① Closure Properties of CFLs

CSCE 355
4/11/2022

Recall: w string — w^R is the reversal of w.

L language — $L^R := \{w^R : w \in L\}$

Prop: If L is a ~~CFL~~ CFL, then L^R is a CFL.

Proof idea: give rule to transform any CFG G into a CFG G^R such that $L(G^R) = L(G)^R$.

G^R has same nonterminals, terminals, & start symbol as G, and $A \to \alpha$ is a production of G iff $A \to \alpha^R$ is a production of G^R.

(Reverse all production bodies of G.)

Prove by induction on the length of derivations (of β) that β is a sentential form of G iff β^R is a sentential form of G^R. . . .

Prop: ~~Closure pp~~ The CFLs are closed under string homomorphic images.

Proof: HW exercise.

Turing machines (TMs)

(2) Recall: A TM is a tuple $\langle Q, \Sigma, \Gamma, \delta, q_0, B, F \rangle$

Q: state set; Σ: input alphabet; Γ: tape alphabet; q_0: start state; B: blank symbol; F: accept state set

$\Sigma \subseteq \Gamma$

$B \in \Gamma \setminus \Sigma$

$q_0 \in Q$ $\quad Q \cap \Gamma = \emptyset$

$F \subseteq Q$

δ is a partial function mapping pairs

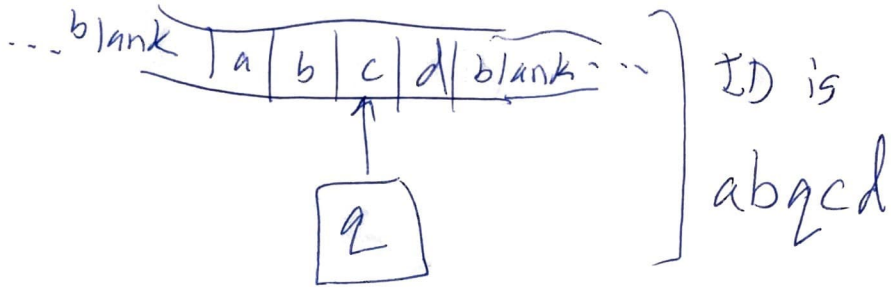
$(q, a) \in Q \times \Gamma$ to triples (r, b, D)

for $r \in Q$, $b \in \Gamma$, $D \in \{L, R\}$.

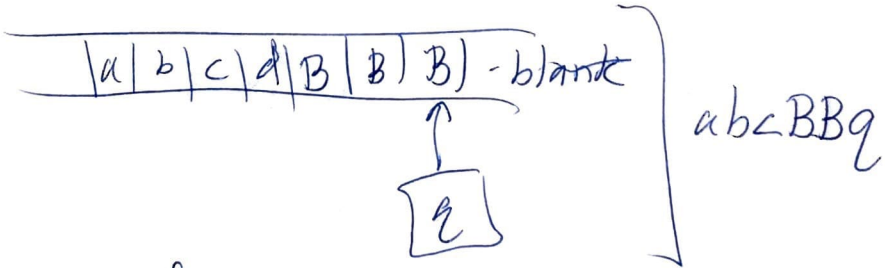
Given a TM $M = \langle \ldots \rangle$ as above, an ID of M (instantaneous description or configuration) is a string of the form wqx where $w, x \in \Gamma^*$ and $q \in Q$.

Intended meaning: M is in state q, the tape ~~const~~ consists of the symbols wx contiguously, surrounded on both sides by all

③ blanks (B) and the head is scanning the symbol just to the right of q. (1st symbol of x, or, if x is empty, a blank symbol).



Must include a contiguous portion of the tape big enough to include all nonblank symbols and the head position



Can always pad an ID with B's on either end: so:

$abq\langle$ same as $Babqc\mathcal{B}BB$

Let $ID_1 := wqx$ for some $q \in Q$, $w, x \in \Gamma^*$ arbitrary. By padding with B's, can

④ assume $x \neq \varepsilon$, $w \neq \varepsilon$. Let $a \in \Gamma$ be the first symbol of x: $x = ay$ (some $y \in \Gamma^*$)

If $\delta(q, a) = (r, b, R)$, then the successor of ID_1 is $ID_2 = wbry$

$wqay$

↓

$wbry$

$ID_1 \vdash ID_2$

If $\delta(q, a) = (r, b, L)$ (some $r \in Q$, $b \in \Gamma$)

and $w = uc$ for some $u \in \Gamma^*$ and $c \in \Gamma$

Then $ID_1 = ucqay$ and its successor is

$ID_2 = urcby$

Turing Machine exactly capture the notion of "computable algorithm"

↑

The Church-Turing thesis

(5) Evidence for the C-T thesis: lots of people have proposed many, many ~~de~~ different models ~~of~~ computation.
All are equivalent to TMs.

TM_add.txt — adds two numbers in binary
TM_compare.txt — compares two numbers in binary.
TM_copy.txt — copies a binary string.

Assume the C-T thesis from now on.
Use the C-T thesis to write algorithms at a high, informal level, argue the there ~~th~~ exists a TM that implements the algorithm.
Also assume that any finite object can be encoded as a string in a "reasonable" way (meaning, the essential basic operations on the object can be implemented computably on the encoded string.
Examples of finitely encodable objects:
natural numbers, (basic ops include, $+, *, -, \div$, $<, \leq$, etc.)

(6) integers,
finite lists of finite objects, (append, prepend, splice, compute length, i'th element)
strings over any alphabet, (encoded with the binary alphabet)

graphs, — enumerate vertices & edges, traverse edges (graph searches)
trees,

Given any finite object O, we'll use $\langle O \rangle$ to denote the (binary) string that is a reasonable encoding of O.

If $O_1, O_2, \ldots, O_n$ are finite objects, use $\langle O_1, O_2, \ldots, O_n \rangle$ to encode the list

DFAs
NFAs
Σ-NFAs

regexes
CFGs
PDAs
TMs

Thm (Turing 1930's): There exists a universal TM U, that is a TM U that, given any input of the form $\langle M, w \rangle$, where M is a TM and w is a string over M's input alphabet,

(7) and behaves exactly as M does on input w.

- If M accepts w, then U accepts $\langle M, w \rangle$
- If " rejects w, " " rejects $\langle M, w \rangle$
- " M loops on w, " " loops on $\langle M, w \rangle$.