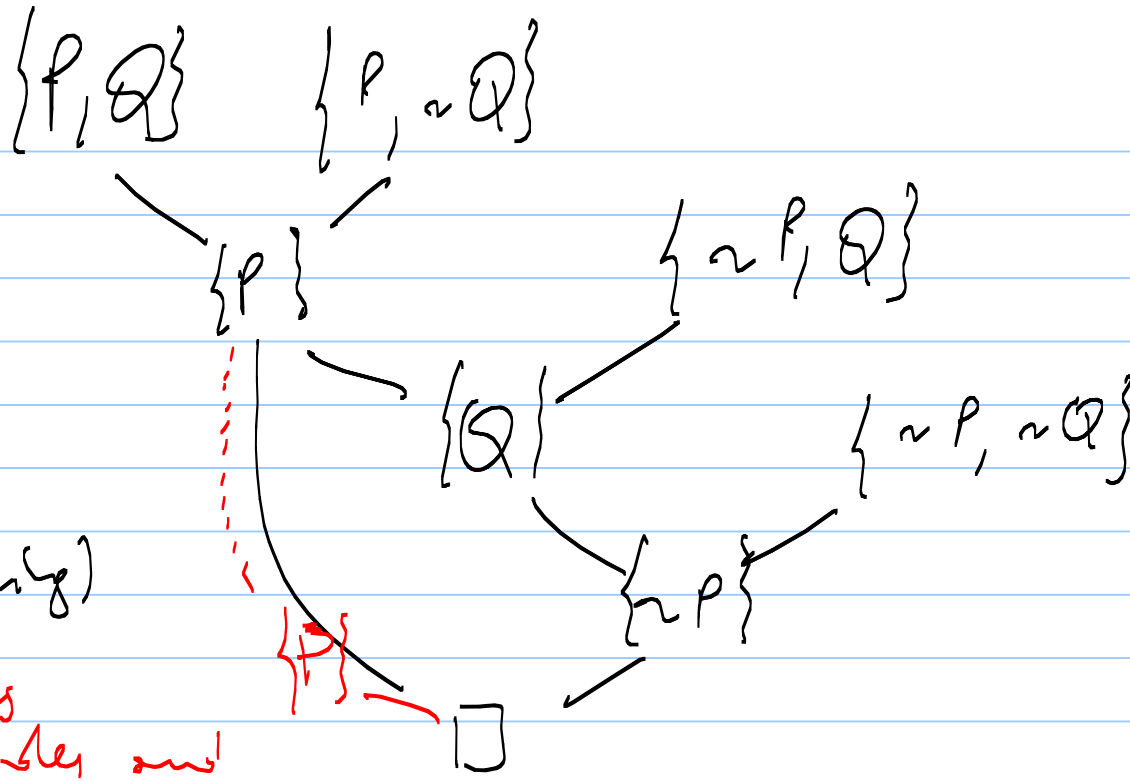
Proof 1:

$$\{P, Q\} \quad \{P, \sim Q\} \qquad \{\sim P, Q\} \quad \{\sim P, \sim Q\}$$

$$\{P\} \qquad\qquad \{\sim P\}$$

$$\square$$

Proof 2:



Note:
this is
a refutation
graph (showing)

(Loveland uses
duplicate nodes and
dashed edges)

Exercise 33 [Schöning]

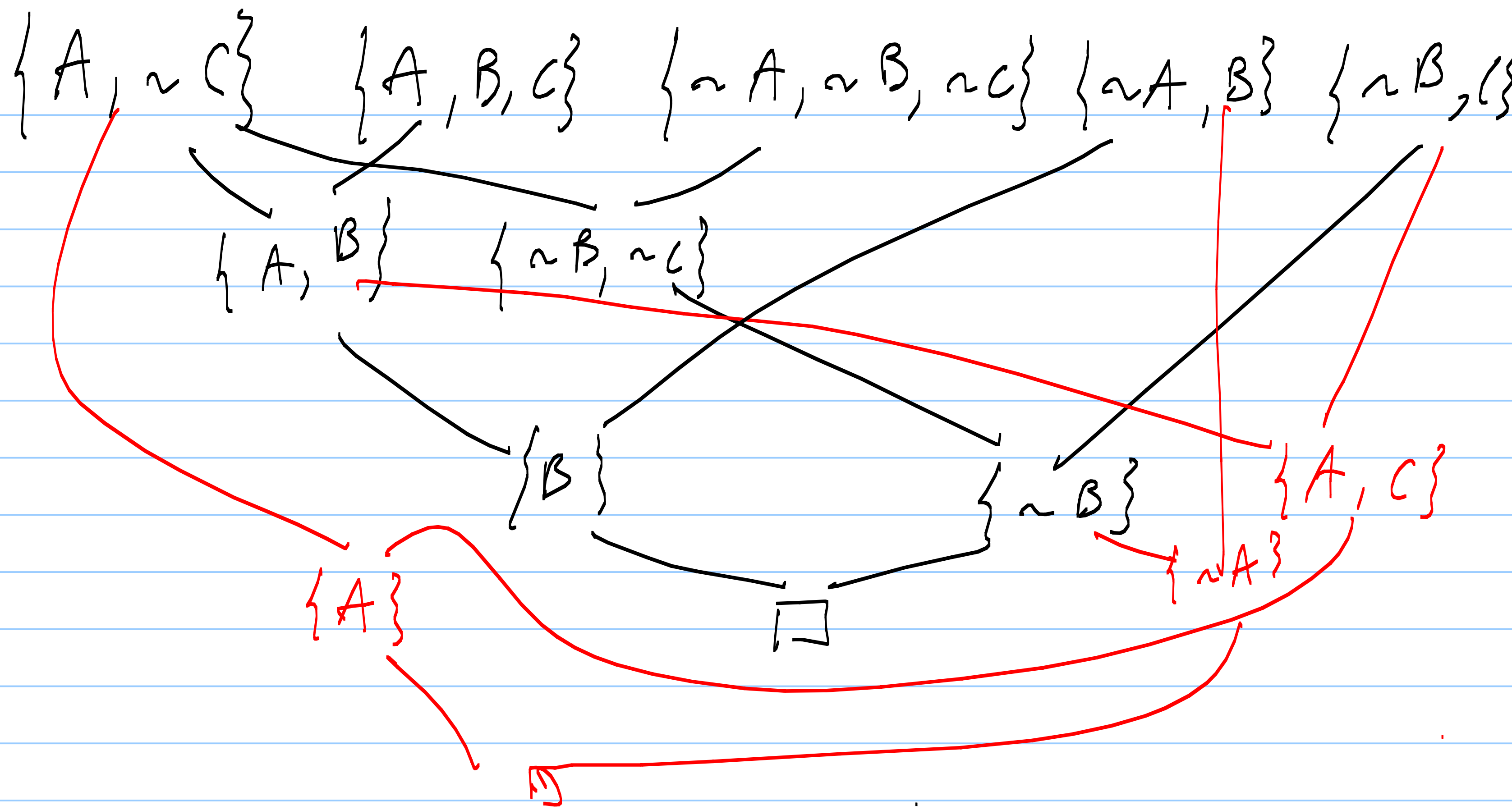
Show that $A \wedge B \wedge C$ is a consequence of the clause set $F = \{\{\sim A, B\}, \{\sim B, C\}, \{A, \sim C\}, \{A, B, C\}\}$

$F \models A \wedge B \wedge C$

Let $F' = (\sim A \vee B) \wedge (\sim B \vee C) \wedge (A \vee \sim C) \wedge (A \vee B \vee C)$

$F' \wedge \sim (A \wedge B \wedge C)$ is unsatisfiable.

$F \cup \{\sim A, \sim B, \sim C\}$ is unsatisfiable



{A, ~C}
{A, B, C}
{~A, ~B, ~C}
{~A, B}
{~B, C}
{A, B}
{~B, ~C}
{B}
{~B}
□
{A, C}
{~A}
{A}
□

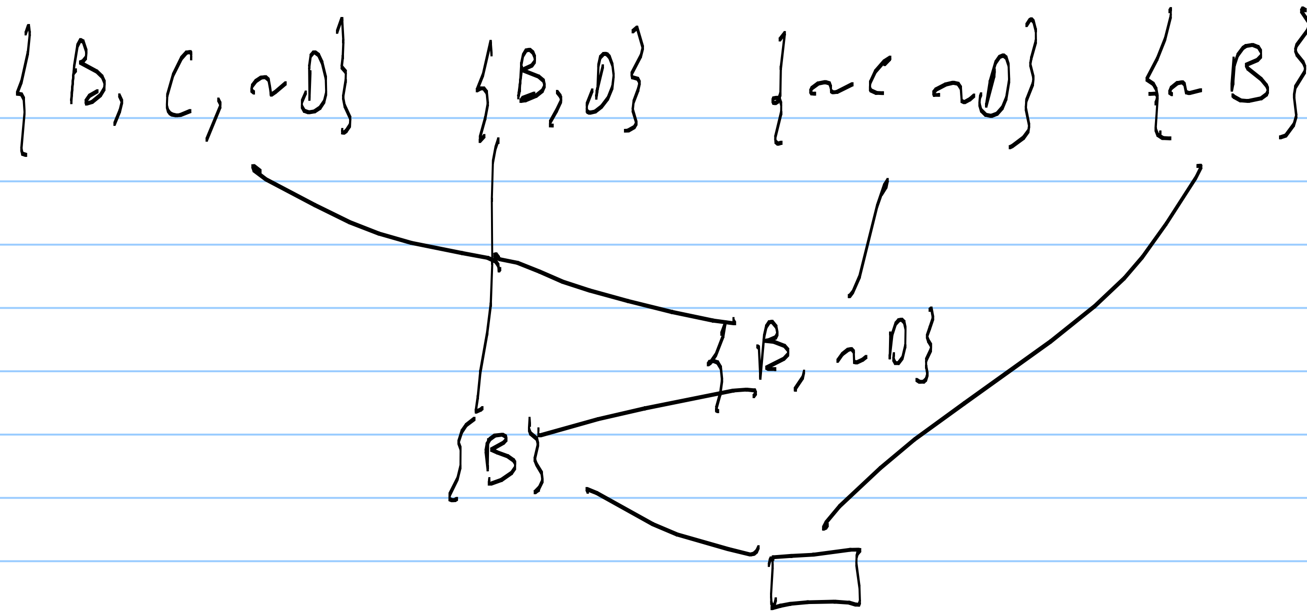
Exercise 34 Show that the following formula is a tautology:

$$F = (\sim B \wedge \sim C \wedge D) \vee (\sim B \wedge \sim D) \vee (C \wedge D) \vee B$$

A formula is a tautology iff its negation is unsatisfiable

$$\sim F = \sim (\sim B \wedge \sim C \wedge D) \wedge \sim (\sim B \wedge \sim D) \wedge \sim (C \wedge D) \wedge \sim B$$

$$\equiv (B \vee C \vee \sim D) \wedge (B \vee D) \wedge (\sim C \vee \sim D) \wedge \sim B$$


{B, C, ~D}
{B, D}
{~C ~D}
{~B}
{B, ~D}
{B}

Poole, Ch. 5

Goal: l1_b ; Show KB ∪ {~l1_b} is unsatisfiable

In computer:

$light1\_broken \leftarrow sw\_up$
$\quad \wedge\ power \wedge unlit\_light1.$

$sw\_up.$

$power \leftarrow lit\_light2.$

$unlit\_light1.$

$lit\_light2.$

In user's mind:

- *light1_broken*: light #1 is broken
- *sw_up*: switch is up
- *power*: there is power in the building
- *unlit_light1*: light #1 isn't lit
- *lit_light2*: light #2 is lit

Conclusion: *light1_broken*

- The computer doesn't know the meaning of the symbols
- The user can interpret the symbol using their meaning