# CSCE 355, Spring 2024, Assignment 3 <br> Due February 12, 2024 at 11:30pm 

1. For the $\epsilon$-NFA of textbook Exercise 2.5.2,

|  | $\epsilon$ | $a$ | $b$ | $c$ |
| ---: | :--- | :--- | :--- | :--- |
| $\rightarrow p$ | $\{q, r\}$ | $\emptyset$ | $\{q\}$ | $\{r\}$ |
| $q$ | $\emptyset$ | $\{p\}$ | $\{r\}$ | $\{p, q\}$ |
| $* r$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

find an equivalent NFA (without $\epsilon$-moves) using the method explained in class. This is also Method 2 described in the COURSE NOTES (link from the course homepage) in Section 10.4.
2. Do Exercise 2.5.3(a): Design an $\epsilon$-NFA for the following language: the set of all strings consisting of zero or more $a$ 's followed by zero or more $b$ 's, followed by zero or more $c$ 's. Try to use $\epsilon$-transitions to simplify your design.
3. Do Problem 2.3 (pp. 81-82). This illustrates a proof by string induction.
4. (a) Show that every regular language is recognized by an $\epsilon$-NFA where out of each state there is no more than one $\epsilon$-transition and no more than one non- $\epsilon$-transition (i.e., a transition on a symbol from the alphabet).
(b) Show that every regular language is recognized by an $\epsilon$-NFA where out of each state there is exactly one $\epsilon$-transition and exactly one non- $\epsilon$-transition (i.e., a transition on a symbol from the alphabet). (A solution to this part is obviously also a solution to the previous part.)
5. Do Exercise 3.1.1(b,c): Write regexes for the following languages:
b) The set of strings of 0 's and 1's whose tenth symbol from the right end is 1 .
c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.
6. (Optional) Do Exercises 3.1.2(b,c) and 3.1.3(a,b,c)
7. Write a regular expression for the language of strings over $\{a, b, c\}$ where no $a$ appears after any $b$ or $c$.
8. Do Exercise 3.2.3: Convert the following DFA to a regular expression, using the stateelimination technique of Section 3.2.2.

$$
\begin{array}{r||l|l} 
& 0 & 1 \\
\hline \hline \rightarrow * p & s & p \\
q & p & s \\
r & r & q \\
s & q & r
\end{array}
$$

9. Do Exercise 3.2.4(c): Convert the following regex to an $\epsilon$-NFA: $\mathbf{0 0}(\mathbf{0}+\mathbf{1})^{*}$.
10. Recall the DFA $D$ we constructed that accepts a binary string iff it has an odd number of 1's:

$$
\begin{array}{r||l|l} 
& 0 & 1 \\
\hline \hline \rightarrow A & A & B \\
* B & B & A
\end{array}
$$

(a) Convert $D$ into an equivalent clean $\epsilon$-NFA using the clean-up procedure in class (add a new start state, a new final state, and some $\epsilon$-transitions).
(b) Use the state elimination method to convert $D$ to a regular expression. Eliminate state $A$ first, then $B$.
11. Same exercise as before, except make $A$ the final state (so that $D$ accepts a string iff it has an even number of 1's).
12. (Optional) Recall the product DFA $P$ that counts an even number of zeros and an odd number of ones:

|  | 0 | 1 |
| ---: | :--- | :--- |
| $\rightarrow E E$ | $O E$ | $E O$ |
| $O E$ | $E E$ | $O O$ |
| $* E O$ | $O O$ | $E E$ |
| $O O$ | $E O$ | $O E$ |

Use the state elimination method to convert $P$ to a regular expression. (To control the complexity, you may wish to define names for intermediate regexes.)
13. Draw the transition diagram of an $\epsilon$-NFA equivalent to the regex $(a+b c)^{*} a a$. You may (but are not required to) contract $\epsilon$-transitions provided it is safe to do so.
14. Write a regular expression for the language of strings over $\{a, b, c\}$ where no $a$ appears after any $b$ or $c$.

