CSCE 355
4/4/2022

PDA ⇝ CFG

$P \longmapsto G$ such that $L(G) = N(P)$

(restricted PDA)
<u>Def</u>: A <u>simplified PDA</u> is a PDA that has only two kinds of transitions:

$\delta(q, a, X)$ ~~$\delta(a, X)$~~ can contain outputs of the form

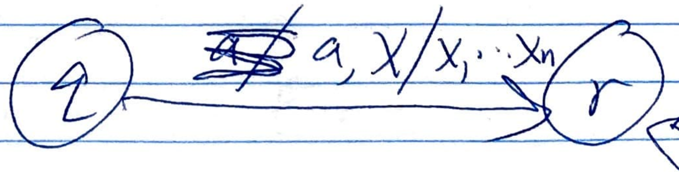
$q \in Q$ $\quad a \in \Sigma \cup \{\varepsilon\}$ $\quad X$: stack symbol

either (r, pop) (equiv to (r, ε)) — r: state

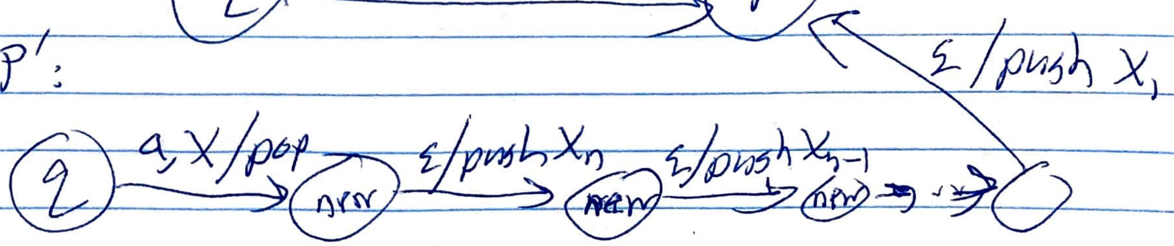
or $(r, \text{push } Y)$ (equiv to (r, YX))

<u>Prop</u>: For every PDA P there exists a restricted PDA P' such that $N(P') = N(P)$.

<u>Proof idea</u>: For an arbitrary transition of P:



In P':



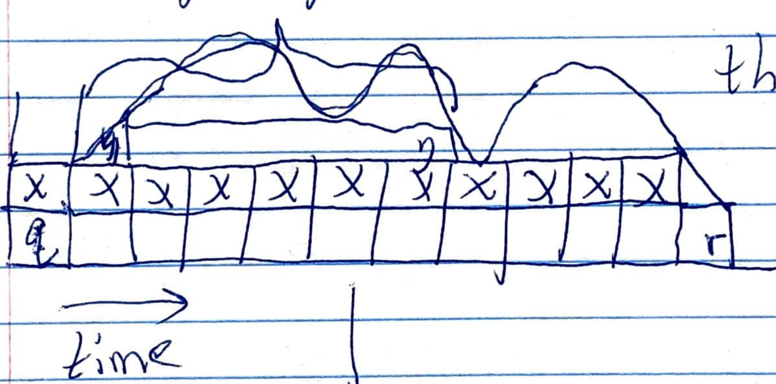
Must handle the case where X is the only thing on the stack by pushing a new bottom-stack marker before simulating P. //

- Suffices to convert a restricted PDA into a CFG.

Consider a PDA computation

$\cancel{(q_0, w)}\ (q_0, w, Z_0) \vdash \cdots \quad \vdash (\quad)$

A <u>segment</u> is a part of the computation whose net effect on the stack is to pop a single symbol. (Top of stack stays there throughout the segment until the last step.)



$\underbrace{[q X r]}_{\text{nonterminal}}$ derives all possible strings that could be read in a segment that starts in state q, ends in state r, and whose net effect is popping X off the stack.

- Start with a restricted PDA

$P = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0 \rangle$ (F is irrelevant)

③

We construct a grammar G as follows:

$$G = \langle V, \Sigma, S, P \rangle$$

where Σ is P's input alphabet

$$V = \{S\} \cup \{[qXr] : q, r \in Q, X \in \Gamma\}$$

With the following productions in P:

Add this prod

$S \rightarrow [q_0 Z_0 r]$ for every $r \in Q$

Then for every transition of the form

$$(r, \underline{pop}) \in \delta(q, a, X)$$

with $q, r \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $X \in \Gamma$

add ~~to~~ the production $[qXr] \rightarrow a$

[1-step segment].

For every transition of the form

$$(r, push\ Y) \in \delta(q, a, X)$$

for $q, r \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $X, Y \in \Gamma$

add the following production for every state $s \in Q$ and $t \in Q$

$$[qXs] \rightarrow \underline{a}\,\underline{[rYs]}\,\underline{[sXt]}$$

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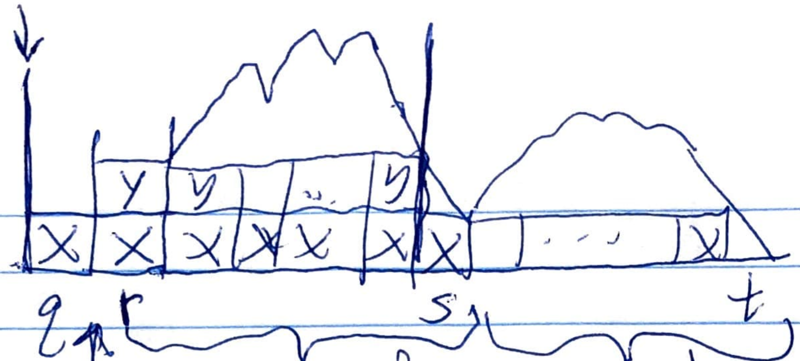
Idea:



read a on input | ~~w~~ read something derivable from $[r y s]$

read something derivable from $[s X t]$

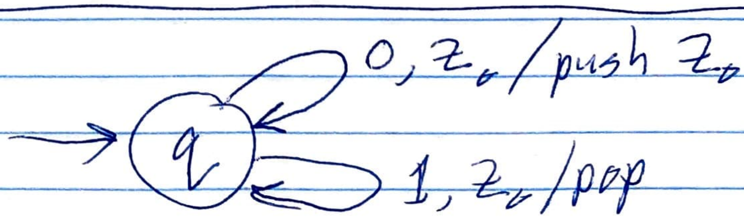
Invariant (proved by induction on the length of any input string (or segment) that a nonterminal

$[q X r] \Longrightarrow^* w$ $\quad (w \in \Sigma^*)$

iff there is a computation sequence of this form $\underbrace{(q, w, X\alpha) \vdash \underbrace{\cdots \vdash}_{X \text{ stays on the stack}} (r, \varepsilon, \alpha)}_{\text{single segment}}$

(for some $\alpha \in \Gamma^*$)

Example:



Grammar G

$S \to [q Z_0 q]$

$[q Z_0 q] \to 0 [q Z_0 q][q Z_0 q]$

$[q Z_0 q] \to 1$

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Let $A := [q Z_0 q]$

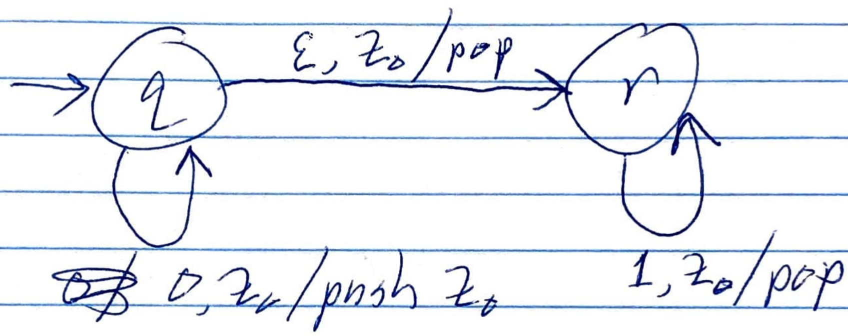
$\cancel{S \to A}$ make A the start ~~sta~~ symbol;

$A \to 1$ $\quad$ $A \to 0AA$

$A \to 0AA$ $\quad$ $A \to 1$

By induction on the length of a derivation, any string in $L(G)$ has one more 1's than 0's.

P:



$N(P) = \{0^n 1^n : n \geq 0\}$

$S \to [q Z_0 q] \mid [q Z_0 r]$

$[q Z_0 q] \to \cancel{0[q Z_0 q][q Z_0 q]} \mid 0[q Z_0 r][r Z_0 q]$

$\cancel{\mid 0[q Z_0 q][q Z_0 r]}$