Homework 6: Radiometric Dating

The two most common examples of exponential functions are radioactive decay and compound interest. These differ mostly in whether the exponent is positive or negative. In the case of compound interest, the value of a savings or other account under compound interest is

$$P = P_0 e^{rt}$$

where P_0 is the initial principal balance, r is the interest rate, and t is the time since the investment began. If one were to do the more sophisiticated math for the loan amortization program, one would approximate the actual amortization schedule with a continuous model using the above equation.

Radioactive decay is the same equation but with a negative exponent. If we start with N_0 atoms of some radioactive isotope, then after t units of time we will have

$$N = N_0 e^{-\lambda t}$$

atoms remaining. Generally, this is all given in terms of the half-life of the isotope, the length of time it takes for N_0 initial atoms of the isotope to decay into $N_0/2$ atoms of the isotope. We set

$$N_0/2 = N_0 e^{-\lambda T}$$

and solve:

$$1/2 = e^{-\lambda T}$$

$$-\ln 2 = -\lambda T$$

$$T = \ln 2_0/\lambda$$

and the the half-life of the isotope is T.

Some sample half-lives are as follows.

Much of the archeological dating that is done with radiometrics is done with carbon-14 dating. Carbon-14 has a half life of 5730 years, which makes it suitable for dating artifacts that contain carbon in the range of a couple of thousand to a couple of tens of thousands of years ago.

Some geologic dating is done with potassium-argon dating. Potassium-40 decays to argon-40 about 11% of the time. Argon is inert, and argon-40 is produced only from potassium-40, and it can be trapped in some mineraals like mica, making it possible to extract the quantity of argon-40 and work

backward to determine how long the rock has been undergoing decay. The half life of potassium-40 is $1.26 \cdot 10^9$ years, making it useful for dating things from some numbers of millions of years to a few billion years old. (Note: I get this from the web. There are several slightly different numbers given, but they are all about this number.)

For even older things, one can use uranium-thorium-lead decay. Uranium-238 has a half life of 4.5 billion years. (I know not much more about this; what I know about carbon-14 and potassium-argon dating comes in part from having worked one summer in a geophysics lab.)

0.1 Your Assignment, due Friday midnight eastern time 7 March 2008 to the departmental dropbox

Write a program to print a "decay amortization schedule" for radioactive decay. The input to the program should be a `String` that is the name of the isotope, an initial number N_0 of atoms of the isotope, and the half-life in years. Your program should print out a table of the number of atoms left in time intervals of 1/20-th of the half-life. For example, if you input the 5730 half life for carbon-14, then you should get 21 lines of output for the times 0, 286.5, 573, 859.5, ..., 5730 years after the start time.

You should print the `String` value for the isotope at the top of the table, and you should use formatted `printf` statements to get the columns to line up.

You can use much of the loan amortization program as a sort of template. The main class should do the input from the user, and there should be a `Decay` class that does the computation and printing of the table.