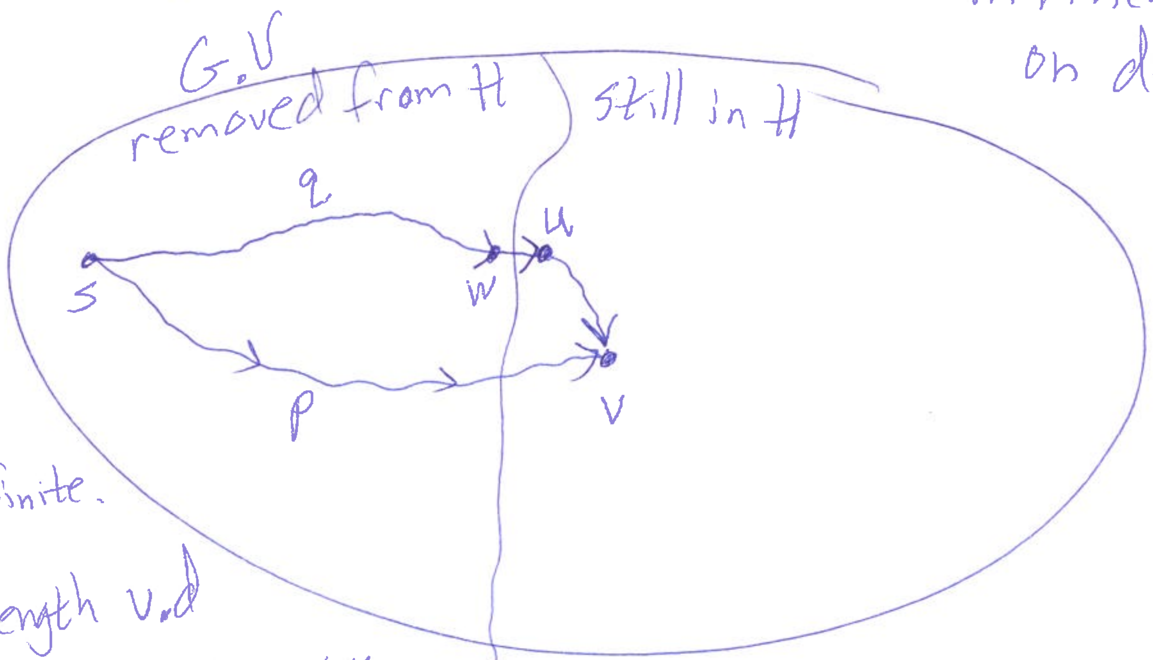
CSCE 750
11/21/2023

Dijkstra's Algo., proof of correctness
NP-completness intro

Dijkstra's Algo: Intermediate stage: H is the minheap on d-values



$s.d = 0$

Assume v.d is finite.

p has length v.d

Just before an ExtractMin operation:

Assume v.d is not the length of a shortest path from s to v. Then let q be an $s \to v$ path whose length is $<$ v.d. Show that v is not removed from ~~the~~ H yet.

Segment of q from $s \to w$ has length w.d, so $u.d \leq w.d + wt(w,u)$

no neg edgeweights so the $u \to v$ ~~path~~ part of q has ≥ 0 length.

Claim: $u.d < v.d$ $\therefore$ $v.d$ is not min in H

length of q is $w.d + wt(w,u) + (\text{length of } u \to v \text{ part})$

Then $\underbrace{u.d}_{\le w.d + wt(w,u)} + \overbrace{(\text{length of } u \to v \text{ part})}^{\ge 0} \le \text{length of } q < v.d$

$u.d \le u.d + (\text{length of } u \to v \text{ part}) \le \text{length of } q$

$< v.d.$

$\therefore$ $u.d < v.d$, so v won't be removed from H next. ☒

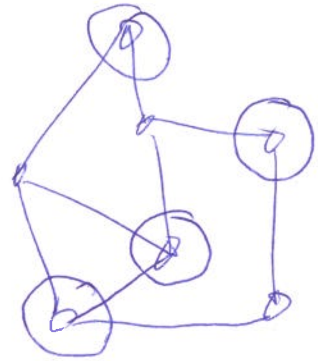
Cor: When a vertex v is removed from H, its d-value is correct (length of a shortest path $s \to v$).

Skipping Bellman-Ford algorithm

NP-completeness

Example: Given a graph G, a vertex cover of G is a set $C \subseteq G.V$ such that every edge in $G.E$ has at least one endpoint in C.

Example:



Nobody ~~kno~~ knows any algo to find a min vertex cover for an arbitrary graph that runs in subexponential time!

Decision version of vertex cover (VC):

Input: A graph G and an integer k.

Question: Does G have a vertex cover of size $\leq k$?

Restrict attention to decision problems:

Presented in the form:

Input: (an instance of the problem)

Question: (a yes/no question about the instance)

A yes-instance is an instance where the answer is "yes"

A no-instance " " " " " " " " " " "no"

Def: G a graph. An independent set in G is a set $I \subseteq G.V$ such that no two vertices in I are adjacent.

Notice: A set C is a vertex cover iff $G.V - C$ is an independent set.
