Exam #1 Review Questions

September 20, 2001

Topics

General Control Configurations

- Jargon MV, CV, DV
- Feedback
- Feedforward
- Cascade

Linear Algebra

- Steady state modeling ($\underline{\Delta y} = \underline{\underline{K}}\,\underline{\Delta u}$)
- Solving $\underline{\underline{A}}\,\underline{x} = \underline{b}$ by row reduction
- Solving $\underline{\underline{A}}\,\underline{x} = \underline{b}$ by calculating $\underline{\underline{A}}^{-1}$
- Matrix multiplication
- Determinant / Eigenvalues of $\underline{\underline{A}}$

Dynamic Modeling (Open-loop)

- Dynamic mass and energy balances

State Space Representation for ODEs

Laplace Transforms

- step, delayed step, impulse
- ramp, sinusoid, exponential
- time delay and Heavyside function
- derivative, integral
- Solving Ordinary Differential Equations (ODEs)
- Step response of First-Order system

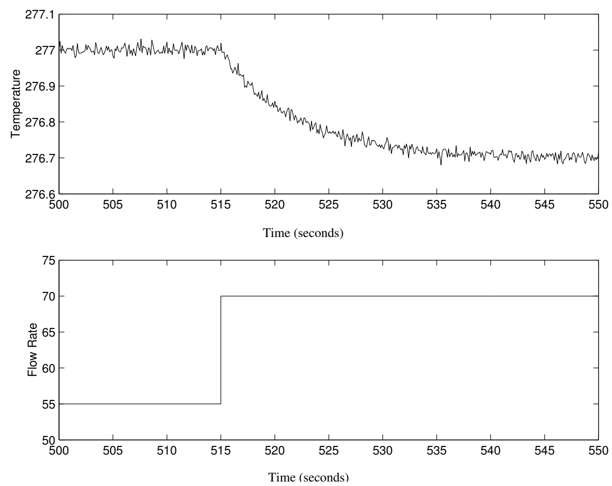
Partial Fraction Expansion

Linearity applied to complex functions

- $f(t) = f_1(t) + f_2(2) \Rightarrow f_1(s) + f_2(s) = f(s)$
- Compound / Composite functions

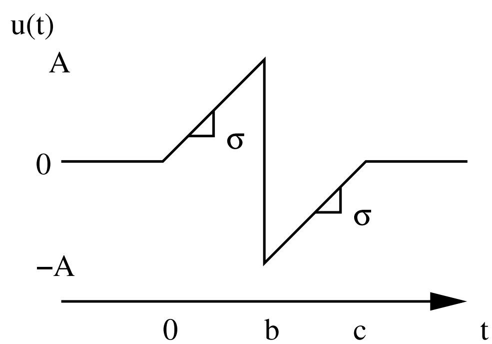
Selected Questions from Exam 1, 2001

1. **(15 pts.)** The Ideal Gas Company is attempting to develop a dynamic process model for a combustion chamber which burns a stream of aqueous liquid waste. The process output and the process input are shown for a input step change. What is the transfer function for this system, assuming it is a first order process?

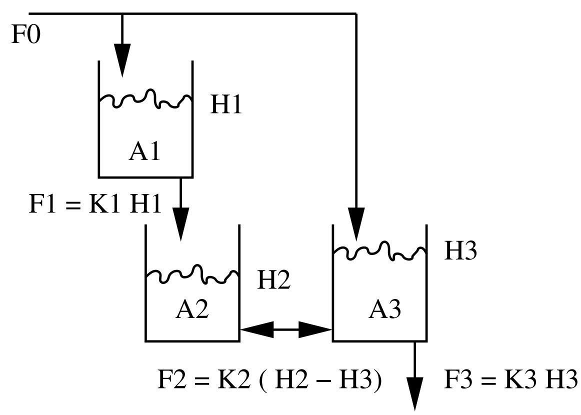


2. **(15 pts.)** What is the Laplace transform $u(s)$ of the following function?

$$u(t) = \begin{cases} 0 & t < \; 0 \\ \sigma t & 0 \le \; t \; < b \\ -2A + \sigma t & b \le \; t \; < c \\ 0 & c \le \; t \end{cases}$$



3. **(25 pts.)** A system consists of three tanks as shown below. The flow rate F_0 can be manipulated. A fraction of the flow rate F_0 into the system goes into tank 1 and the rest of the flow enters into tank 3 as shown. The fraction of flow F_0 into tank 1 is γ, with $0 \le \gamma \le 1$ and γ remaining constant. The flow rate from tank 1 to tank 2 is given as $F_1 = k_1 h_1$. The flow rate into tank 3 from tank 2 is $F_2 = k_2(h_2 - h_3)$. The flow rate out of tank 3 is $F_3 = k_3 h_3$. The constant cross sectional tank areas are A_1, A_2, and A_3, respectively.

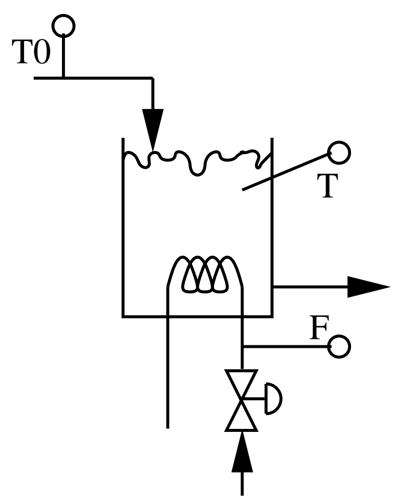


a. Derive the differential equation model for the system.

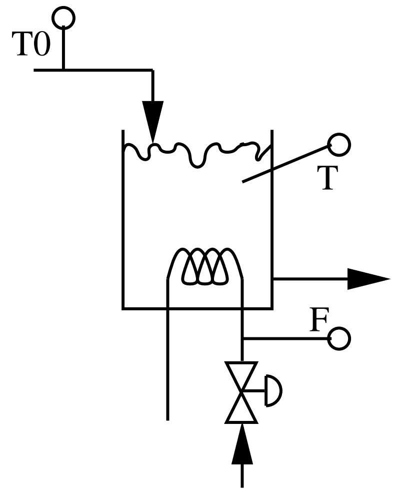
b. Put your differential equation model into State Space form ($\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}u$, $y = \underline{c}^T x$) for the system, given that $u = F_0$, $y = h_3$, and $\underline{x}$ with $\underline{x}$:

$$\underline{x} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

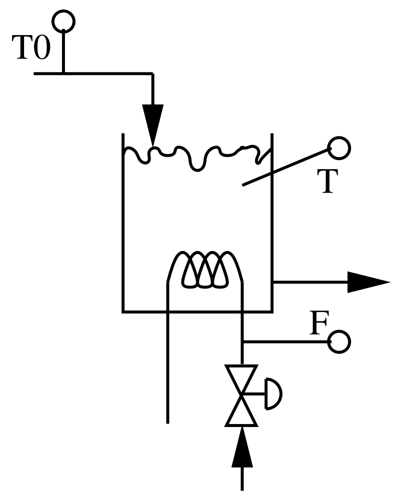
4. **(20 pts.)** For the following system, steam is used to heat the liquid in a constant volume tank. The available measurements include the temperature of the liquid in the tank, the temperature of the feed flowing into the tank, and the steam flow rate. The steam valve can be manipulated. It is desired to regulate the temperature of the exit flow from the tank at a constant value.



a. In the figure above, draw a feedback control loop for the system



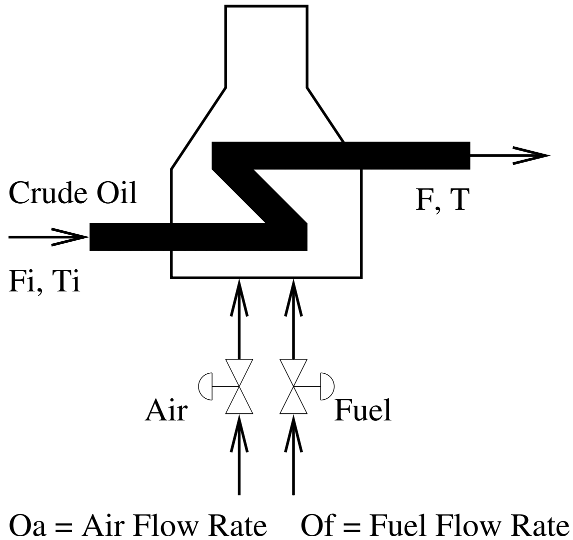
b. In the figure above, draw a feed forward control loop, assuming the feed temperature acts as the disturbance.



c. In the figure above, assuming the steam flow rate varies unpredictably, draw a cascade configuration using two feedback controllers.

Quiz 1, Practice, 2001

1. **(4 pts.)** A preheater furnace is used increase the temperature of crude oil from T_i to T, the target value. The preheated hot crude oil is then sent down stream to a reactor. The crude oil enters the furnace at the flow rate F and leaves at the same rate. Fuel and air are mixed and burned in the furnace to heat the crude oil. See diagram below.



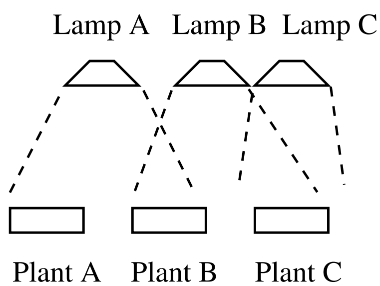
Construct two different feedback control configurations. Also, construct two different feedforward control configurations. Clearly label what is measured and what is manipulated.

3. **(2 pts.)** What are the eigenvalues of the following matrix?

$$\begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

Quiz 1, 2001

1. **(4 pts.)** An agricultural process requires that trays of plants be maintained at specified temperatures. Three lamps are used to warm three plant trays as seen below.



A 3x3 steady state model is desired relating the change in voltages ΔV_i (for each lamp i) to the change in plant temperature ΔT_j (for each plant j). It is known that increasing the voltage for Lamp A by 1 volt increases the temperature of Plant A by 3.3 degrees and increases the temperature of Plant B by 2.1 degrees. Increasing Lamp B voltage by 1 volt increases both Plant B and Plant C by 2 degrees. Increasing Lamp C voltage by 1 volt increases the temperature of Plant C by 4 degrees.

Develop a model in the form $\underline{\underline{A}}\underline{x} = \underline{b}$ and identify $\underline{\underline{A}}$, $\underline{x}$, and $\underline{b}$.

You may want to check your model by assuming arbitrary values for the change in lamp voltages, then verifying the expected change in plant temperatures.

3. **(2 pts.)** a. What is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -6 & 7 \\ -1 & -2 & 3 \end{bmatrix}$$

Quiz 1, 2002

You must develop a model of paper machine sheet forming process. A simple schematic is shown below. A feed stream of pulp (wood fibers and water) is sprayed onto a moving screen (conveyor belt). As the screen moves, water drains out of the pulp, through the screen. At the product end of the paper machine, the pulp is effectively just wet paper.

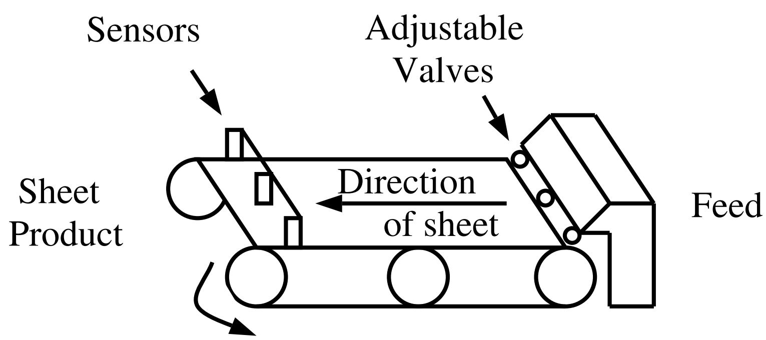


Diagram of a sheet forming process.

Three valves are available to adjust the flowrate of pulp when the pulp concentrations change. Three sensors measure the thickness of the wet paper. Increasing the valves on the edge by 1% (v_1 and v_3) increases the thickness in the corresponding paper location by 2mm. A 1% increase in v_1 and v_3 will also decrease the thickness in the center position by 1 mm. A 1% increase in v_2 will increase the thickness in the center by 3 mm and reduce the edge thickness by 0.5mm.

1. **(1pt)** What are the controlled variables, manipulated variables, and disturbances for this paper making process?

2. **(3pts)** Develop a model of this process relating $\underline{\Delta s}$ and $\underline{\Delta v}$.

3. **(2pts)** Put your model in the form $\underline{\underline{A}}\underline{x} = \underline{b}$ and clearly identify $\underline{\underline{A}}$, $\underline{x}$, and $\underline{b}$.

4. **(2pts)** What is the determinant of the following matrix??

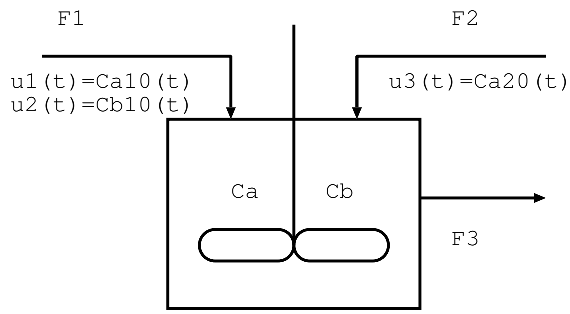
$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 4 \\ 1 & 3 & 1 \end{bmatrix}$$

5. **(2pts)** What are the eigenvalues of the following matrix?

$$\begin{bmatrix} -5 & -2 \\ 3 & -10 \end{bmatrix}$$

Quiz2, 2002

1. At the Ideal Gas Company, you are in charge of operating a reactant mixing system. Your boss wants a dynamic model of the system to be used for process control and process optimization. The constant volume mixing tank has two feed streams with constant volumetric flowrates of F_1 and F_2. Feed stream 1 contains both species A and species B, while stream 2 only contains species A. You can modify the initial concentrations of the two species coming into the tank system, $u_1(t) = C_{A10}(t)$, $u_2(t) = C_{B10}(t)$, $u_3(t) = C_{A20}(t)$. At the exit stream, due to instrumentation limitations, you can only measure the total concentration of both components, $y(t) = C_A(t) + C_B(t)$.

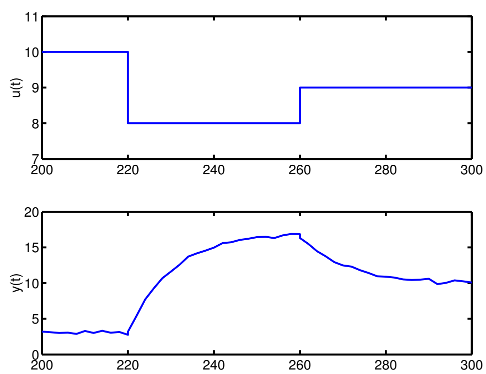


a. **(4 points)** What is the dynamic mass balance describing the concentrations of species A and species B at the exit of the mixing tank?

b. **(4 points)** Put your model in state space form. Clearly identify $\underline{x}$, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$. Example state space form:

$$\begin{aligned} \dot{x} &= \underline{\underline{A}}x + \underline{\underline{B}}u \\ y &= \underline{\underline{C}}x + \underline{\underline{D}}u \end{aligned}$$

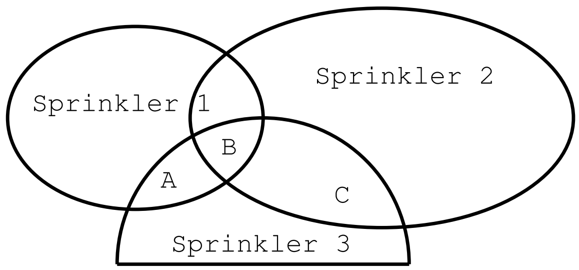
2. **(2 points)** After running some step tests for your system varying $u_1(t)$ and measuring the output $y(t)$ you have the following process data. Identify the approximate process gain for this Single-Input-Single-Output system.



3. Bonus - Dr. Gatzke has a flower bed with three sprinkler heads. In one minute, sprinkler 1 delivers $2\,mm$ of water to its coverage area, sprinkler 2 delivers $0.5\,mm$ of water to its coverage area, and sprinkler 3 delivers $4\,mm$ of water to its coverage area. Plant A is covered by sprinkler 1 and 3, plant B is covered by all sprinkler, and plant C is covered by sprinkler 2 and 3. The system is currently set to operate at normal operating program times. Develop a steady state model relating possible changes in sprinkler operating times to changes in amount of water delivered to each plant. Put you model in the form:

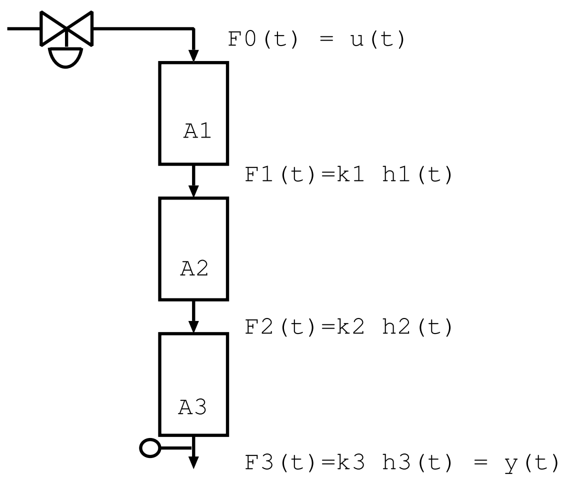
$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

3b. Assume that plant A needs an additional $2\,mm$ of water, plant B needs $1\,mm$ less, and Plant C is fine the way it is. How does the problem change? What changes to the sprinkler operating times would make this change? Solve using Row Reduction Methods.



EXAM Practice problems, 2002

1. A series of tanks are shown below. You can manipulate $F_0(t)$ and you can measure the flow rate out of tank 3, $F_3(t)$.



a. Assuming constant density, develop a mass balance for the system.

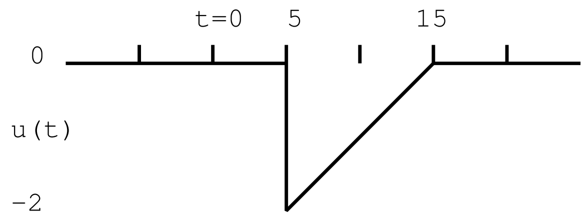
b. Put your model in state space form. Clearly identify $\underline{x}$, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$. Example state space form:

$$\begin{aligned}\dot{x} &= \underline{\underline{A}}x + \underline{\underline{B}}u \\ y &= \underline{\underline{C}}x + \underline{\underline{D}}u\end{aligned}$$

c. What are the eigenvalues of your $\underline{\underline{A}}$ matrix from your system?

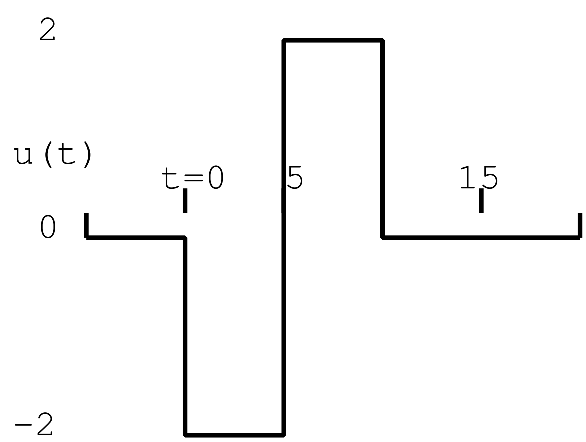
d. From part (a.) take the Laplace transform of your dynamic model assuming the tanks are empty initially. Sove the three equations for the relationship between $y(s)$ and $u(s)$.

2a.Express the following function as a simple function of time (You may need to use the heaviside function multiplied by other functions.)



b. Establish the Laplace transform of the function, $u(s)$.

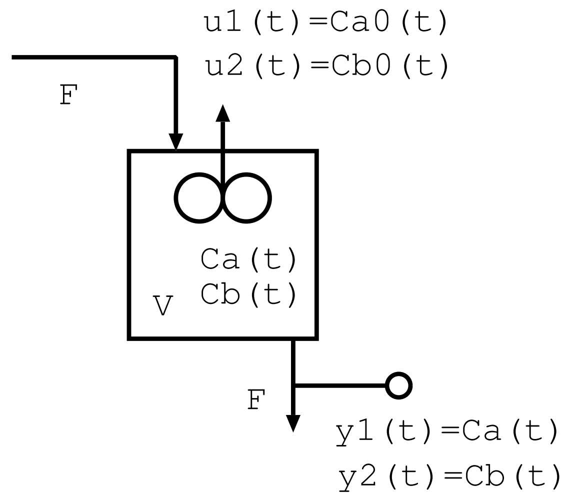
3a.Express the following function as a simple function of time (You may need to use the heaviside function multiplied by other functions.)



b. Establish the Laplace transform of the function, $u(s)$.

c. Assuming this function $u(t)$ is the input to a first-order system, $g(s) = \frac{5}{10s+1}$, $y(s) = g(s)u(s)$. Establish $y(s)$ and $y(t)$.

4. Assuming a constant volume mixing tank for two species, A and B. Assuming you can change the inlet concentrations of A and B and measure the outlet concentrations of A and B, develop a dynamic mass balance and put your equations in state space form.


u1(t)=Ca0(t)
u2(t)=Cb0(t)
F
Ca(t)
Cb(t)
V
F
y1(t)=Ca(t)
y2(t)=Cb(t)