COMPLEXITY ABSTRACTS 2002. Vol XII

Abstract

This is a collection of one page abstracts of recent results of interest to the Complexity community. The purpose of this document is to spread this information, not to judge the truth or interest of the results therein.

TABLE OF CONTENTS

Power from Random Strings

Eric Allender, Rutgers Univesity (allender@cs.rutgers.edu)
Harry Burhman, CWI and University of Amsterdam (buhrman@cwi.nl)
Michal Koucký, Rutgers Univesity (mkoucky@paul.rutgers.edu)
Dieter van Melkebeek, University of Wisconsin (dieter@cs.wisc.edu)
Detlef Ronneburger, Rutgers Univesity (detlef@paul.rutgers.edu)

Abstract Number 02-1

We consider sets of strings with high Kolmogorov complexity, mainly in resource-bounded settings but also in the traditional recursion-theoretic sense. We present efficient reductions, showing that these sets are hard and complete for various complexity classes.
In particular, in addition to the usual Kolmogorov complexity measure K, we consider the time-bounded Kolmogorov complexity measure KT that was introduced by Allender [All01], as well as a space-bounded measure KS, and Levin's time-bounded Kolmogorov complexity Kt [Lev84]. Let $R_{\mathrm{K}}, R_{\mathrm{KT}}, R_{\mathrm{KS}}, R_{\mathrm{Kt}}$ be the sets of strings x having complexity at least $|x|/2$, according to each of these measures. Our main results are:

- R_{KS} and R_{Kt} are complete for PSPACE and EXP, respectively, under P/poly-truth-table reductions.
- EXP= $\mathrm{NP}^{R_{\mathrm{Kt}}}$.
- PSPACE= $\mathrm{ZPP}^{R_{\mathrm{KS}}} \subseteq \mathrm{P}^{R_{\mathrm{K}}}$.
- The Discrete Log is in $\mathrm{BPP}^{R_{\mathrm{KT}}}$.

The core idea to prove such a hardness result for a class $\mathcal{C}$ is to argue that the output of a pseudo random generator G_f, built from a complete function $f \in \mathcal{C}$, has low Kolmogorov complexity with respect to a measure μ. Thus R_μ can be used as a distiguisher for G_f which in turn yields an efficient routine relative to R_μ computing f. The hardness results for EXP and PSPACE rely on nonrelativizing proof techniques. Our techniques also allow us to show that all recursively-enumerable sets are reducible to R_{K} via P/poly-truth-table reductions. Our hardness result for PSPACE gives rise to fairly natural problems that are complete for PSPACE under $\leq_{\mathrm{T}}^{\mathrm{p}}$ reductions, but not under $\leq_{\mathrm{T}}^{\log}$ reductions. In spite of the EXP- and PSPACE-completeness of R_{Kt} and R_{KS}, it remains unknown if either of these problems is in logspace.

A full paper can be obtained by contacting any of the authors. It will also be available through ECCC (www.eccc.uni-trier.de/eccc/).

On the Enumerability of the Determinant and the Rank

Alina Beygelzimer and *Mitsunori Ogihara*,
Department of Computer Science, University of Rochester, Rochester, NY 14627, USA, ({beygel,ogihara}@cs.rochester.edu)

Abstract Number 02-2

We investigate the complexity of enumerative approximation of two elementary problems in linear algebra, computing the rank and the determinant of a matrix. In particular, we show that if there exists an enumerator that, given a matrix, outputs a list of constantly many numbers, one of which is guaranteed to be the rank of the matrix, then it can be determined in AC^0 (with oracle access to the enumerator) which of these numbers is the rank. Thus, for example, if the enumerator is an FL function, then the problem of computing the rank is in FL. The result holds for matrices over any commutative ring whose size grows at most polynomially with the size of the matrix. The existence of such an enumerator also implies a slightly stronger collapse of the exact counting logspace hierarchy.

For the determinant function we establish the following two results: (1) If the determinant is poly-enumerable in logspace, then it can be computed exactly in FL. (2) For any prime p, if computing the determinant modulo p is $(p-1)$-enumerable in FL, then computing the determinant modulo p can be done in FL.

These results give a new perspective on the approximability of many elementary linear algebra problems equivalent to computing the rank or the determinant. Due to the close connection between the determinant function and #L, and between the rank function and $C_=L$, our results might yield a better understanding of the power of counting in logspace and the relationships among the complexity classes sandwiched between NL and uniform TC^1.

A full paper is available as ECCC Report TR02-016.

Equivalence and Isomorphism for Boolean Constraint Satisfaction

Elmar Böhler, Theoretische Informatik, Universität Würzburg, Am Hubland, 97074 Würzburg, Germany. (boehler@informatik.uni-wuerzburg.de)
Edith Hemaspaandra, Department of Computer Science, Rochester Institute of Technology, Rochester, NY 14623 (eh@cs.rit.edu)
Steffen Reith, Rosengassse 10, 97070 Würzburg, Germany. (streit@streit.cc)
Heribert Vollmer, Abteilung Theoretische Informatik, Universität Hannover, Appelstr. 4, 30167 Hannover, Germany. (vollmer@cs.uni-hannover.de)

Abstract Number 02-3

A Boolean constraint satisfaction instance is a conjunction of constraint applications, where the allowed constraints are drawn from a fixed set $\mathcal{C}$ of Boolean functions. We consider the problem of determining whether two given constraint satisfaction instances are equivalent and prove a *Dichotomy Theorem* by showing that for all sets $\mathcal{C}$ of allowed constraints, this problem is either polynomial-time solvable or coNP-complete, and we give a simple criterion to determine which case holds.
A more general problem addressed in this paper is the isomorphism problem, the problem of determining whether there exists a renaming of the variables that makes two given constraint satisfaction instances equivalent in the above sense. We prove that this problem is coNP-hard if the corresponding equivalence problem is coNP-hard, and polynomial-time many-one reducible to the graph isomorphism problem in all other cases.

A full paper is available by email to the authors.

Competing Provers Yield Improved Karp–Lipton Collapse Results

Jin-Yi Cai, Venkat Chakaravarthy, Department of Computer Sciences, University of Wisconsin, Madison, WI 53706, USA. ({jyc,venkat}@cs.wisc.edu)
Lane Hemaspaandra, Mitsunori Ogihara, Department of Computer Science, University of Rochester, Rochester, NY 14627, USA. ({lane,mitsu}@cs.rochester.edu)

Abstract Number 02-4

We show that if a language A is self-reducible and has polynomial size circuits then $\mathrm{S}_2^A = \mathrm{S}_2$. Using this result, we also show that (i) if $\mathrm{NP} \subseteq (\mathrm{NP} \cap \mathrm{coNP})/\mathrm{poly}$ then the polynomial hierarchy PH collapses to $\mathrm{S}_2[\mathrm{NP} \cap \mathrm{coNP}]$; (ii) if $\mathrm{NP} \subseteq \mathrm{coNP}/\mathrm{poly}$ then $\mathrm{PH} = \mathrm{S}_2^{\mathrm{NP}}$. Under the same assumptions, the previous best known collapses were to $\mathrm{ZPP}^{\mathrm{NP}}$ and $\mathrm{ZPP}^{\mathrm{NP}^{\mathrm{NP}}}$. It is known that $\mathrm{S}_2 \subseteq \mathrm{ZPP}^{\mathrm{NP}}$. This result and its relativized version show that the new collapses indeed improve the known results. We also obtain new collapse consequences under the assumption that classes such as UP, FewP and CP have polynomial size circuits. Our techniques also apply to a wide range of classes having self-reducible complete sets.

A full paper is available by email to venkat@cs.wisc.edu.

New Results on Monotone Dualization and Generating Hypergraph Transversals

Thomas Eiter, Institut für Informationssysteme, Technische Universität Wien, Favoritenstraße 9-11, 1040 Wien, AUSTRIA, (`eiter@kr.tuwien.ac.at`),
Georg Gottlob, Institut für Informationssysteme, Technische Universität Wien, Favoritenstraße 9-11, 1040 Wien, AUSTRIA, (`gottlobr@dbai.tuwien.ac.at`),
Kazuhisa Makino, Division of Systems Science, Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, JAPAN, (`makino@sys.es.osaka-u.ac.jp`)

Abstract Number 02-5

We consider the problem of dualizing a monotone CNF (equivalently, computing all minimal transversals of a hypergraph), whose associated decision problem is a prominent open problem in NP-completeness. We present a number of new polynomial time resp. output-polynomial time results for significant cases, which largely advance the tractability frontier and improve on previous results. Furthermore, we show that duality of two monotone CNFs can be disproved with limited nondeterminism. More precisely, this is feasible in polynomial time with $O(\chi(n) \cdot \log n)$ suitably guessed bits, where $\chi(n)$ is given by $\chi(n)^{\chi(n)} = n$; note that $\chi(n) = o(\log n)$. This result sheds new light on the complexity of this important problem.

Keywords: Dualization, hypergraphs, transversal computation, output-polynomial algorithms, combinatorial enumeration, treewidth, hypergraph acyclicity, limited nondeterminism.

A full paper is available at CoRR http://arXiv.org/abs/cs.DS/0204009 and as Technical Report INFSYS RR-1843-02-05, Institut für Informationssysteme, Technische Universität Wien, April 2002. A short version of this paper appears in Proc. *ACM STOC 2002*.

PP-lowness and a simple definition of AWPP

Stephen A. Fenner, Computer Science and Engineering Department, University of South Carolina, Columbia, SC 29208 USA, (fenner@cse.sc.edu)

Abstract Number 02-6

We show that the counting class **AWPP** is more robust than previously thought. (**AWPP** was defined by Li, who showed that all **AWPP** languages are low for **PP**; Fortnow and Rogers showed that **BQP** $\subseteq$ **AWPP**.) Namely, we show that a language L is in **AWPP** if and only if there is a GapP function f and a polynomial p such that, for all x of length n,

$$\begin{aligned} x \in L &\Rightarrow 2/3 \leq f(x)/2^{p(n)} \leq 1, \\ x \notin L &\Rightarrow 0 \leq f(x)/2^{p(n)} \leq 1/3. \end{aligned}$$

Thus if L satisfies the conditions above, then L is **PP**-low. Our results also imply that **AWPP** is a Σ_2^0 definable class. We further show that **AWPP** $\subseteq$ **APP** (also defined by Li and known to contain only **PP**-low sets).
Our results are reminiscent of amplifying certainty in probabilistic computation.

A full paper is available by email to fenner@cse.sc.edu

Avg Case Complexity of a Simul. Comm Problem

Lance Fortnow, NEC (`fortnow@research.nj.nec.com`)
William Gasarch, University of Maryland (`gasarch@cs.umd.edu`)

Abstract Number 02-7

DEF: Let $f : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$. Alice has x, Bob has y, and they want to convey to Carol what $f(x,y)$ is. A *Simultaneous Message Protocol* P for f is a triple of functions $m_A : \{0,1\}^n \rightarrow \{0,1\}^*$, $m_B : \{0,1\}^n \rightarrow \{0,1\}^*$, and $g_C : \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}$ such that, for all $x, y \in \{0,1\}^n$,

$$g_C(m_A(x), m_B(y)) = f(x,y).$$

The *complexity of* P is $\max\{|m_1(x)|, |m_2(y)| \mid (x,y) \in \{0,1\}^n \times \{0,1\}^n\}$. We denote this SM_P. The *average case complexity of* P is

$$\sum_{(x,y)\in\{0,1\}^n\times\{0,1\}^n} \frac{1}{2^{2n}} \max\{|m_1(x)|, |m_2(y)|\}$$

We denote this AVGSM_P.
The *Simultaneous Message Complexity of* f (henceforth $\mathrm{SM}(f)$) is the min over all SM protcols P for f of SM_P. The *Average Simultaneous Message Complexity of* f (henceforth $\mathrm{AVGSM}(f)$) is the min over all SM protcols P for f of AVGSM_P.
KNOWN: Let M_f be the matrix of f. Let r be the number of distinct rows and c be the number of distinct columns. It is easy to show that $\mathrm{SM}(f) = \max\{\log r, \log c\}$.
For the rest of this note we view strings in $\{0,1\}^n$ as numbers between 0 and $2^n - 1$.
Consider the function $f : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$ defined by

$$f(x,y) = \begin{cases} 1 & \text{if } x \leq y; \\ 0 & \text{if } x > y. \end{cases}$$

Clearly $\mathrm{SM}(f) \leq n$ by the protocol where Alice sends x and Bob sends y. By the above KNOWN, $\mathrm{SM}(f) = n$.
We show, using Kolg complexity, that

$$\mathrm{AVGSM}(f) \geq n - O(1).$$

A full paper is available from gasarch@cs.umd.edu.

Recognizing When Heuristics Can Approximate Minimum Vertex Covers Is Complete for Parallel Access to NP

Edith Hemaspaandra, Department of Computer Science, Rochester Institute of Technology, Rochester, NY 14627, USA, (eh@cs.rit.edu)
Jörg Rothe, Abteilung für Informatik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, GERMANY. (rothe@cs.uni-duesseldorf.de)
Holger Spakowski, Abteilung für Informatik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, GERMANY. (spakowsk@cs.uni-duesseldorf.de)

Abstract Number 02-8

For both the edge deletion heuristic and the maximum-degree greedy heuristic, we study the problem of recognizing those graphs for which that heuristic can approximate the size of a minimum vertex cover within a constant factor of r, where r is a fixed rational number. Our main results are that these problems are complete for the class of problems solvable via parallel access to NP. To achieve these main results, we also show that the restriction of the vertex cover problem to those graphs for which either of these heuristics can find an optimal solution still is NP-hard.

To be presented at WG 2002. A full paper is available as Technical Report cs.CC/0110025, Computing Research Repository (CoRR), October 2001 at http://xxx.lanl.gov/abs/cs.CC/0110025.

Complete Problems for Dynamic Complexity Classes

William Hesse, Neil Immerman
Department of Computer Science, University of Massachusetts, Amherst MA 01003-9264
whesse@cs.umass.edu, immerman@cs.umass.edu

Abstract Number 02-9

Complexity theory has focused on static complexity classes, which measure the complexity of checking – upon presentation of an entire input – whether the input satisfies a certain property. For most modern applications of computers it is more appropriate to model the process as a dynamic one. There is a large set of data that is repeatedly modified by users over a period of time.

We present a new model of dynamic problems, namely parameterized sequences of regular languages. This results in a definition of Dyn-$\mathcal{C}$ for any (static) complexity class $\mathcal{C}$. These definitions are consistent with, but extend the corresponding definitions in [Patnaik-Immerman 97]. It is also natural from this point of view to define a dynamic reduction as a uniform sequence of bounded homomorphisms. Using our new formulation of dynamic computation, we present the first complete problems for dynamic complexity classes including Dyn-FO and Dyn-ThC^0. Such problems had long been sought.

The first problem that we show complete for Dyn-FO is a single step circuit value problem (SSCV). However, SSCV, is not complete via the weakest version of reductions that we describe. To overcome this problem we construct a first-order formula ζ which is universal for all first-order formulas. The formula ζ can emulate any first-order formula of syntactic depth k by being iterated k times. An equivalent way of stating this is that we have constructed a complete first-order quantifier block (QBC).

Using ζ, we define the problem CSSCV which is the same as SSCV except that it is initialized with the first-order definable AC^0 circuits corresponding to ζ. We show that CSSCV is complete for Dyn-FO via our weakest reductions: bounded first-order homomorphisms. We then construct other Dyn-FO-complete problems including a monotone version of SSCV, and an iterated Boolean matrix multiplication problem. Finally we present some complete problems for other classes including Dyn-ThC^0.

This paper will appear in LICS '02. It is available at http://www.cs.umass.edu/∼whesse

Reachability in Graphs with Bounded Independence Number

Arfst Nickelsen and Till Tantau, Technische Universität Berlin, Fakultät IV, 10623 Berlin, Germany (`nicke@cs.tu-berlin.de`)

Abstract Number 02-10

We study the reachability problem for finite directed graphs whose independence number is bounded by some constant k. This generalises the reachability problem for tournaments. We show that the problem is first-order definable for all k. In contrast, the reachability problem for many other types of finite graphs, including dags and trees, is not first-order definable. We show that first-order definability does not carry over to reachability in infinite graphs with bounded independence number. We prove that the number of strongly connected components in a graph with bounded independence number can be computed using TC^0-circuits, but cannot be computed using AC^0-circuits.

We also study the succinct version of the problem and show that it is Π_2^P-complete for all k. It remains Π_2^P-complete even for tournaments.

A full paper is available by email to {`nicke,tantau`}`@cs.tu-berlin.de`

Exact Complexity of Exact-Four-Colorability

Jörg Rothe, Abteilung für Informatik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, GERMANY. (rothe@cs.uni-duesseldorf.de)

Abstract Number 02-11

Let M_k be a given set that consists of k noncontiguous positive integers. Define Exact-M_k-Colorability to be the problem of determining whether $\chi(G)$, the chromatic number of a given graph G, equals one of the k elements of the set M_k exactly. In 1987, Wagner [Wag87] proved that Exact-M_k-Colorability is $\mathrm{BH}_{2k}(\mathrm{NP})$-complete, where $M_k = \{6k+1, 6k+3, \ldots, 8k-1\}$ and $\mathrm{BH}_{2k}(\mathrm{NP})$ is the $2k$th level of the boolean hierarchy over NP. In particular, for $k = 1$, it is DP-complete to determine whether $\chi(G) = 7$, where $\mathrm{DP} = \mathrm{BH}_2(\mathrm{NP})$. Wagner raised the question of how small the numbers in a k-element set M_k can be chosen such that Exact-M_k-Colorability still is $\mathrm{BH}_{2k}(\mathrm{NP})$-complete. In particular, for $k = 1$, he asked if it is DP-complete to determine whether $\chi(G) = 4$.

In this note, we solve this question of Wagner and determine the precise threshold $t \in \{4, 5, 6, 7\}$ for which the problem Exact-$\{t\}$-Colorability jumps from NP to DP-completeness: It is DP-complete to determine whether $\chi(G) = 4$, yet Exact-$\{3\}$-Colorability is in NP. More generally, for each $k \geq 1$, we show that Exact-M_k-Colorability is $\mathrm{BH}_{2k}(\mathrm{NP})$-complete for $M_k = \{3k+1, 3k+3, \ldots, 5k-1\}$.

References

[Wag87] K. Wagner. More complicated questions about maxima and minima, and some closures of NP. *Theoretical Computer Science*, 51:53–80, 1987.

To be presented at IFIP-TCS 2002. A full paper is available as Technical Report cs.CC/0109018, Computing Research Repository (CoRR), September 2001 at http://xxx.lanl.gov/abs/cs.CC/0109018.

Exact Complexity of the Winner Problem for Young Elections

Jörg Rothe, Abteilung für Informatik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, GERMANY, (`rothe@cs.uni-duesseldorf.de`)
Holger Spakowski, Abteilung für Informatik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, GERMANY, (`spakowsk@cs.uni-duesseldorf.de`)
Jörg Vogel, Institut für Informatik, Friedrich-Schiller-Universität Jena, 07740 Jena, GERMANY. (`vogel@minet.uni-jena.de`)

Abstract Number 02-12

In 1977, Young proposed a voting scheme that extends the Condorcet Principle based on the fewest possible number of voters whose removal yields a Condorcet winner. We prove that both the winner and the ranking problem for Young elections is complete for $\mathrm{P}_{||}^{\mathrm{NP}}$, the class of problems solvable in polynomial time by parallel access to NP. Analogous results for Lewis Carroll's 1876 voting scheme were recently established by Hemaspaandra et al. In contrast, we prove that the winner and ranking problems in Fishburn's homogeneous variant of Carroll's voting scheme can be solved efficiently by linear programming.

To be presented at IFIP-TCS 2002. A full paper is available as Technical Report cs.CC/0112021, Computing Research Repository (CoRR), December 2001 at http://xxx.lanl.gov/abs/cs.CC/0112021.