CSCE 750

11/21/08

Graph G (undirected)

$G=(V,E)$

Cost function

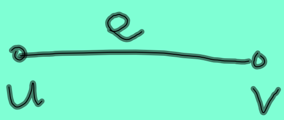
$c: E \to \mathbb{R}$

$c(e)$ is cost of e

$e=(u,v)$

write as $c(u,v)$

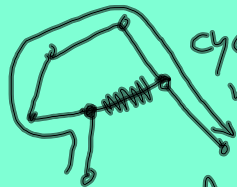
Connected subgraph of G



$c(e)$ of installing wire between u & V.

Assume $c(e) \geq 0$

Connected subgraph of G with minimum cost.



cycle

<u>no</u> <u>cycles</u>

<u>connected</u>:

A tree is a connected acyclic graph.

Minimum Spanning Tree

(MST)

least cost $\left(\sum_{e \text{ in tree}} c(e)\right)$

includes all vertices

2 algos for MST (greedy)

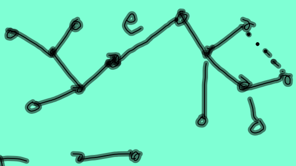
Kruskal's algo & Prim's algo

Kruskal's algo:

1. Put edges into a min heap Q by cost
2. Set of edges T
   $T := \emptyset$.
3. WHILE Q not empty DO
   remove min cost edge e from Q
4. If e does not form a cycle with T
   then $T := T \cup \{e\}$

// invariant: T is acyclic

else ignore e

T: 

e merges 2 connected components into one

Return T

For test: Use disjoint set system.

Sets are connected components (sets of vertices)

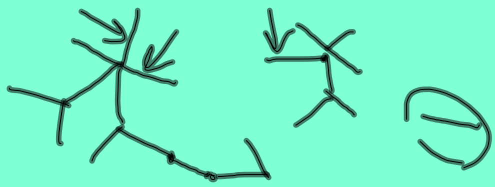
MakeSet(x) — makes $\{x\}$

→ Union(x,y) — merges set containing x with set containing y (original sets are destroyed)

(Find)

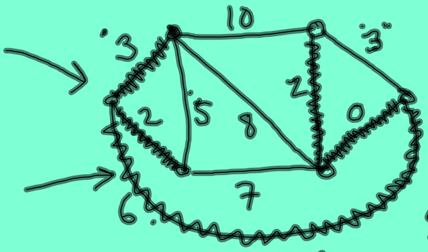
SameSet(x,y) — tests whether x,y are in same set

T:



Return T

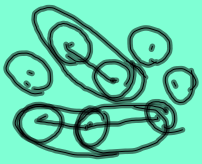
(V, T)



$|V| = n$

stop when $n-1$ edges added to T

$(V, \emptyset)$

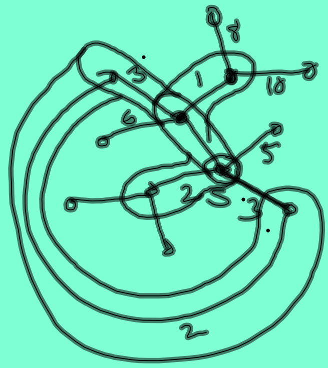


Fact: any tree on n vertices has exactly $n-1$ edges.

Prim's Algo: Instance of graph search

start vertex s

grow single tree out from s. $\underbrace{T}_{\text{includes vertices and edges}}$



$T := (\underset{V[T]}{\{s\}}, \underset{E[T]}{\emptyset})$

WHILE there is an edge $e=(u,v)$ where

$u \in V[T]$ and

$v \notin V[T]$ THEN

let e be such an edge with min cost

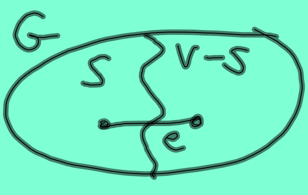
$V[T] := V[T] \cup \{v\}$

$E[T] := E[T] \cup \{e\}$

END while

Return T

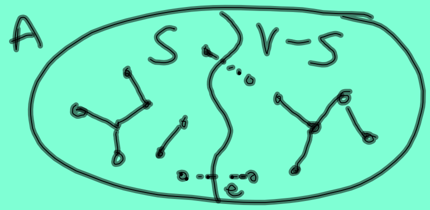
A <u>cut</u> in $G=(V,E)$ is a partition $(S, V-S)$ where S & $V-S$ are nonempty subsets of V.



Edge e crosses cut if one endpt is in S and the other is in V-S.

Let A be a set of edges. The cut (S,V-S) respects A if no edge in A crosses the cut.

An edge e is safe for A if there is a cut (S, V-S) that respects A and e is a least cost edge crossing the cut.



Thm: If A is a subset of some MST, and e is safe for A, then $A \cup \{e\}$ is also a subset of an MST.

Proof: Let $A \subseteq T$ where T is an MST. Let e be safe for A.

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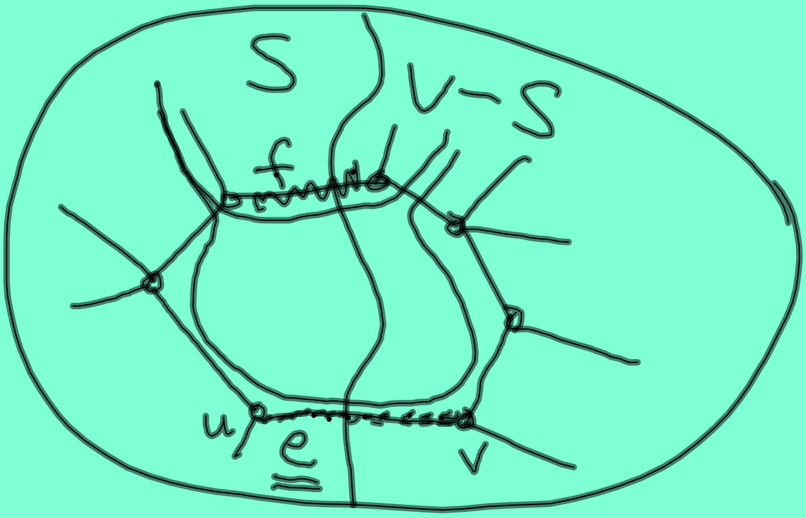
e is least cost edge crossing some cut $(S, V-S)$ respecting A.

If $e \in T$, done.

So assume $e \notin T$.



Let $T' := (T - \{f\}) \cup \{e\}$

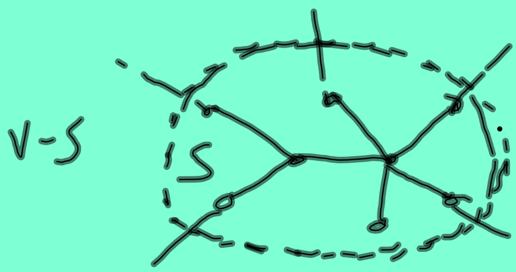
$A \cup \{e\} \subseteq T'$ & T' is connected.

So T' is a spanning tree

$c(e) \leq c(f)$

$$\mathrm{cost}(T') \leq \mathrm{cost}(T)$$

$$=$$



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