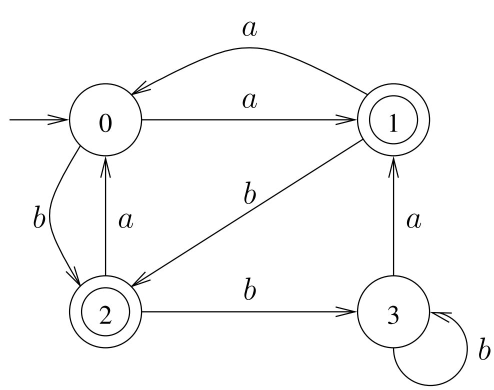
CSCE 355, Spring 2024, Assignment 2
Due January 24, 2024 at 11:30pm

NOTE the following definitions:

- The *binary alphabet* is the set $\{\texttt{0}, \texttt{1}\}$.
- A *binary string* is any string over the binary alphabet.
- If w is any string, then w^R (the *reversal* of w) is w written backwards, that is, comprising the symbols of w in reverse order.
- A string x is a *prefix* of a string y iff there exists a string z such that $y = xz$.
- A string x is a *suffix* of a string y iff there exists a string z such that $y = zx$.
- A string x is a *substring* of a string y if there exist strings w, z such that $y = wxz$.

1. Consider the following DFA:



   (a) For each of the strings below, say which state the DFA is in after reading the string, and say whether or not the DFA accepts the string.

   $aaa \quad bb \quad bbb \quad abab \quad bbbbbbbbbbbbbbbaaa \quad \varepsilon \quad aabbbbababbaaabbaabbababbbb$

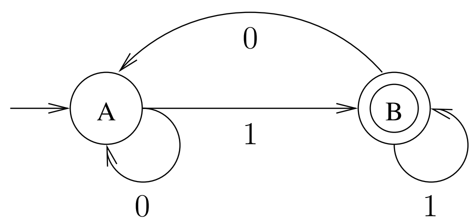
   (b) Give two different strings of length 4 that each make the DFA go from state 0 to state 1.

2. Draw a DFA with alphabet $\{0, 1\}$ that accepts a binary string x iff x has odd length, i.e., iff $|x|$ is odd.

3. Let A be the DFA given by the following tabular form:

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

(A accepts a binary string iff it represents a multiple of 3.) Recall the DFA described in class (here we'll call it B) that accepts a binary string iff the string ends with 1:



Recall the product construction from class. Draw the diagram for the product of A and B so the resulting DFA recognizes the language $L(A) \cap L(B)$.

4. Describe a DFA B that accepts a string over the alphabet $\{a, b, c\}$ iff its first and last symbols are different.

5. Consider the following two languages over the alphabet $\{a, b\}$:

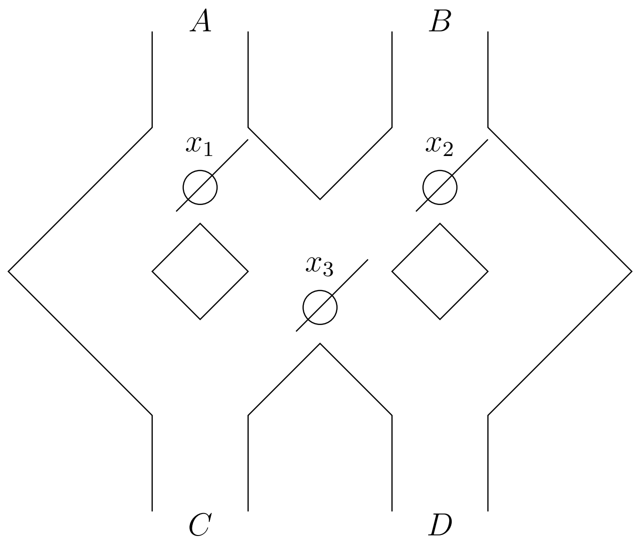
$$L_1 = \{w \mid w \text{ is either the empty string or ends with } b\} \,,$$
$$L_2 = \{w \mid \text{there is a } b \text{ followed by an } a \text{ somewhere in } w\} \,.$$

   (a) Draw a 2-state DFA recognizing L_1 and a 3-state DFA recognizing L_2.

   (b) Using your answer and the product construction, draw a DFA recognizing $L_1 \cap L_2$. Do *not* perform any optimizations (e.g., removing unreachable states or transitions, or merging indistinguishable states).

6. Give the transition diagram for a DFA over the alphabet $\Sigma = \{a, b, c\}$ that accepts a string w iff w contains ab as a substring but does not contain abb as a substring. What is the least number of states you need?

7. (Optional) This exercise is adapted from Exercise 2.2.1 on pages 52–53, which is formulated somewhat vaguely. Consider the marble-rolling toy (redrawn from Figure 2.8):



A marble is dropped at A or B. Levers x_1, x_2, and x_3 cause the marble to fall either to the left or to the right. whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Model this toy as a finite automaton. An input to the atomaton is a string over the alphabet $\{A, B\}$, which represents a sequence of marbles being dropped into the toy. The toy is initially in the configuration above before any marbles are dropped (so that the first ball will exit at C regardless of where it is dropped). Say that a sequence of marble drops is *accepted* exactly in the case that if one additional marble were to be dropped in, it would go out through D regardless of where it was dropped.