2011-02-01

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Theorem 9.7 (Yasuhara) (simplified version)
Let P be a propositional theory (based on the language $L(P_1)$) whose rules of inference include modus ponens. If the theorems of P include all tautologies and a contradiction, then for all (formulas) $A \in F(P_1)$, $\vdash_P A$.

<u>Proof</u>

Let C be a contradiction that is a theorem of P.

(1) $\vdash_p C$ C is a theorem

(2) $\vdash_p \sim C$ $\sim C$ is a tautology

(3) $\vdash_p (\sim C \supset (C \supset A))$ Lemma 9.1 (3)

(comment: $(\neg C \wedge C) \rightarrow A$)

(4) $\vdash_p (C \supset A)$ m.p. on 2, 3

(5) $\vdash_p A$ m.p. on 1, 4

$\square$

Theorem 9.7 (Yasuhara) (Full version)

Let P be a propositional theory (based on the language $L(P_1)$) whose rules of inference include modus ponens. If the theorems of P include more than those formulas that are tautologies, then for all $A \in F(P_1)$, $\vdash_P A$.

Proof.

$(C \supset A)$ holds because either A holds or $\sim C$ holds

C	A	$C \supset A$
t	t	t
t	f	f
f	t	t
f	f	t

(By "holds" we mean: it has value t in its truth table.)

In the first case (A holds and C holds)

$B_1, \ldots, B_k \vdash_p (C \supset A)$ using the construction of Lemma 9.2

$B_1, \ldots, B_k \vdash_p C$ C is a theorem

$B_1, \ldots, B_k \vdash_p A$ by m. p.

In the second case ($\sim C$ holds)

$B_1 \ldots B_k \vdash_p (C \supset A)$ using the construction of Lemma 9.2

$B_1, \ldots, B_k \vdash_p C$ C is a theorem

$B_1, \dots, B_k \vdash_p A$ by m.p.

Use the same argument as in Thm. 9.5 (main part) and conclude

$$\vdash A.$$

$\square$

(The propositional calculus is Post-complete)

Schöning, Ch. 1

To show that $A \to B$, show that $A \wedge \neg B$ is unsatisfiable.