CSCE 355
1/24/2024

ε-NFAs

Last time:

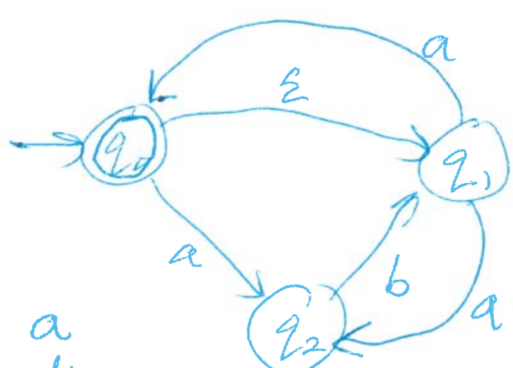
$r \in \delta(q, \varepsilon)$

Def: An ε-NFA is a tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are the same as with an NFA or DFA, and

$$\delta : Q \times \underbrace{(\Sigma \cup \{\varepsilon\})}_{} \longrightarrow 2^Q$$

Σ_ε = set of strings of length 0 or 1

$\Sigma = \{a, b\}$



Tabular form

	a	b	ε
q_0	$\{q_2\}$	$\emptyset$	$\{q_1\}$
q_1	$\{q_0, q_2\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\{q_1\}$	$\emptyset$

Input: a
accepted!

$$\underbrace{q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{a} q_0}_{\text{unique accepting path}}$$

<u>Def:</u> Let $A := \langle Q, \Sigma, \delta, q_0, F \rangle$ be an ε-NFA, and let $w \in \Sigma^*$ be a string. A <u>computation</u> (or path) of A on input w is a sequence of states $s_0, s_1, \ldots, s_n \in Q$ $(n \geq 0)$ such that there exist $w_1, w_2, \ldots, w_n \in \Sigma_\varepsilon$ such that

1) $w = w_1 w_2 \cdots w_n$,
2) $s_0 = q_0$, and
3) for all i, $1 \leq i \leq n$,

$$s_i \in \delta(s_{i-1}, w_i)$$

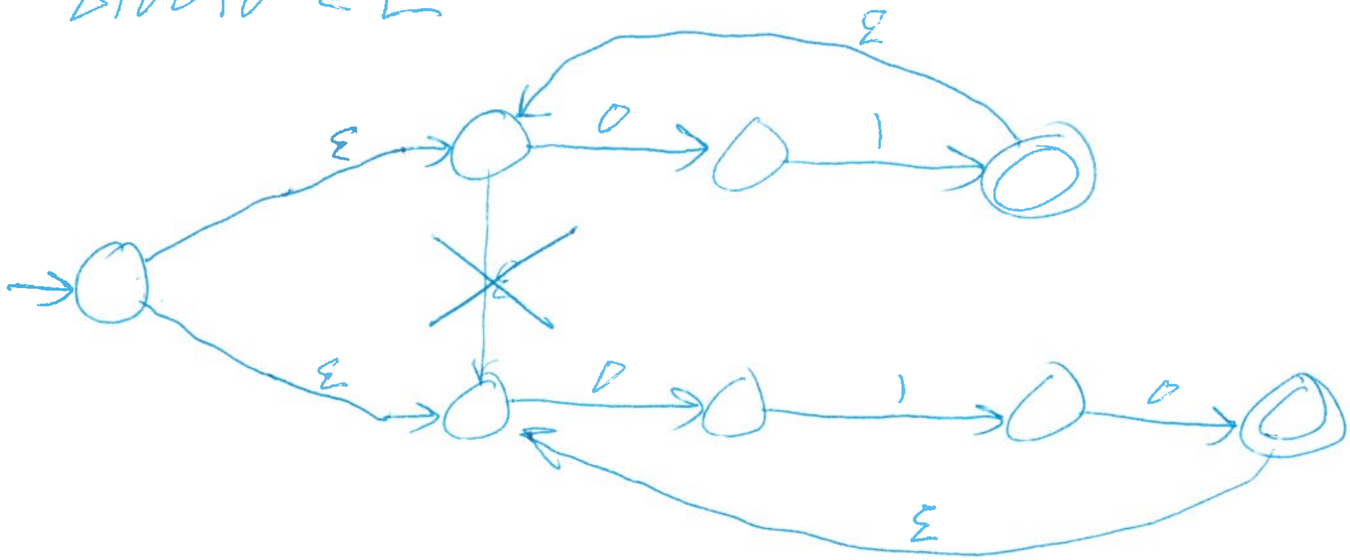
<u>End state</u>, <u>acceptance</u> are same as with an NFA.

<u>Example</u> where ε-moves come in handy: $\Sigma = \{0, 1\}$

$L := \{ w \in \Sigma^* :$ <u>either</u> w is 01 repeated one or more times <u>or</u> the ~~string~~ w is the string 010 repeated one or more times $\}$

$010101 \in L$ $\quad$ $01010 \notin L$

$010010 \in L$



Observe: Every NFA has an equivalent ε-NFA [same transition diagram]

Theorem: For every ε-NFA there is an equivalent (i.e., recognizing the same language) NFA with the same state set.

Proof: By construction. Let $A := \langle Q, \Sigma, \delta, q_0, F \rangle$ be an ε-NFA. We construct an equivalent NFA $B := \langle Q, \Sigma, \delta', q_0, F' \rangle$ in 3-stages:

Stage 1:

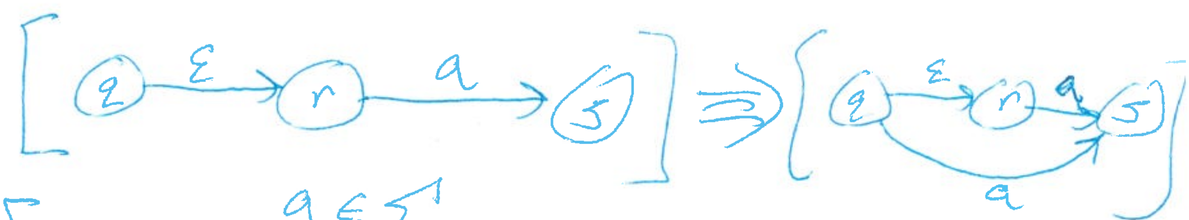
$F' := F$

while there exist states $q, r \in Q$

such that $q \notin F'$ and $r \in F'$
and $r \in \delta(q, \varepsilon)$:

$F' := F' \cup \{q\}$ // make q accepting

end-while

Stage 2: 

$\delta' := \delta$ $\quad a \in \Sigma$

while there exist states $q, r, s \in Q$ and $a \in \Sigma$
such that

$r \in \delta'(q, \varepsilon)$,

$s \in \delta'(r, a)$, and

$s \notin \delta'(q, a)$

$\delta'(q,a) := \delta'(q,a) \cup \{s\}$

// add $q \xrightarrow{a} s$ transition

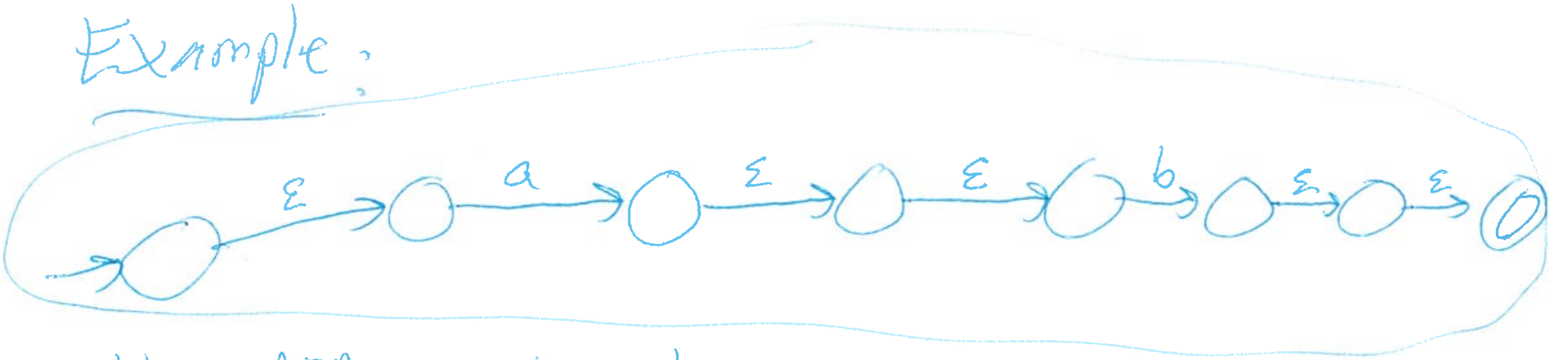
Stage 3: //Remove all ε-transitions

foreach $q \in Q$:

$\delta'(q, \varepsilon) := \emptyset$

~~Stage 1 example~~

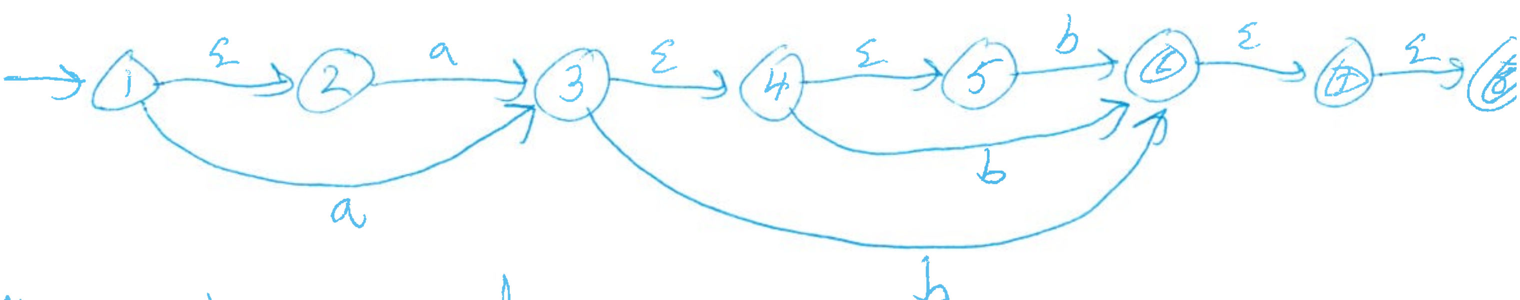
Example:



This ε-NFA accepts ab

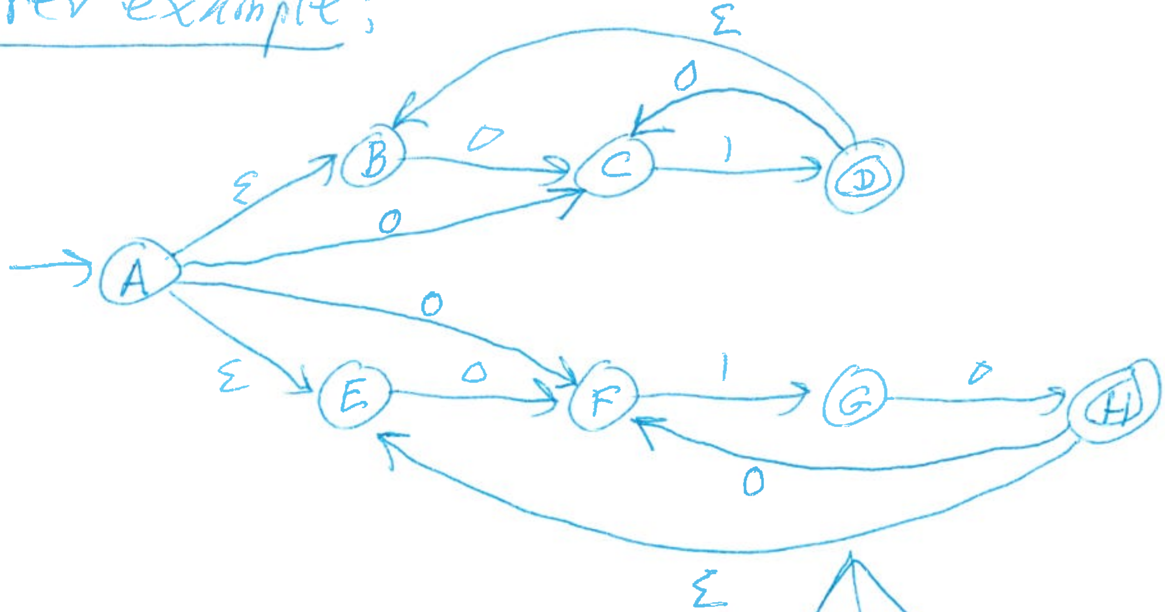
$\Downarrow$ stage 1

$\Downarrow$ stage 2



Now: ab accepted without using ε-moves:

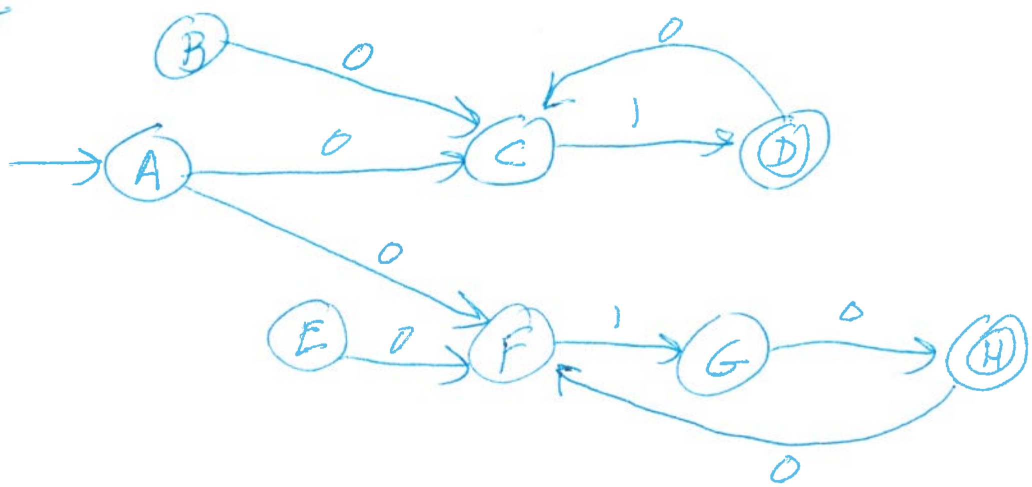
Prev example:



stage 1: nothing to do

stage 2: done

stage 3:



"Stage 4": Remove any states unreachable from the start state: