Structures Abstracts 1993. Vol III

Abstract

This is a collection of one page abstracts of recent results of interest to the Structural Complexity community. The purpose of this document is to spread this information, not to judge the truth or interest of the results therein.

List of Titles of Abstracts

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Relationships Among PL, #L, and the Determinant

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Abstract Number 93-1

Recent results by Toda, Vinay, Damm, and Valiant have shown that the complexity of the determinant is characterized by the complexity of counting the number of accepting computations of a nondeterministic logspace-bounded machine. (This class of functions is known as #L.) Using that characterization, we give a very simple proof of a theorem of Jung, showing that probabilistic logspace-bounded (PL) machines lose none of their computational power if they are restricted to run in polynomial time.
We also present new results comparing and contrasting the classes of functions reducible to PL, #L, and the determinant, using various notions of reducibility. In particular, we show that the sets AC^0 reducible to the determinant and to PL define logspace-analogues of the counting hierarchy, and we show that LogFew is in PL.

A full paper is available.

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Time-Space Tradeoffs for Undirected Graph Traversal

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Abstract Number 93-2

We prove time-space tradeoffs for traversing undirected graphs, using a variety of structured models that are all variants of Cook and Rackoff's "Jumping Automata for Graphs." Our strongest tradeoff is a quadratic lower bound on the product of time and space for graph traversal. For example, achieving linear time requires linear space, implying that depth-first search is optimal. Since our bound in fact applies to nondeterministic algorithms for *non*connectivity, it also implies that closure under complementation of nondeterministic space-bounded complexity classes is achieved only at the expense of increased time. To demonstrate that these structured models are realistic, we also investigate their power. In addition to admitting well known algorithms such as depth-first search and random walk, we show that one simple variant of this model is nearly as powerful as a Turing machine. Specifically, for general undirected graph problems, it can simulate a Turing machine with only a constant factor increase in space and a polynomial factor increase in time.

A full paper (Univ. of Washington Dept. of CS&E Technical Report 93-02-01, February, 1993, 46pp) is available by anonymous ftp from cs.washington.edu (128.95.1.4), file tr/1993/02/UW-CSE-93-02-01.PS.Z; by request from tr-request@cs.washington.edu; or from the authors.

This paper presents new results, as well as significantly strengthened versions of most of the results previously reported (in an extended abstract with the same title) in the *Proceedings of the 31st Annual IEEE Symposium on Foundations of Computer Science*, St. Louis, MO, Oct. 1990, pages 429–438.

Frequency Computations and Bounded Queries

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Abstract Number 93-3

Let A be a set and $m, n \in N$. A function f is in $freq(A, m, n)$ if $f(x_1, \ldots, x_n) \in \{0,1\}^n$ and agrees with $A(x_1) \cdots A(x_n)$ in at least m places. We investigate exactly how many queries are needed to compute a function in $freq(A, m, n)$ for several nonrecursive sets A. We obtain matching upper and lower bounds for several types of sets including the Halting set and any semirecursive set. For superterse sets we have matching upper and lower bounds of a curious nature: We need $\lceil \log K(n, n-m) \rceil$ queries, but cannot do it in $\lceil \log K(n, n-m) \rceil - 1$ queries to any set, where $K(n, r)$ is the number of spheres of radius r that are needed to cover $\{0,1\}^n$. This is curious since we do not know what $K(n, r)$ is, though we know it is recursive.

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The Θ-operator and the low hierarchy

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Abstract Number 93-4

Long and Sheu introduced a refinement of the low hierarchy based on the Θ-levels of the polynomial time hierarchy which gives a deeper sight of the internal structure of NP. We show $\Theta(\Theta_k^P) = \Theta_k^P$ and as a consequence, we get easily the Θ-lowness results given by Long and Sheu. Besides, we clarify the situation of the classes in $L_2^{P,\Delta}$ for which their membership to $L_2^{P,\Theta}$ was not clear. Concretely, we show $NP \cap E_T(TALLY) \subseteq L_2^{P,\Theta}$ if and only if the tally set that gives the Turing equivalence is in Θ_2^P.

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Bounded Queries and Approximation

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Abstract Number 93-5

In [ALM$^+$92] it was shown that (assuming P$\neq$NP) there exists an ϵ such that no polynomial time algorithm will, on input G, approximate $\omega(G)$ (the size of the largest clique in G) with ratio n^ϵ. We investigate the situation if a small number of queries to SAT are allowed. It is easy to see that if $\log\log_k n$ queries to SAT are allowed then CLIQUE can be approximated with ratio k. We show that there exists a constant c such that $\log\log_k n - c$ queries to SAT will not suffice (unless P=NP).

We have also investigated letting the ratio be (1) $\log n$, (2) $n^{\frac{1}{k}}$, and (3) other functions. In all cases we have upper and lower bounds that differ by the same additive constant c. The upper bounds can be improved by better algorithms that approximate clique. The lower bounds are near optimal in the sense that lower bounds that are better by more than 1 query can only be achieved by improving the results in [ALM$^+$92].

We have also investigated Graph Coloring and other problems and have obtained similar results.

References

[ALM$^+$92] S. Arora, C. Lund, R. Motwani, M. Sudan, and M. Szegedy. Proof verification and hardness of approximation problems. In *Proceedings of the IEEE Symposium on Foundations of Computer Science*, pages 14–23, 1992.

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A note on relativing random reductions from NP to $\oplus P$

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Abstract Number 93-6
Although it is well known that NP randomly reduces to $\oplus P$, several researchers have constructed relativized worlds where NP does not deterministically reduce to $\oplus P$. On the other hand, it is also known that for all oracle worlds, $FewP \subseteq \oplus P$, i.e, in all relativized settings, when the number of accepting paths of an NP machine is categorically bounded by a polynomial, a deterministic reduction to $\oplus P$ is possible. A natural question which arises is: to what extent can we increase the categorical bound on the number of accepting paths and still obtain a relativizing deterministic reduction to $\oplus P$? In this note we provide a definitive answer to this question. We show that if the categorical bound is Z then $\Theta(\log Z - c \log n)$ (for any constant c) is a tight bound on the number of random bits used by relativizing randomized reductions to $\oplus P$. Thus, when Z is polynomial then deterministic reductions are possible; for larger Z, no relativizing deterministic reductions are possible and we provide relativizing reductions which use the minimum number of random bits.

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An Exact Characterization of $\mathbf{FP}_{\|}^{\mathrm{NP}}$

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Abstract Number 93-7

$\mathrm{FP}_{\|}^{\mathrm{NP}}$ is the class of all functions computable in polynomial time with a polynomial number of queries to an NP oracle but the queries must be made in parallel in one round. It is a natural functional analogue of $\mathrm{P}_{\|}^{\mathrm{NP}}$, a famous complexity class of sets. Besides of this naturalness, much recent work has shown that $\mathrm{FP}_{\|}^{\mathrm{NP}}$ is a very important complexity class of functions. For example, the authors showed that $\mathrm{FP}_{\|}^{\mathrm{NP}}$ plus randomness captures the complexity of computing optimal solutions for unweighted NP optimization problems. These known results not only show the importance of $\mathrm{FP}_{\|}^{\mathrm{NP}}$ but also urge us to find natural and simple characterizations of $\mathrm{FP}_{\|}^{\mathrm{NP}}$. Unfortunately, no such characterization is known in the literature. This lackness often makes it difficult to imagine the computational power of $\mathrm{FP}_{\|}^{\mathrm{NP}}$ away from its primitive definition.

In this paper, we show a characterization of $\mathrm{FP}_{\|}^{\mathrm{NP}}$ via those functions that compute the supremum of solutions for NP-decidable binary relations. Let $R \subseteq \{0,1\}^* \times \{0,1\}^*$ be a binary relation. We define Sup_R to be the function which computes, given $x \in \{0,1\}^*$, the supremum of the set $\{s : \langle x, s\rangle \in R\}$. We first show that each function in $\mathrm{FP}_{\|}^{\mathrm{NP}}$ can be decomposed into a Sup_R function followed by polynomial-time computation, where R is a P-decidable binary relation. Based on this characterization, we then show many natural relations R for which Sup_R is complete for $\mathrm{FP}_{\|}^{\mathrm{NP}}$ under metric reductions. For example, we show that computing the edges contained in one or more Hamiltonian circuits of a given graph is complete for $\mathrm{FP}_{\|}^{\mathrm{NP}}$ under metric reductions. Of special interest are the proofs for the completeness results. To show the completeness results, we discover several structural properties of relations R in terms of which the $\mathrm{FP}_{\|}^{\mathrm{NP}}$-completeness of Sup_R can be deduced. This approach has two advantages. In the first place, it explicitly exhibits the factors in relations R that result in the $\mathrm{FP}_{\|}^{\mathrm{NP}}$-completeness of Sup_R. In the second place, it enables us to prove the completeness results for numerous relations R in a systematic way.

A full paper is available by email to Zhi-Zhong Chen (chen@info.mie-u.ac.jp).

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Sensitivity of Boolean Functions, Abstract Harmonic Analysis, and Circuit Complexity

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Abstract Number 93-8

We exploit the notion of sensitivity of Boolean functions to find complexity results. We first analyze the distribution of the average sensitivity over the set of all the n-ary Boolean functions, and we use this to prove that only a negligible fraction of all the Boolean functions belongs to AC^0 and to find a natural extension of the class AC^0. We then use harmonic analysis on the cube to study how the average sensitivity of a Boolean function propagates if the function corresponds, e.g., to an oracle available to compute another function. To do this, we analyze the sensitivity of the composition of Boolean functions. More precisely, we find the linear transformation relating the Fourier coefficients of a Boolean function f to the Fourier coefficients of a Boolean function $h = f(g_1, \dots, g_m)$, where the g_i's are Boolean functions. We give an exact evaluation of the norm of the matrix of the transformation. We also find upper bounds on the sensitivity of h with respect to the sensitivity of f and of the g_i's. This technique is particularly amenable to analyze relativized complexity classes, to find properties of sets that reduce to sets with given sensitivity, and to estimate the size of small depth circuits as a function of the sensitivity. We find the relation existing between a known complexity measure for symmetric functions and the average sensitivity. We use this relation to prove that the average sensitivity of symmetric functions in AC^0 decreases exponentially. This is, in the special case of symmetric functions, a substantial improvement over a recently proposed characterization. We also prove that a family of Boolean functions has exponentially decreasing sensitivity if and only if the associated set is almost sparse or co-sparse. This allows us to conclude that sets in AC^0 whose characteristic function is symmetric are almost sparse or co-sparse. We use this result to find a tight estimate to the number of symmetric functions in AC^0. We then show that sets with a given sensitivity can not be complete in NC^1 under certain special reductions. In particular we use the notion of sensitivity to find another proof of the fact that $MAJORITY$ is not complete for NC^1 under projections. We furthermore obtain a simple expression for the average sensitivity of functions computable by read-once formulas.

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Some Steps Towards the Conciliation of Approximation and Optimization Theory:
A New Suggested Approximation Measure and Preliminary Results

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Abstract Number 93-9

It is commonly known that a convenient way to express combinatorial problems is to write them down as integer linear programs. In this context the theory of the polynomial approximation of NP-complete problems seems to be inadequate to capture the nature of the treated problems, as it is not able to capture in a global manner the equivalence, with respect to their approximation, of all the ways of expressing a given problem in terms of an integer program. In fact, as we show, the way a combinatorial problem is expressed influences on the approximation results one can obtain for it.
We discuss some anomalies of the theory for the approximation of the NP-complete problems that seems to ignore the equivalence of all the ways an optimization problem can be expressed. We firstly define formally the notions of an optimization problem as well as the one of equivalence among such problems, equivalence including more particularly some intuitive equivalences as the several ways of expressing an optimization problem (for example, by translating or affining the objectif function) or yet some evident equivalences between maximization and minimization problems (for example the equivalence between *minimum vertex cover* and *maximum independent set*), that are not respected in the frame of the conventional approximation theory. We impose as principal axiom on the approximation the respect of this equivalence and we prove that the approximation ratio as a two-variable function can not verify this axiom. We define then a new approximation ratio, three-variable function, coherent to this equivalence and under the choice of the variables the new ratio is introduced by an axiomatic approach. This ratio has, between other, the advantage to uniformize the variation of its values in both minimization and maximization case. Finally, using the new ratio, we prove approximation results for a number of combinatorial problems.

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A Kernel for Function Definable Classes and Its Relation to Lowness

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Abstract Number 93-10

In 1991 Fenner, Fortnow, and Kurtz published an article about Gap-definable classes. As a generalisation we define $\mathcal{F}$-definability for an arbitrary class of functions $\mathcal{F}$. We show that every $\mathcal{F}$-definable class of languages already contains a kernel $\mathrm{U}\mathcal{F} \cap \text{co-U}\mathcal{F}$, and if $\mathcal{F}$ is closed under subtraction from one then the kernel itself is $\mathcal{F}$-definable.

Moreover, we investigate lowness of function classes, where we specialize on classes basing on the class P with oracles from an arbitrary class $\mathcal{C}$ which are transformed by an operator $\mathcal{O} \in \{\#, \mathrm{Gap}, \mathrm{span}, \mathrm{F}\}$ into function classes $\mathcal{O}\mathrm{P}^{\mathcal{C}}$. We prove for each of the four kinds of function classes that exactly the kernel $\mathrm{U}\mathcal{O}\mathrm{P}^{\mathcal{C}} \cap \text{co-U}\mathcal{O}\mathrm{P}^{\mathcal{C}}$ of the classes of sets, which can be defined via functions from $\mathcal{O}\mathrm{P}^{\mathcal{C}}$, is low. And the kernel is equal to the intersection of all $\mathcal{O}\mathrm{P}^{\mathcal{C}}$-definable language classes.

Next we restrict the allowed oracle classes to classes from the polynomial time counting hierarchy. We observe different ways to characterize classes in the counting hierarchy. Then we look at #-, Gap-, span-, and F-lowness. These cases fall into two groups: the low classes for function classes produced by #, Gap, and span can be expressed in a way that does not depend on P anymore, while the similar result for F-classes would imply the closure under complement for NP.

A full paper is available.

Graph Rigidity is Nonadaptively Checkable

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Abstract Number 93-11

In 1989, Blum and Kannan described a theoretical model of program checking. Roughly, a language L has a program checker if given a program $\mathcal{P}$ purporting to compute L and an instance x, the checker can determine that either $\mathcal{P}$ gives the correct answer to x or that $\mathcal{P}$ gives an incorrect answer on some input. As an example, Blum and Kannan exhibited a program checker for graph isomorphism.

Graph isomorphism served as the prototypical example for the design of program checkers. An important open question about the checker for graph isomorphism is whether the checker for graph isomorphism can be made nonadaptive, i.e. whether the checker can ask the program all the questions it needs to ask in one round and not make the questions depend on answers to previous questions. In the case that a program says that two graphs are not isomorphic, such a nonadaptive checker is known. However, the problem seems difficult when the program says that the two graphs are isomorphic.

While the solution of this problem looks difficult, we instead look to the problem of graph rigidity. Graph rigidity is the following problem: Given a graph $G = (V, E)$ is its automorphism group trivial, i.e. are there no automorphisms of the graph besides the identity automorphism? Besides the fact that it is a natural decision problem, the problem is also interesting for complexity-theoretic reasons. It is known (by an easy reduction) to be no harder than graph isomorphism, but the question of whether it is no easier than graph isomorphism has not been settled.

In this paper we show that there is a nonadaptive checker for the problem of graph rigidity. This implies that if there is a many-one reduction from graph isomorphism to graph rigidity, then there would be a nonadaptive checker for isomorphism as well.

A preliminary paper is available. Received May 1993.

Oracles, Proofs and Checking

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Abstract Number 93-12

We look at various oracle issues about interactive proofs and proof checking and show the following results:

- The PCP$(\log n, 1)$ = NP result does not relativize in a strong way.
- Any oracle that would collapse PCP$(\log n, 1)$ and EXP would imply P $\neq$ NP in the unrelativized world.
- We create a reasonable oracle access mechanism under which "local-checkability" relativizes.
- The IP = PSPACE result holds relative to algebraic extensions.

A preliminary paper is available. Received May 1993.

Frontiers of Feasible and Probabilistic Feasible Boolean Manipulation with Branching Programs

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Abstract Number 93-13

A central issue in the solution of many computer aided design problems is to find concise representations for circuit designs and their functional specification. Recently, a restricted type of branching programs (so–called OBDD's) proved to be extremely useful for representing Boolean functions for various CAD applications. Unfortunatelly, many circuits of practical interest provably require OBDD–representations of exponential size. In the paper we systematically study the question up to what extend more concise branching program based representations can be successfully used in symbolic Boolean manipulation, too. We prove, in very general settings:

- The frontier of efficient (deterministic) symbolic Boolean manipulation in terms of branching programs are free branching programs (so–called FBDD's).
- The frontier of efficient probabilistic manipulation in terms of branching programs are free MOD-2 branching programs (so–called MOD-2–FBDD's).

Since FBDD's and MOD-2-FBDD's are generally more – sometimes even exponentially more – succinct than OBDD–representations our results make accessible more succinct types of branching programs as data structures for practical purposes. (An FBDD–package is already available, a MOD-2–FBDD–packagas is in preparation.)
On the other side, we show that the solution of basic tasks in Boolean manipulation for less restricted types of branching programs becomes NP–hard.

(An extended abstract is included in the proceedings of STACS'93. A full paper is available as technical report.)

Efficient Analysis and Manipulation of OBDD's can be Extend to FBDD's

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Abstract Number 93-14

OBDD's are the state–of–the–art data structure for Boolean function manipulation since basic tasks of Boolean manipulation such as testing equivalence, satisfiability, or tautology, and performing single Boolean synthesis steps can be done efficiently. In the paper we propose to work with a more general data structure, so–called FBDD's since

- FBDD's are generally more (sometimes even exponentially more) succinct than OBDDs,
- FBDD's provide, similar as OBDD's, canonical representations of Boolean functions, and
- in terms of FBDD's basic tasks of Boolean manipulation can be performed similarly efficient as in terms of OBDD's.

The power of the FBDD–concept is demonstrated by showing that the verification of the benchmark circuit design for the hidden weighted bit function HWB proposed by Bryant can be carried out efficiently in terms of FBDD's while, for principial reasons, this is impossible in terms of OBDD's.

(The full paper is available as technical report.) Received May 1993.

The Classes Low for PP

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Abstract Number 93-15
This paper studies the classes which are low for PP. Recently Toda proved that the class WPP is low for PP. We show that there is a more general counting class which is low for PP and contains all low classes for PP we know. On the other hand, the exact low class of PP is not comparable with PP and $C_=P$ relative to some oracles.
We prove that every sparse co_$C_=P$ set is low for PP which improves a result of Köbler, Schöning, Toda and Torán that every sparse NP set is low for PP.
We also give a new characterization of PP.
We show that if class $\mathcal{C}$ is closed under join, complementation, and polynomial time conjunctive reducibility, then $\#P^{\mathcal{C}} = \# \cdot \mathcal{C}$ and $\mathrm{GapP}^{\mathcal{C}} = \mathrm{Gap} \cdot \mathcal{C}$. where $\# \cdot \mathcal{C}$ denotes the function class such that $f \in \# \cdot \mathcal{C}$ iff $\exists B \in \mathcal{C}$, a polynomial p such that

$$f(x) = ||\{y \in \{0,1\}^{p(|x|)} \mid (x,y) \in B\}||$$

$\mathrm{Gap} \cdot \mathcal{C}$ denotes the function class such that $g \in \mathrm{Gap} \cdot \mathcal{C}$ iff $\exists B \in \mathcal{C}$, a polynomial p such that

$$g(x) = ||\{y \in \{0,1\}^{p(|x|)} \mid (x,y) \in B\}|| - ||\{y \in \{0,1\}^{p(|x|)} \mid (x,y) \notin B\}||$$

This generalizes a number of results such as $\#P^{PH} = \# \cdot PH$ [Toda and Watanabe], $\oplus P^{\oplus P} = \oplus P$ [Papadimetrial and Zaches]

Lower Bounds, Hierarchies and Closure Properties for Sublogarithmic Space Classes[1]

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Abstract Number 93-16

Several logarithmic lower bounds are shown for space-bounded alternating Turing machines (ATMs) recognizing specific languages. This generalizes our previous results presented in [*Separating the lower levels of the sublogarithmic space hierarchy,* Proc. 10. STACS, Würzburg, 1993, 16-27]. The basic proof method is an extension of the $1^n \mapsto 1^{n+n!}$ technique to alternating TMs. Let llog denote the logarithmic function log iterated twice, and $\Sigma_k Space(S)$, $\Pi_k Space(S)$ be the complexity classes defined by S-space-bounded ATMs that alternate at most $k-1$ times and start in an existential, resp. universal state. Our first result shows that for each $k > 1$ the sets

$$\Sigma_k Space(\text{llog}) \setminus \Pi_k Space(o(\log)) \quad \text{and}$$
$$\Pi_k Space(\text{llog}) \setminus \Sigma_k Space(o(\log))$$

are both not empty. This implies that for each $S \in \Omega(\text{llog}) \cap o(\log)$ the classes

$$\begin{aligned} \Sigma_1 Space(S) &\subset \Sigma_2 Space(S) \subset \Sigma_3 Space(S) \subset \dots \\ &\subset \Sigma_k Space(S) \subset \Sigma_{k+1} Space(S) \subset \dots \end{aligned}$$

form an infinite hierarchy.

Furthermore, this separation is extended to space classes defined by ATMs with a nonconstant alternation bound A provided that the product $A \cdot S$ grows sublogarithmically.

Combining these lower bounds with our techniques developed to investigate closure properties of such classes we obtain that for any $S \in \Omega(\text{llog}) \cap o(\log)$ and all $k > 1$ $\Sigma_k Space(S)$ and $\Pi_k Space(S)$ are not closed under complementation and concatenation. Moreover, $\Sigma_k Space(S)$ is not closed under intersection, and $\Pi_k Space(S)$ is not closed under union.

In our previous paper we have also discussed the question whether for sublogarithmic bounds S the property that $\Pi_k Space(S)$ is the complement of $\Sigma_k Space(S)$ is fulfilled. This is a nontrivial problem since there is no obvious way how to detect infinite computation paths. Here, we generalize Sipser's result on halting space-bound computations for sublogarithimic space bounded deterministic TMs [*Halting space-bounded computations,* Theoret. Comput. Sci. 10, 1980, 335-338] to ATMs with a constant number of alternations that recognize bounded languages. For the class of Z-bounded languages with $Z \leq \exp S$ we obtain the equality

$$co\text{-}\Sigma_k Space(S) = \Pi_k Space(S) .$$

[1]A complete version of the paper is available.

Finally, we consider the space requirement for the recognition of nonregular context-free languages. Alt, Geffert and Mehlhorn have recently shown a logarithmic lower bound for nondeterministic TMs [*A lower bound for the nondeterministic space complexity of context-free recognition,* Inform. Process. Lett. 42, 1992, 25-27]. We improve this result obtaining the same lower bound for ATMs. Thus this last result shows that even alternations do not increase the power of sublogarithmic machines substantially.

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On the linear time hierarchy

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Abstract Number 93-17
The linear time hierarchy is the class of sets accepted by linear time bounded alternating Turing machines making at most a constant number of alternations. Similar time hierarchies can also be defined for other time constructible functions. We review and give new insights on the known result that separates level 0 and level 1 of the linear time hierarchy. We give a new surprisingly short proof of the recent result by Gupta that separates level 0 and level 2 of any hierarchy based on a time constructible function.

A full paper is available.

On bounded truth table reducibilities from counting classes to sparse sets

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Abstract Number 93-18

Ogiwara and Lozano showed that if a one word-decreasing self-reducible set A reduces to a sparse set via a bounded-truth table reduction, then A is in P. While simplifying their proof, we actually also improve the efficiency of the algorithm solving A, leading to various new corollaries about counting classes. For example, if there is a sparse set which is $\leq^{\mathrm{P}}_{\log\text{-}tt}$-hard for PP, then $\mathrm{PP} \subseteq \mathrm{DTIME}(2^{\mathrm{polylog}})$.

A full paper will be available in July.

On MOD and Generalised MOD Classes

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Abstract Number 93-19
The Mod_kP classes consist of sets accepted by polynomial-time nondeterministic Turing machines, where a string is said to be accepted if the number of accepting paths is not a multiple of k. Properties of such classes have been investigated in [Her90, BGH90]. Several "nice" properties of such classes have been known to hold only when k is prime.

In this paper we propose two modifications to the acceptance criteria for Mod-classes. Under one modification, a string is accepted iff the number of accepting paths is co-prime to k; this gives the classes $ModR_kP$. A further restriction on the number of accepting paths so that it is always either co-prime to k or an exact multiple of k gives the classes $ModRZ_kP$. When k is prime, both these classes coincide with the Mod_kP class.

We give normal form characterisations of the $ModR_kP$ and $ModRZ_kP$ classes, for *all* values of k. These normal forms allow us to characterise the $ModR_kP$ classes as the complements of Mod_kP classes, and to establish that the $ModRZ_kP$ classes are low for Mod_kP and $ModRZ_kP$. They also indicate that the $ModRZ_kP$ classes are the Mod generalisations of the class UP, where there is at most one accepting path for each input.

We show that if k and j are co-prime, then there are relativised worlds where $ModRZ_kP$ and $ModRZ_jP$ are distinct. Also, if k is not prime, then there is a relativised world where $ModRZ_kP$ and Mod_kP are distinct.

In [KT93], the class of languages $ModP$ is defined as a generalisation of the Mod_pP classes, for p prime. We consider such generalised classes under the new acceptance criteria described above. We show that $ModKP$ and $ModRP$ correspond exactly to the $\leq^p_{ctt}$ and $\leq^p_{dtt}$ closures, respectively, of $ModP$. This gives a neater alternative definition of the class Mod_*P defined in [KT93]. We also show that $ModRZP$ coincides exactly with $ModP$, giving an equivalent formulation of $ModP$ where the burden of promise is shifted from the FP function (it need not return a prime number) to the $\#P$ function.

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[KT93] J. Köbler and S. Toda, On the Power of Generalized MOD-Classes in: *Eighth Annual Conference on Structure in Complexity theory* (1993).

A full version of this paper will shortly be available as a technical report. Contact the first author at `meena@imsc.ernet.in`.

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A Note on Möbius Functions and the Communication Complexity of the Graph–Accessibility–Problem

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Abstract Number 93-20

An important motivation to investigate the graph–accessibility–problem (GAP_N) is its property to provide natural complete problems for logarithmic space–bounded complexity classes. Following an approach proposed by Hajnal, Maass, Turan, Lovasz, and Saks we investigate the communication complexity of GAP_N by means of some combinatorial invariants. In detail, we construct a lattice $\mathcal{L}_{GAP_N}$ whose number of Möbius elements gives a lower bound on the communication complexity of GAP_N .

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Approximability Preserving Reductions for NP-Complete Problems

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Abstract Number 93-21

We conceive some approximability preserving (continuous) reductions among a number of combinatorial problems. The preservation of the approximation ratio along these reductions establishes a kind of continuity between the involved problems, continuity resulting to a possible transfer (up to multiplication by the so called expansion of the reduction) of positive, negative or conditional results along chains of such reductions. We are, more particularly interested in continuity of reductions in the interior of hierarchies of combinatorial problems as well as, given that approximability preserving reductions are composable and transitive, we explore the continuity of reductions between members of different hierarchies. For a combinatorial problem, a hierarchy is obtained when we impose additional restrictions on its instances. We construct first a hierarchy for *set covering* problem and we explore the completeness of its members under reductions that preserve their approximation ratio (*continuous reductions*). We construct also a hierarchy for *hitting set* and we give continuity results for it. Moreover, we relate another NP-complete database query optimization problem, the *minimal join problem* to set covering hierarchy by proposing a reduction and by proving its continuity. After, we establish hierarchies for the *vertex covering by cliques* problem as well as for the *vertex covering* problem. For the members of each hierarchy we investigate the continuity of reductions relating them. Also, we propose such reductions connecting problems from distinct hierarchies. So, subclasses of NP-complete problems related by approximability preserving reductions are systematically constructed.

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On Set Size Comparison Problem

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Abstract Number 93-22

For any polynomial time computable binary predicate R and any constant $c > 0$, the following problem is called *Promised Set Size Comparison* (in short, Promised SSC) specified by R and c.

Instance: x_1 and x_2 such that $|x_1| = |x_2|$.

Promise: Either $\|Y(x_1)\| \geq c\|Y(x_2)\|$ or $\|Y(x_2)\| \geq c\|Y(x_1)\|$.
Where $Y(x) = \{y \in \Sigma^* : R(x,y)\}$.

Question: $\|Y(x_1)\| \geq \|Y(x_2)\|$?

Remark. We assume that $R(x,y)$ holds only if $|x| = |y|$.

Without promise, the problem is in general as hard as #P problems. (That is, all SSC are polynomial time solvable if and only if all #P functions are polynomial time computable.) On the other hand, it is shown in [1] that every Promised SSC is $\mathrm{BP} \cdot \Theta_2^{\mathrm{P}}$ solvable, and every Θ_2^{P} problem is $\leq_{\mathrm{m}}^{\mathrm{P}}$-reducible to some Promised SSC. We investigate the average case complexity of Promised SSC, and showed that if all distributed NP problems are polynomial time solvable in average (i.e., distNP $\subseteq$ aveP), then every Promised SSC is polynomial time solvable in average under any polynomial time computable distribution. As a corollary (to our proof), we can give a direct proof for the fact [2,3] that dist-NP $\subseteq$ ave-P $\Rightarrow$ dist-Θ_2^{P} $\subseteq$ ave-P.

A full paper (in Japanese) is available as Technical Report of IEICE COMP92-80 (1993).

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On The Computational Power of Neural Networks

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Abstract Number 93-23

This paper deals with finite networks which consist of interconnections of synchronously evolving processors. Each processor updates its state by applying a "sigmoidal" scalar nonlinearity to a linear combination of the previous states of all units. The use of sigmoids allows potentially infinite power, providing an appealing model of analog computation.
We prove that one may simulate all Turing Machines by rational nets. In particular, one can do this in linear time, and there is a net made up of about 1,000 processors which computes a universal partial-recursive function. Products (high order nets) are not required, contrary to what had been stated in the literature. Furthermore, we assert a similar theorem about non-deterministic Turing Machines. Consequences for undecidability and complexity issues about nets are discussed too.

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Analog Computation Via Neural Networks

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Abstract Number 93-24

We pursue a particular approach to analog computation, based on dynamical systems of the type used in neural networks research.
Our systems have a fixed structure, invariant in time, corresponding to an unchanging number of "neurons". If allowed exponential time for computation, they turn out to have unbounded power. However, under polynomial-time constraints there are limits on their capabilities, though being more powerful than Turing Machines. (A similar but more restricted model was shown to be polynomial-time equivalent to classical digital computation in our previous work. Moreover, there is a precise correspondence between nets and standard non-uniform circuits with equivalent resources, and as a consequence one has lower bound constraints on what they can compute. This relationship is perhaps surprising since our analog devices do not change in any manner with input size.
We note that these networks are not likely to solve polynomially NP-hard problems, as the equality "P = NP" in our model implies the almost complete collapse of the standard polynomial hierarchy.
In contrast to classical computational models, the models studied here exhibit at least some robustness with respect to noise and implementation errors.

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On Sets Bounded Truth-Table Reducible to P-selective Sets

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Abstract Number 93-25
For some subclass $\mathcal{R}$ of P/poly and for some uniform complexity class $\mathcal{C}$, we have been able to prove that $\mathrm{NP} \subseteq \mathcal{R} \implies \mathrm{NP} \subseteq \mathcal{C}$. Note that $\mathrm{P/poly} = \mathrm{R}^{\mathrm{P}}_{\mathrm{T}}(\mathrm{SPARSE})$, where $\mathrm{R}^{\mathrm{P}}_{\mathrm{T}}(\mathrm{SPARSE})$ is the class of sets polynomial time Turing reducible to some sparse set. Thus, we obtain subclasses of P/poly by restricting the type of reduciblity to sparse sets. The following is an example for the kind of results we are interested in.

$$\mathrm{NP} \subseteq \mathrm{R}^{\mathrm{P}}_{\mathrm{btt}}(\mathrm{SPARSE}) \implies \mathrm{NP} \subseteq \mathrm{P},$$

where $\mathrm{R}^{\mathrm{P}}_{\mathrm{btt}}(\mathrm{SPARSE})$ is the class of sets bounded truth-table reducible to some sparse set. (Due to the limited space, we omit the references to the previous results mentioned here.) Also, we have $\mathrm{P/poly} = \mathrm{R}^{\mathrm{P}}_{\mathrm{T}}(\mathrm{SELECT})$, where $\mathrm{R}^{\mathrm{P}}_{\mathrm{T}}(\mathrm{SELECT})$ is the class of sets polynomial time Turing reducible to some P-selective set. Thus, by restricting the type of reduciblity to P-selective sets, we get again subclasses of P/poly. Here, it is known that

$$\mathrm{NP} \subseteq \mathrm{R}^{\mathrm{P}}_{1\text{-tt}}(\mathrm{SELECT}) \implies \mathrm{NP} \subseteq \mathrm{P} \text{ and}$$
$$\mathrm{NP} \subseteq \mathrm{R}^{\mathrm{P}}_{\mathrm{tt}}(\mathrm{SELECT}) \implies \mathrm{NP} \subseteq \mathrm{R}.$$

In this paper, we prove the following result.

$$\mathrm{NP} \subseteq \mathrm{R}^{\mathrm{P}}_{\mathrm{btt}}(\mathrm{SELECT}) \implies \mathrm{NP} \subseteq \mathrm{DTIME}(2^{n^{O(1/\sqrt{\log n})}}).$$

The result is extended to some unbounded reducibilities such as $\leq^{\mathrm{P}}_{(\log n)^{O(1)}}$-reducibility.

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On Closure Properties of GapP

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Abstract Number 93-26

We study the closure properties of the function classes GapP and GapP_+, where GapP_+ are the positive functions in GapP. We characterize the property of GapP_+ being closed under decrement and of GapP being closed under maximum, minimum, or median by seemingly implausible collapses among complexity classes; thereby giving evidence that these function classes don't have these closure properties.
We show a similar result concerning operations we call *bit cancellation* and *bit insertion*: Given a function $f \in \mathrm{GapP}_+$ and a polynomial-time computable function κ. Then we ask whether the function $f^*(x)$ that is obtained from $f(x)$ by canceling the $\kappa(x)$-th bit in the binary representation of $f(x)$, or whether the function $f^+(x)$ that is obtained from $f(x)$ by inserting a bit at position $\kappa(x)$ in the binary representation of $f(x)$, is also in GapP_+. These operations can be seen as generalizations of division and multiplication. We give necessary conditions and sufficient conditions for GapP_+ being closed under bit cancellation and bit insertion, respectively.

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On Different Completeness Notions for Function Classes

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Abstract Number 93-27
Krentel defined a function f to be metrically reducible to function h (in symbols: $f \leq_{met}^{\mathrm{FP}} h$) if and only if there exist two functions $g, g' \in \mathrm{FP}$ such that for all x, $f(x) = g'(x, h(g(x)))$. We define f to be functionally many-one reducible to h (in symbols: $f \leq_m^{\mathrm{FP}} h$) if and only if there exists a function $g \in \mathrm{FP}$ such that for all x, $f(x) = h(g(x))$. We consider these notions in the context of the median classes $\mathrm{M\overline{e}dP}$, MedP, and MidP (see the abstract *The complexity of finding middle elements* in this volume). We show that all three classes share the same hard problems under $\leq_{met}^{\mathrm{FP}}$ which are exactly those problems hard for $\mathrm{FP}^{\#\mathrm{P}}$ under $\leq_{met}^{\mathrm{FP}}$. Thus, all problems considered by Toda (The Complexity of Finding Medians, FOCS 1990), related to the well known NP-complete decision problems satisfiability, Hamiltonian cycle, knapsack, and so on, are hard for all three classes under $\leq_{met}^{\mathrm{FP}}$. On the other hand, we show that these problems and slight modifications of them are *complete* under $\leq_m^{\mathrm{FP}}$ in different of the mentioned median classes. Additionally, since the considered classes are closed under $\leq_m^{\mathrm{FP}}$, we conclude that if any problem complete for MedP under $\leq_m^{\mathrm{FP}}$ is in $\mathrm{M\overline{e}dP}$ or MidP, then the counting hierarchy collapses; and if any problem complete for MidP under $\leq_m^{\mathrm{FP}}$ is in $\mathrm{M\overline{e}dP}$ or MedP, then the polynomial time hierarchy is in PP. Thus the assumption that the median classes share the same hard problems under $\leq_m^{\mathrm{FP}}$ has the same unlikely consequences.

A paper is under preparation.

The Complexity of Finding Middle Elements

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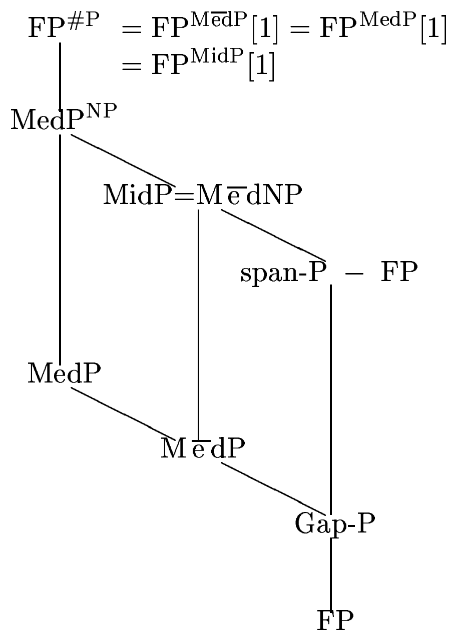
Abstract Number 93-28

Seinosuke Toda (The Complexity of Finding Medians, FOCS 1990) introduced the class MidP of functions that yield the middle element in the *set* of output values over all paths of nondeterministic polynomial time Turing machines. We define two related classes: MedP consists of those functions that yield the middle element in the ordered *sequence* of output values of nondeterministic polynomial time Turing machines (i.e. we take into account that elements may occur with multiplicites greater than one). $\mathrm{M\overline{e}dP}$ consists of those functions that yield the middle element of all accepting paths (in some resonable encoding) of nondeterministic polynomial time Turing machines. We establish the inclusionships shown in the figure between these classes, the relativized classes $\mathrm{M\overline{e}dNP}$ and $\mathrm{MedP}^{\mathrm{NP}}$, and the well-known classes #P, Gap-P, and span-P.



Moreover, we apply the operators C, U, and X (which transform the class #P into the classes PP, UP, and XP, respectively) to all of the above classes of functions and determine the resulting classes of sets. This enables us to show that (unless PH $\subseteq$ PP or the counting hierarchy collapses to P^{PP}) all inclusions given in the above figure are strict.

A full paper is available.

A Note on Set Bit Enumeration

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Abstract Number 93-29
An m-set-bit-enumerator for a function f is a source of information that, in response the query: "How many set bits ('1's) are there in the binary representation of $f(x)$?", computes a list of m numbers, one of which is the correct value. This paper extends previous results, for optimization functions, on the hardness of set-bit-enumeration, to the class #P. Out main theorem states:

> If polynomial time computable $n^{\frac{1}{2}-\epsilon}$-set-bit-enumerators exist for all #P functions, then $\#\mathrm{P} = \mathrm{PF}$.

A full paper is not yet available.

On the Amount of Nondeterminism and the Power of Verifying

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Abstract Number 93-30
The relationship between nondeterminism and other computational resources is studied based on a special interactive-proof system model *GC*. Let $s(n)$ be a function and $\mathcal{C}$ be a complexity class. Define $GC(s(n),\mathcal{C})$ to be the class of languages that are accepted by verifiers in $\mathcal{C}$ that can make an extra $O(s(n))$ amount of nondeterminism. Our main results are (1) A systematic technique is developed to show that for many functions $s(n)$ and for many complexity classes $\mathcal{C}$, the class $GC(s(n),\mathcal{C})$ has natural complete languages; (2) The class Π_h^0 of languages accepted by $\log$-time alternating Turing machines making h alternations is precisely the class of languages accepted by uniform families of circuits of depth h; (3) The classes $GC(s(n),\Pi_h^0)$, $h \geq 1$, characterize precisely the fixed-parameter intractability of *NP*-hard optimization problems. In particular, the $(2h)$th level $W[2h]$ of W-hierarchy introduced by Downey and Fellows collapses if and only if $GC(s(n),\Pi_{2h}^0) \subseteq P$ for some $s(n) = \omega(\log n)$.

A Preliminary version of this paper is to appear in the *Proceedings of Mathematical Foundations of Computer Science Symposium, Gdansk, Poland, 1993.* A full paper is under preparation.

On Fixed-Parameter Tractability and Approximability of *NP*-hard Optimization Problems

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Abstract Number 93-31

Fixed-parameter tractability and approximability of *NP*-hard optimization problems are studied based on a model $GC(s(n), \Pi_k^L)$. Our main results are (1) a class of *NP*-hard optimization problems, including Dominating-Set and Zero-One Integer-Programming, are fixed-parameter tractable if and only if $GC(s(n), \Pi_2^L) \subseteq P$ for some $s(n) \in \omega(\log n)$; (2) Most approximable *NP*-hard optimization problems are fixed-parameter tractable. In particular, the class *MAX NP* is fixed-parameter tractable; (3) a class of optimization problems do not have fully polynomial time approximation scheme unless $GC(s(n), \Pi_k^L) \subseteq P$ for some $s(n) \in \omega(\log n)$ and for some $k > 1$; and (4) every fixed-parameter tractable optimization problem can be approximated in polynomial time to a non-trivial ratio.

A Preliminary version of this paper is to appear in the *Proceedings of the 2nd Israel Symposium on Theory of Computing and Systems, Natanya, Israel, 1993.* A full paper is under preparation.

Random Debators and the Hardness of Approximating Stochastic Functions

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Abstract Number 93-32

A *probabilistically checkable debate system* (PCDS) for a language L consists of a probabilistic polynomial-time verifier V and a debate between Player 1, who claims that the input x is in L, and Player 0, who claims that the input x is not in L. It is known that there is a PCDS for L in which V flips $O(\log n)$ coins and reads $O(1)$ bits of the debate if and only if L is in PSPACE (cf. [Condon *et al.*, Proc. 25th ACM Symposium on Theory of Computing, 1993]). In this paper, we restrict attention to RPCDS's, which are PCDS's in which Player 0 follows a very simple strategy: Whenever it is his turn, he chooses uniformly at random from the set of legal moves. We prove the following result, which is best possible: L has an RPCDS in which the verifier flips $O(\log n)$ coins and reads $O(1)$ bits of the debate if and only if L is in PSPACE.

This new characterization of PSPACE is used to show that certain *stochastic* PSPACE-hard functions are as hard to approximate closely as they are to compute exactly. Examples of such functions include optimization versions of Dynamic Graph Reliability, Stochastic Satisfiability, Dynamic Markov Process, Stochastic Coloring, Stochastic Generalized Geography, and other "games against nature" of the type introduced in [Papadimitriou, J. Comput. System Sci., 31 (1985), pp. 288–301].

An extended abstract is available.

On the Correlation of Symmetric Functions

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Abstract Number 93-33

The correlation between two Boolean functions of n inputs is defined as the number of times the functions agree minus the number of times they disagree, all divided by 2^n. In this paper we compute, in closed from, the correlation between any two *symmetric, periodic* Boolean functions. An important corollary of our main result is that if one of the functions has a small, odd period and the other is the parity function, the correlation is exponentially small in n. This results in an improvement of a result of Smolensky restricted to symmetric Boolean functions: If q is odd, the fraction of agreement between parity and a circuit consisting of a Mod_q gate over AND-gates of small fan-in is $\frac{1}{2} + 2^{-n^{\Omega(1)}}$, if the function computed by the sum of the AND-gates is symmetric. In addition we find that for a large class of symmetric functions the correlation with parity is *identically* zero for infinitely many n. We characterize exactly those symmetric Boolean functions having this property.

A full paper is available. Received May 1993.