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CSCE 750
9/13/2022

Randomized algorithms (analysis)

- correct with high probability
- fast with high probability

Monte Carlo algo: Efficient & correct with high probability

Las Vegas algo — Never gives a wrong answer, but may fail, or run a long time, but prob of either is low

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Quicksort (Las Vegas) $A[p, -, r]$

- Partition the array
  - choose a pivot value $A[q]$ for some q, $p \le q \le r$
  - Arrange elements so that $A[q] \le$ pivot is to the left of the pivot and $A[q] \ge$ pivot to the right
- Recurse to left of pivot,
- Recuse to right of the pivot

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worst-case analysis given by the recurrence:
To sort n items

$$T(n) = \max_{0 \le q \le n-1} \left(T(q) + T(n-q-1) \right) + \underbrace{\Theta(n)}_{\text{partition}}$$

By subst method,

$$T(n) = O(n^2)$$

Assume $T(m) \le cm^2$ for $m < n$

$$T(n) = an + \max_{0 \le q < n} \left(T(q) + T(n-q-1) \right)$$

$$\le an + \max_q \left(cq^2 + c(n-q-1)^2 \right)$$

$$= an + \max_{q=0 \text{ or } q=n-1} \{ c(n-1)^2, c(n-1)^2 \}$$

$$= an + c(n-1)^2$$

$$= an + cn^2 - 2cn + c$$

$$= cn^2 + \underbrace{an - 2cn + c}_{\le 0}$$

$$\le cn^2 \quad \text{for } n \ge 1 \text{ and } c \ge a$$

Lower bound (worst_case) is the same, so

$$T(n) = \Theta(n^2)$$

How to choose the pivot:
- 1st element?,
- last element?,
- middle element?,
- median of these 3

Randomized Quicksort — choose the pivot uniformly at random among the elements of the array.

Expected time of Randomized quicksort (for a worst-case array of length n) is $\Theta(n \lg n)$.

Average case vs. worst-case expected time

Former is the average taken over all inputs of a given size, randomly chosen according to some prior distribution

Expected time is average time over the algorithm's random choices (any fixed input)

W.C. Expected time corresponds to the maximum expected time for any input of a given size

Most cases, expected time only depends on the size of the input

True for Randomized Quicksort