① Today: the editing problem (EP) is undecidable.

CSCE 355
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Def: An editing system (simplified universal grammar) is ~~as~~ a pair $\langle \Sigma, P \rangle$ where

- Σ is an alphabet
- P is a finite set of expressions of the form,

$$x \to y$$ } production, or an allowed edit

where $x, y \in \Sigma^*$.

Let $S = \langle \Sigma, P \rangle$ be an editing system and let w be a string, $w \in \Sigma^*$, and $w' \in \Sigma^*$. We say that w edits to w' in one step ($w \Rightarrow w'$) iff with respect to S

$$\underbrace{u \mid x \mid v}_{w} \qquad \underbrace{u \mid y \mid v}_{w'}$$

There exist $x \to y \in P$ and strings $u, v \in \Sigma^*$ such that $w = uxv$ and

$w' = uyv$

② Say that w edits to w' (in any number of steps) iff there exist $w_1, \ldots, w_n$ $(n \geq 0)$ such that

$$w \Rightarrow w_1 \Rightarrow \cdots \Rightarrow w_n = w'$$

(or $w = w'$). We write $w \Rightarrow^* w'$

Editing Problem (EP):

Given a pair $\langle S, w \rangle$ where $S = \langle \Sigma, P \rangle$ is an editing system and $w \in \Sigma^*$, is it the case that $w \Rightarrow^* \varepsilon$?

Ex: Consider $\Sigma = \{0,1\}$

$$P = \{10 \to 001,\ 10 \to 011,\ 000011111 \to \varepsilon\}$$

$w = 110$.

Theorem: EP is undecidable.

Proof: Suppose EP is decidable. Then we can decide the Halting Problem. $\therefore$ EP is undecidable.

Idea: Let $M = \langle Q, \Sigma, \Gamma, \delta, q_0, B, F \rangle$ be a TM, and let $w \in \Sigma^*$.

Build (in a computable way) an editing system ~~$S = \langle \Sigma, P \rangle$~~

③ $S = \langle \Delta, P \rangle$ and a string $x \in \Delta^*$ such that

M halts on input w, iff $x \Rightarrow^* \varepsilon$ via S.

so any decider for EP can answer whether M halts on input w.

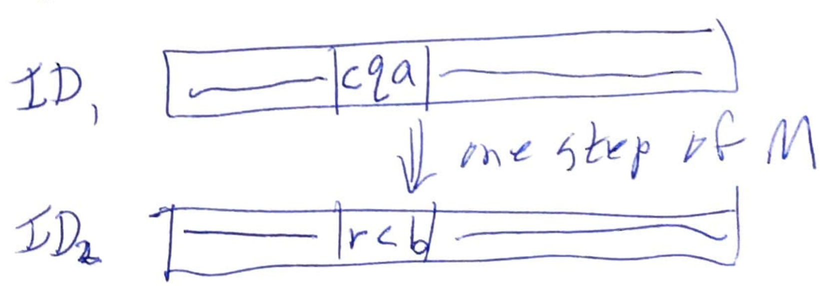
WLOG, $Q \cap \Gamma = \emptyset$ and there is a special state q_{halt} so that all undefined transitions transition to q_{halt} instead.

So M halts exactly when it enters state q_{halt}.

Construction of $S = \langle \Delta, P \rangle$ and $x \in \Delta^*$:

$\Delta := \Gamma \cup Q \cup \{\$\}$ where $\$ \notin \Gamma \cup Q$

$x := \$ q_0 w \$$



(4) P contains the following productions:

1. For every state $q \in Q$, $q \neq q_{halt}$ and tape symbol $a \in \Gamma$, ~~add to P~~ such that

$$\delta(q, a) = (r, b, R) \qquad \begin{pmatrix} r \in Q \\ b \in \Gamma \end{pmatrix}$$

add the ~~the~~ production

$$qa \to br \quad \text{to } P$$

2. For every $q \in Q$, $q \neq q_{halt}$ and $a, c \in \Gamma$ such that

$$\delta(q, a) = (r, b, L) \qquad \begin{pmatrix} \text{some} \\ r \in Q \\ b \in \Gamma \end{pmatrix}$$

Add

$$cqa \to rcb$$

to P.

3. For every $q \in Q$, $q \neq q_{halt}$ add these to P

$$\$q \to \$Bq \qquad \text{(pad left)}$$

$$q\$ \to qB\$ \qquad \text{(pad right)}$$

(5) For every $a \in \Gamma$, add

$$q_{halt}\,a \to q_{halt}$$

$$a\,q_{halt} \to q_{halt}$$

$$\$\,q_{halt}\,\$ \to \varepsilon$$

M halts on input w iff

$$\underbrace{\$\,q_0\,w\,\$}_{x} \Rightarrow^* \$ \leadsto q_{halt} \text{———} \$$$

$$\Rightarrow^* \$\,q_{halt}\,\$ \Rightarrow \varepsilon$$

Some restrictions of EP:

1. For every $x \to y$ in P, must have $|x| \geq |y|$ decidable: build a digraph
2. For every $x \to y$ in P, $|x| \leq 2$

undecidable: Replace above $\$q_{halt}\$ \to \varepsilon$

with $\$q_{halt} \to q_{halt}$

$q_{halt}\$ \to \varepsilon$

Replace $cqa \to rcb$ with

$qa \to \hat{r}b$ and $c\hat{r} \to rc$ $(\hat{r} \notin Q)$

3. For every $x \to y$ in P, $|x| \le 1$

ALL_{CFG}:

Given a CFG G with terminal alphabet Σ,
is $L(G) = \Sigma^*$?

<u>Thm</u>: ALL_{CFG} is undecidable.

<u>Proof Idea</u>: Assume ALL_{CFG} is decidable. Can decide, given a TM M and input string w, whether M halts on input w as follows:

1. Build (in a computable way) a PDA P which accepts all strings except ~~those~~ the string that encodes (in a particular way) the halting computation of M on w
2. Convert (in a computable way) a grammar G equivalent to the PDA P.