

CSCE 750
9/7/2023

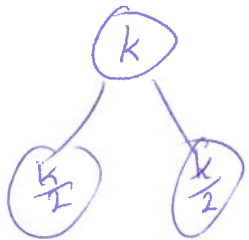
Tree method
Change of Variables
Master Theorem

①

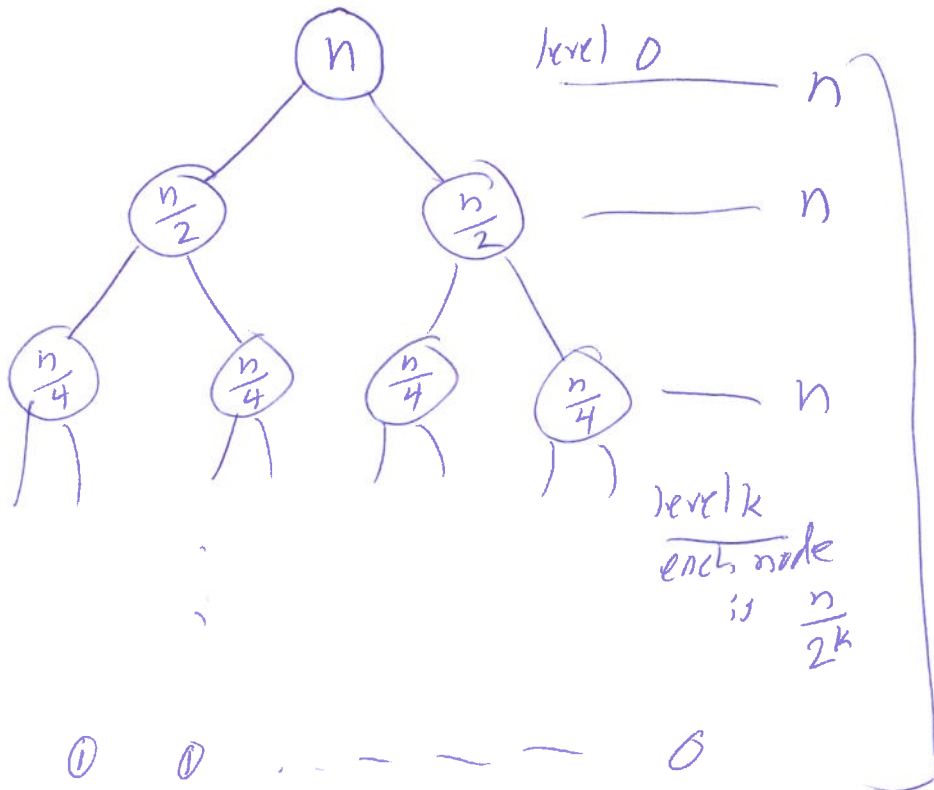
Tree Method

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

Anywhere in the tree:



parent = sum of children



+

$$\Theta(n \text{ (number of levels)}) = \Theta(n \lg n)$$

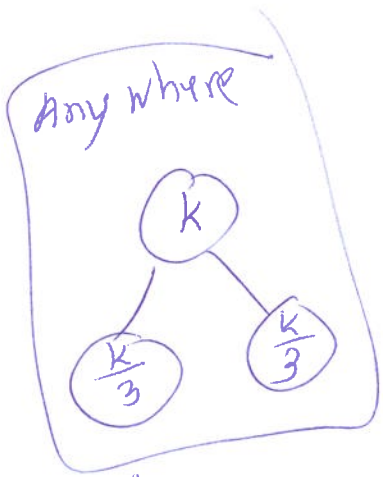
$$\therefore \frac{n}{2^k} = 1$$

$$n = 2^k$$

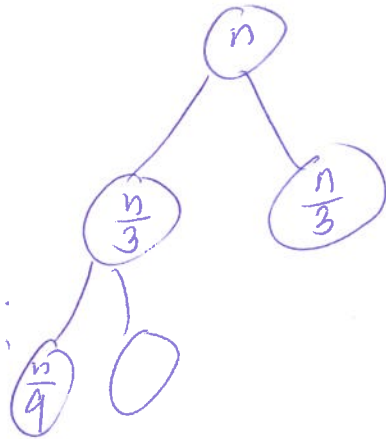
$$\lg n = \lg 2^k = k$$

$$T(n) = 2T\left(\frac{n}{3}\right) + n$$

(2)



sum of children = $\frac{2}{3}$ parent



—	n
—	$\frac{2}{3}n$
—	$\frac{4}{9}n$
—	$\left(\frac{2}{3}\right)^i n$

+ 0 - - - 0

~~Upper bound~~

$$T(n) = \sum_{i=0}^{\lfloor \log_3 n \rfloor} n \left(\frac{2}{3}\right)^i$$

Upper bound:

$$T(n) = \sum_{i=0}^{\lfloor \log_3 n \rfloor} n \left(\frac{2}{3}\right)^i = n \sum_{i=0}^{\lfloor \log_3 n \rfloor} \left(\frac{2}{3}\right)^i$$

$$\leq n \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i = n \frac{1}{1 - \frac{2}{3}} = 3n$$

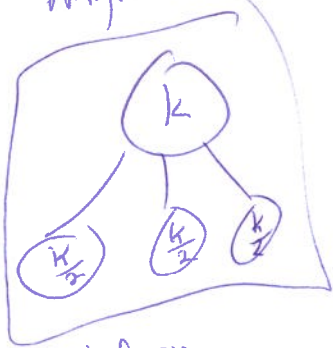
Lower bound: $T(n) \geq n$

first term of the sum!

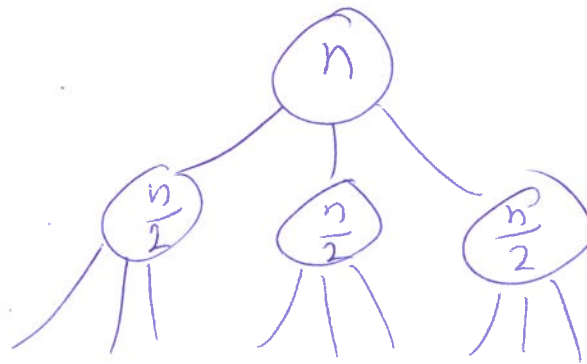
$\therefore T(n) = \Theta(n)$

$$T(n) = 3T\left(\frac{n}{2}\right) + n \quad \left| \begin{array}{l} \text{fast mult} \\ \text{rechnerisch} \end{array} \right. \textcircled{3}$$

Anywhere



3 children
= $\frac{3}{2}$ parent



$$\begin{array}{l} \text{--- } n \\ \text{--- } \frac{3}{2}n \\ \text{--- } \frac{9}{4}n \\ \vdots \\ \text{level } i \text{ --- } \left(\frac{3}{2}\right)^i n \end{array}$$

+

$$T(n) = \sum_{i=0}^{\lfloor \lg n \rfloor} n \left(\frac{3}{2}\right)^i = n \sum_{i=0}^{\lfloor \lg n \rfloor} \left(\frac{3}{2}\right)^i$$

$$= n \left(\frac{\left(\frac{3}{2}\right)^{\lfloor \lg n \rfloor + 1} - 1}{\frac{3}{2} - 1} \right) = \textcircled{+} \left(n \left(\frac{3}{2}\right)^{\lfloor \lg n \rfloor + 1} \right)$$

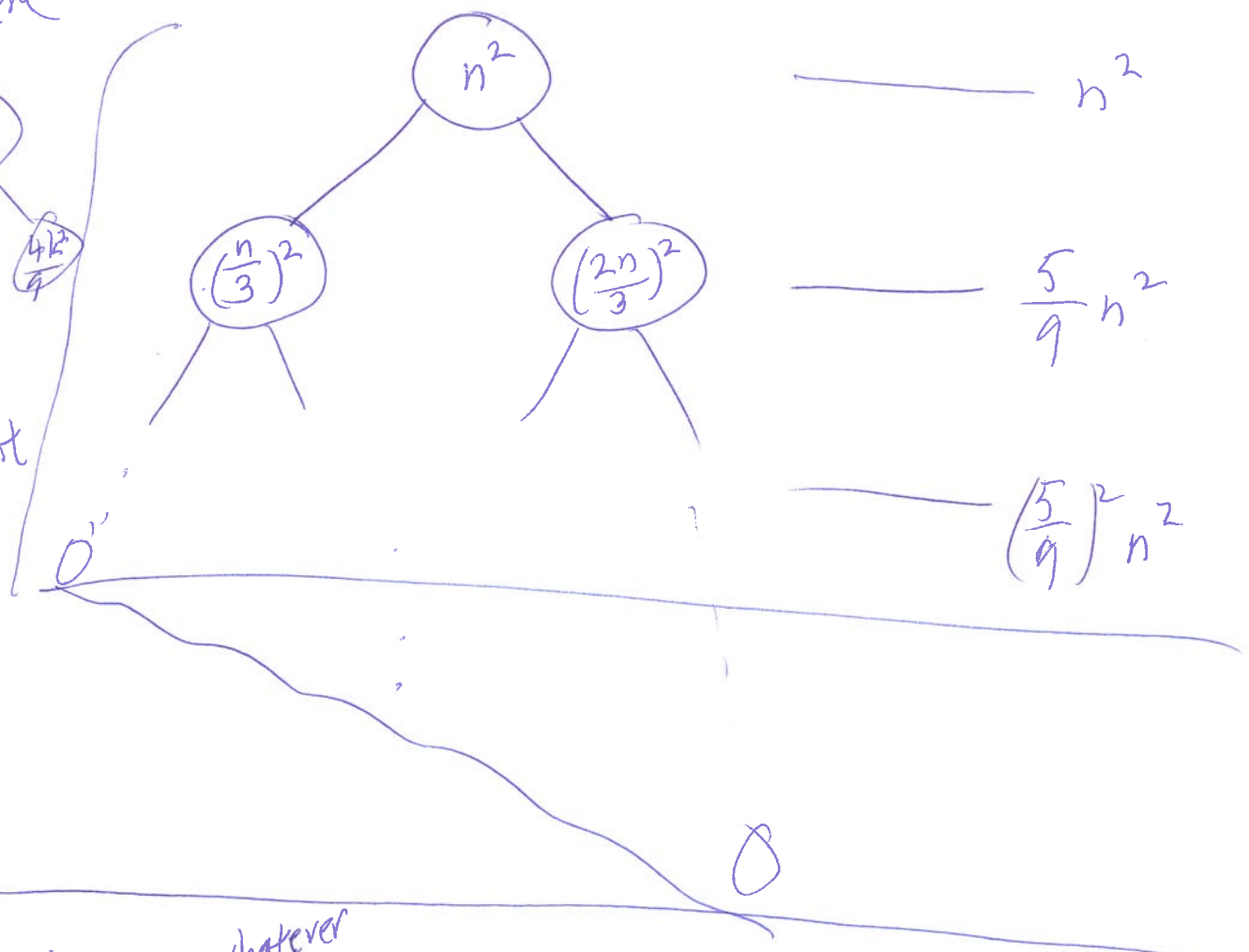
$$= \textcircled{+} \left(n \left(\frac{3}{2}\right)^{\lg n} \right) = \textcircled{+} \left(n \cdot n^{\lg\left(\frac{3}{2}\right)} \right)$$

$$= \textcircled{+} \left(n \cdot n^{\lg 3 - \lg 2} \right) = \textcircled{+} \left(n^{\lg 3} \right)$$

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n^2$$

Amynrbene

\sum children
= $\frac{5}{9}$ parent



u.B.:

$$T(n) = \sum_{i=0}^{\text{whatever}} n^2 \left(\frac{5}{9}\right)^i \leq n^2 \sum_{i=0}^{\infty} \left(\frac{5}{9}\right)^i = O(n^2)$$

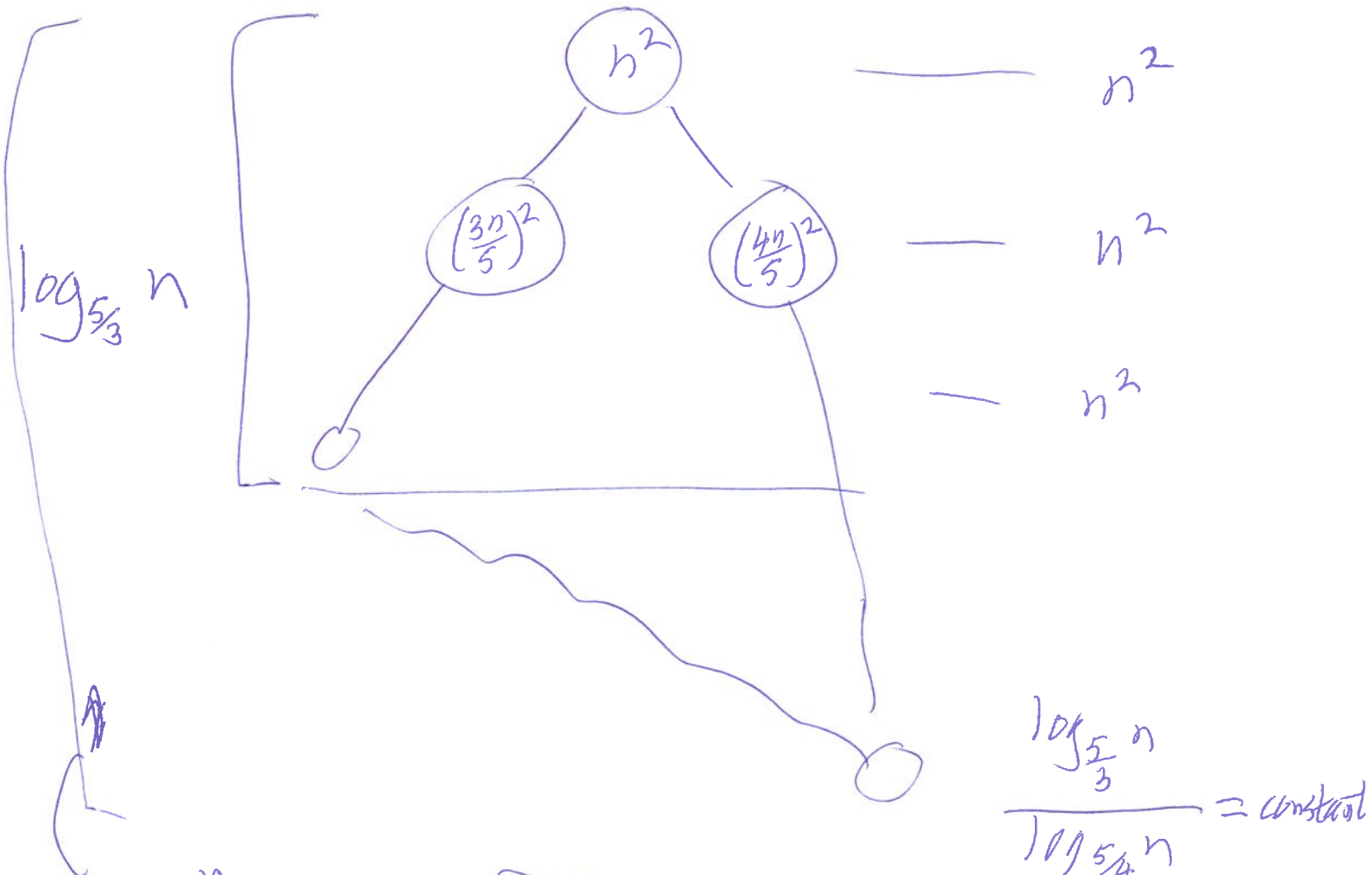
l.B.:

$$T(n) \geq n^2$$

$$\therefore T(n) = \Theta(n^2)$$

$$T(n) = T\left(\frac{3n}{5}\right) + T\left(\frac{4n}{5}\right) + n^2$$

(5)



$\log_{5/3} n$

$\log_{5/4} n$

$$n^2 \sqrt{\log_{5/3} n} \leq T(n) \leq n^2 \sqrt{\log_{5/4} n}$$

$$\therefore T(n) = n^2 \lg n \quad (+) \quad (n^2 \lg n)$$