Yasuhara Ch 9 (Prop. Calculus)

Note Title 2011-01-18

$(p_1 \supset (p_2 \supset p_1))$ $((\sim p_1 \supset \sim p_2) \supset (p_2 \supset p_1))$

$(p_1 \supset p_2)$

$(A \supset (B \supset A))$ axiom 1 (axiom scheme or schema)

$((A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C)))$ axiom 2

$((\sim A \supset \sim B) \supset (B \supset A))$ axiom 3

The (only) rule of inference of P_0 is

$\{A, (A \supset B)\} \to B$ (modus ponens)
mp

$\vdash_{P_0} A$ means: there is a derivation (proof) of A using the axioms & rule of inference of P_0

Q 3 Try proving $\vdash_{P_0} (A \supset A)$

(1) $(A \supset (B \supset A))$ axiom 1

(2) $((A \supset (A \supset A)) \supset (A \supset A) \supset (A \supset A))$ axiom 2

(3) $(A \supset A) \supset (A \supset A)$ m.p. on

Dead!

(1) $((A \supset (B \supset A)) \supset ((A \supset B) \supset (A \supset A)))$ axiom 2

(2) $(A \supset (B \supset A))$ axiom 1

(3) $((A \supset B) \supset (A \supset A))$ mp

(4) $(A \supset (A \supset A))$ axiom 1

See p. 188 top for a good proof.

Exercise 9.5

$A \vdash_{P_0} A$? Yes!

We would like $A \vdash_{P_0} A$ to imply

$\vdash_{P_0} (A \supset A)$.

Theorem 9.1. If $B_1, \ldots, B_{n-1}, B_n \vdash_{P_0} C$

then $B_1, \ldots, B_{n-1} \vdash_{P_0} (B_n \supset C)$

(The deduction theorem)

$n = 1$, C is an axiom

(1) $B_1, \ldots, B_{n-1}, \vdash_{P_0} C$ (b/c C is an axiom)

(2) $B_1, \ldots, B_{n-1} \vdash_{P_0} (C \supset (B_K \supset C))$ axiom 1

$(3)\ B_1, \ldots, B_{k-1} \vdash_{P_0} (B_k \supset C)$ MP on (1), (2)