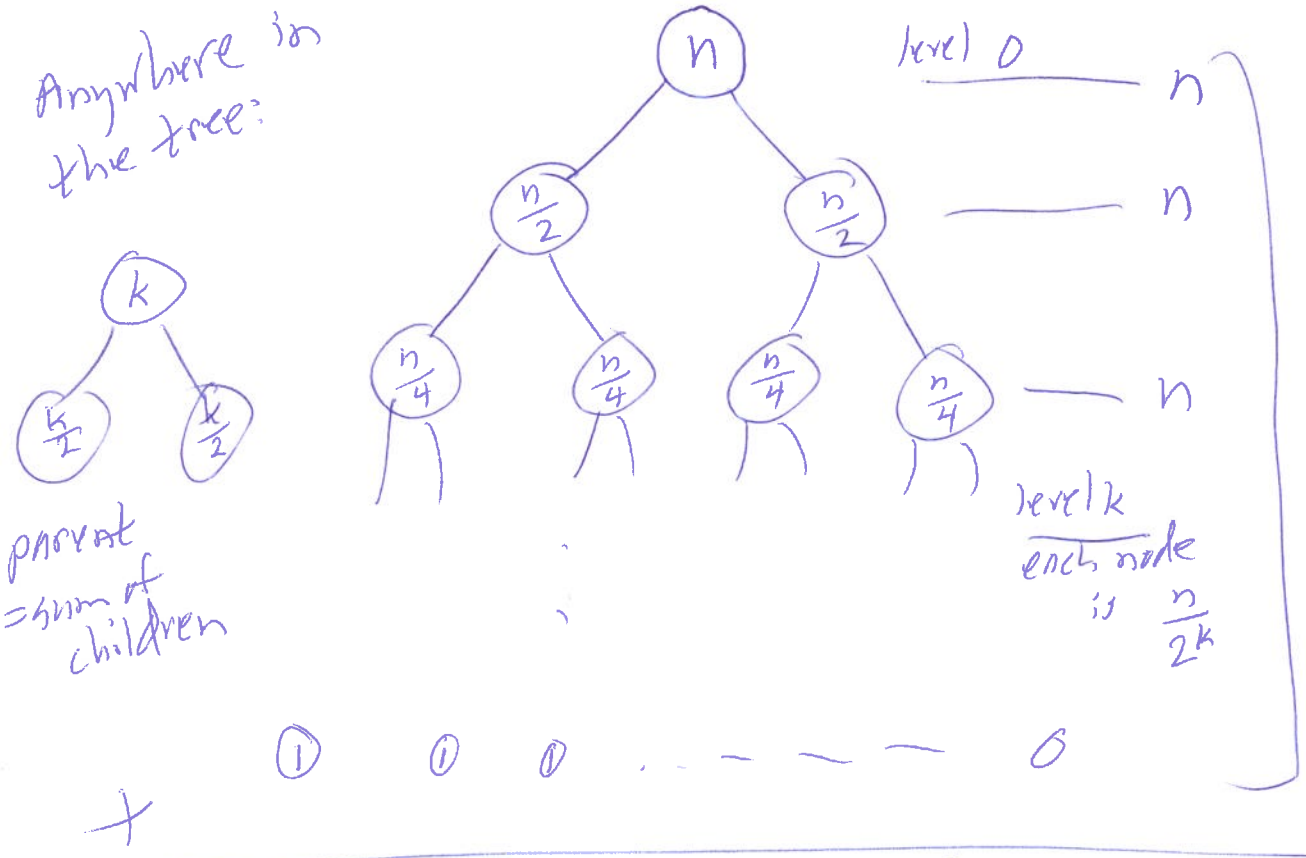
CSCE 750
9/7/2023

Tree method
Change of Variables
Master Theorem

Tree Method

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$



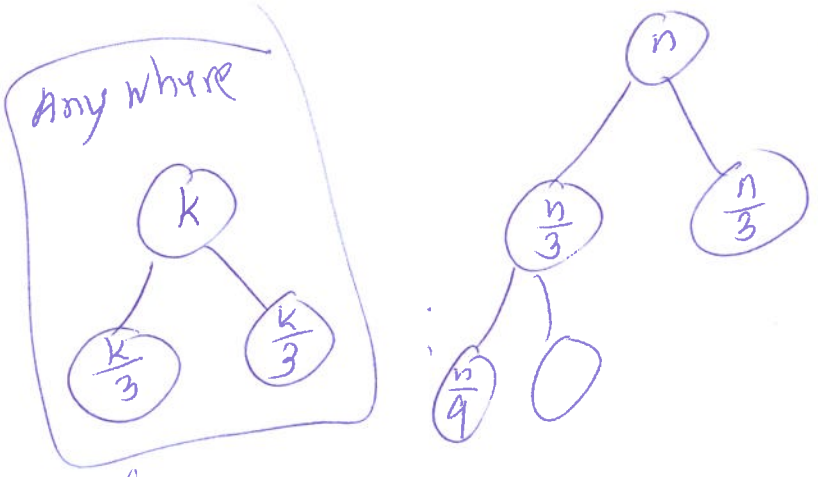
$\Theta(n \cdot (\text{number of levels})) = \Theta(n \lg n)$

? $\frac{n}{2^k} = 1$

$n = 2^k$

$\lg n = \lg 2^k = k$

$$T(n) = 2T\left(\frac{n}{3}\right) + n$$



sum of children = $\frac{2}{3}$ parent

~~Upper bound~~

$$T(n) = \sum_{i=0}^{\lfloor \log_3 n \rfloor} n\left(\frac{2}{3}\right)^i$$

Upper bound:

$$T(n) = \sum_{i=0}^{\lfloor \log_3 n \rfloor} n\left(\frac{2}{3}\right)^i = n \sum_{i=0}^{\lfloor \log_3 n \rfloor} \left(\frac{2}{3}\right)^i$$

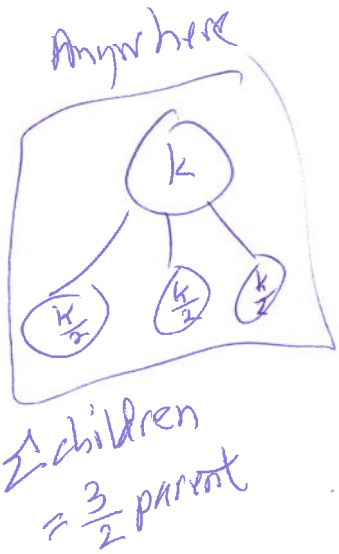
$$\leq n \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i = n \frac{1}{1-\frac{2}{3}} = 3n$$

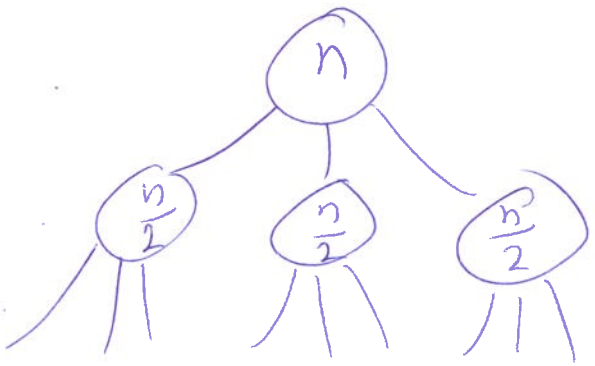
Lower bound: $T(n) \geq n$

first term of the sum!

$\therefore T(n) = \Theta(n)$

$T(n) = 3T\left(\frac{n}{2}\right) + n$ | fast mult recurrence (3)





— n

— $\frac{3}{2}n$

— $\frac{9}{4}n$

⋮

level i — $\left(\frac{3}{2}\right)^i n$

+

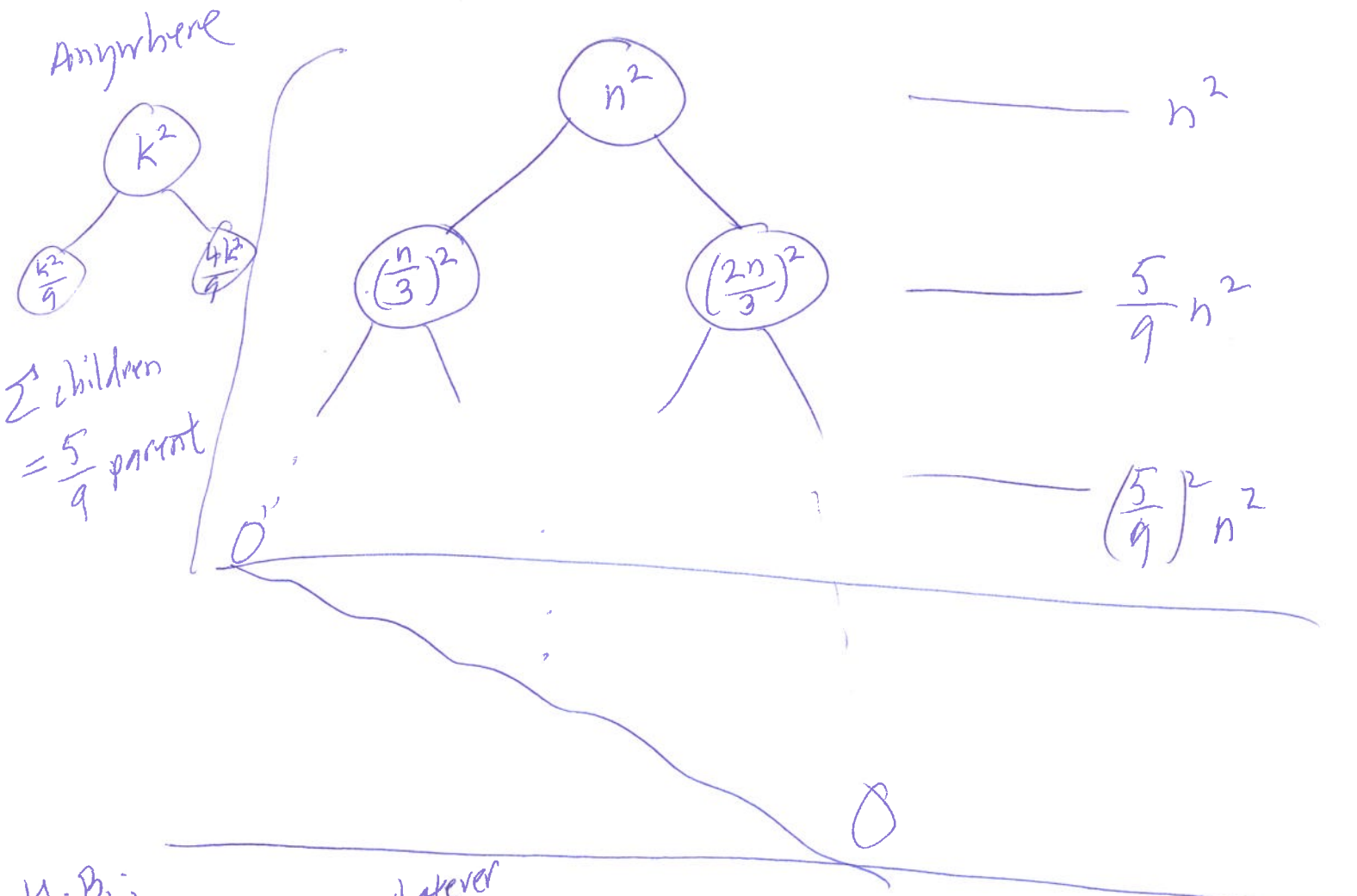
$$T(n) = \sum_{i=0}^{\lfloor \lg n \rfloor - 1} n\left(\frac{3}{2}\right)^i = n \sum_{i=0}^{\lfloor \lg n \rfloor - 1} \left(\frac{3}{2}\right)^i$$

$$= n\left(\frac{\left(\frac{3}{2}\right)^{\lfloor \lg n \rfloor + 1} - 1}{\frac{3}{2} - 1}\right) = \Theta\left(n\left(\frac{3}{2}\right)^{\lfloor \lg n \rfloor + 1}\right)$$

$$= \Theta\left(n\left(\frac{3}{2}\right)^{\lg n}\right) = \Theta\left(n \cdot n^{\lg\left(\frac{3}{2}\right)}\right)$$

$$= \Theta\left(n \cdot n^{\lg 3 - \lg 2}\right) = \Theta\left(n^{\lg 3}\right)$$

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n^2$$



U.B.:

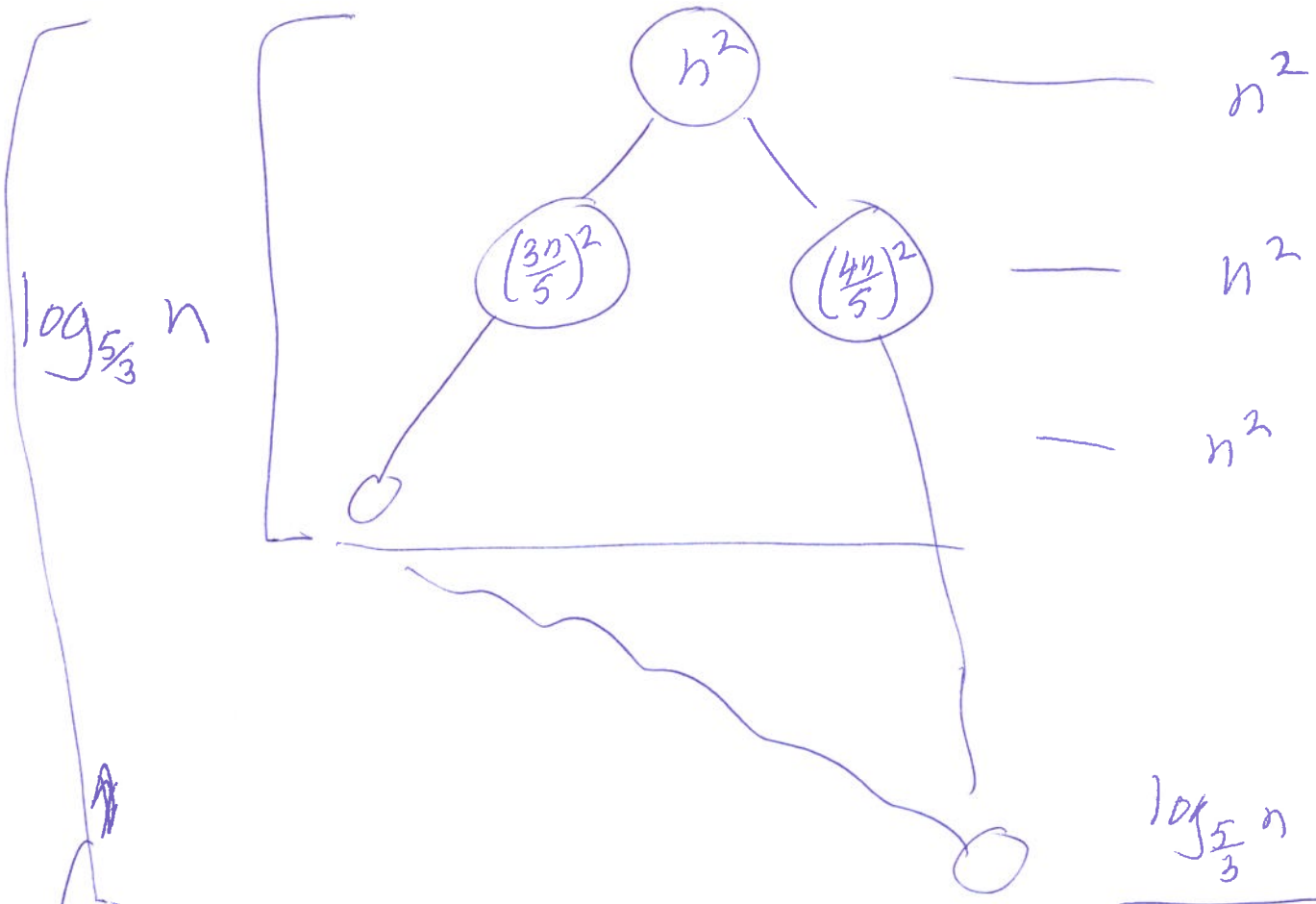
$$T(n) = \sum_{i=0}^{\text{whatever}} n^2\left(\frac{5}{9}\right)^i \leq n^2 \sum_{i=0}^{\infty}\left(\frac{5}{9}\right)^i = O(n^2)$$

L.B.:

$$T(n) \geq n^2$$

$$\therefore T(n) = \Theta(n^2).$$

$$T(n) = T\left(\frac{3n}{5}\right) + T\left(\frac{4n}{5}\right) + n^2$$



$$\frac{\log_{\frac{5}{3}} n}{\log_{5/4} n} = \text{constant}$$

$$n^2 \log_{\frac{5}{3}} n \leq T(n) \leq n^2 \log_{5/4} n$$

$$\therefore T(n) = \cancel{n^2 \lg n} \ \Theta(n^2 \lg n)$$