CSCE 551
Summer 2006
Final Exam

Do all problems, putting your answers on separate sheets of paper. There are 110 points total in the exam. Full credit for graduate students is 85 points; full credit for undergrads is 70 points. The rest is extra credit. You have 135 minutes. You may use, without proof, any results from the book or from class.

1. (15 points) Show that if A is regular, then the language

$$\mathrm{PREFIX}(A) = \{w \in \Sigma^* \mid (\exists z \in \Sigma^*) wz \in A\}$$

is also regular. [Hint: If there is an n-state DFA recognizing A, then there also is an n state DFA recognizing PREFIX(A).]

2. (10 points) Show that every Turing-recognizable language is recognized by a Turing machine that never rejects any input, i.e., the machine always either accepts or loops. Your proof should include low-level detail about the machine's states and transition function.

3. (15 points) Recall that the *range* of a function $f : A \to B$ is defined as

$$\{f(x) \mid x \in A\}.$$

Recall that every nonempty Turing-recognizable language is the range of some computable function. Using any method you like, precisely describe a computable function $f : \Sigma^* \to \Sigma^*$ whose range is exactly A_{TM}. A high-level description will suffice. [Hint: Fix a "fall-back" string $x \in A_{\mathrm{TM}}$. Consider inputs to f of the form $\langle M, w, t \rangle$ where M is a TM, w is a string over M's input alphabet, and t is a nonnegative integer. Don't forget to say what f does with inputs not of this form.]

4. (10 points) Show that the language

$$A = \{\langle M, w \rangle \mid M \text{ is a DFA, } w \text{ is a string, and } w^n \in L(M) \text{ for all } n \geq 0\}$$

is in P. (Here, "w^n" means $ww \cdots w$ (n times).)

5. (15 points) Show that the language

$$B = \{\langle M, w\rangle \mid M \text{ is an NFA, } w \text{ is a string, and } w^n \in L(M) \text{ for all } n \geq 0\}$$

is in PSPACE. [Hint: You may wish first to compute a number N such that $\langle M, w\rangle \in B$ iff $w^n \in L(M)$ for all $0 \leq n \leq N$.]

6. (15 points total) Let F be the quantified Boolean formula

$$F = \exists x_1 \forall x_2 \exists x_3 \forall x_4 [(x_2 \vee \overline{x_3} \vee x_4) \wedge (\overline{x_1} \vee \overline{x_4}) \wedge (x_1 \vee \overline{x_3} \vee \overline{x_4})].$$

   (a) (10 points) Using the polynomial-time mapping reduction from TQBF to GG (Generalized Geography) given in the book or in class, give the instance of GG (i.e., the graph) corresponding to F. You may either draw the graph, or else give a list of vertices and a list edges.

   (b) (5 points) Is F true? Explain.

7. (15 points) Show that if $\overline{\text{SAT}} \in \mathbf{NP}$, then $\mathbf{NP} = \text{co}\mathbf{NP}$. (Recall that $\text{co}\mathbf{NP} = \{\overline{A} \mid A \in \mathbf{NP}\}$.) [Hint: To show this, the only thing you need to know about SAT is that it is **NP**-complete (under p-time m-reductions, of course).]

8. (15 points) Let $\Sigma = \{0, 1\}$ and suppose that $f : \Sigma^* \to \Sigma^*$ is some polynomial-time computable function. For each $n \geq 0$, let f_n be f restricted to the domain Σ^n of all binary strings of length n. Show that the language $L = \{0^n \mid n \geq 0 \ \& \ \text{range}(f_n) = \Sigma^n\}$ is in co**NP**. [Hint: use the Pigeon Hole Principle.]