# 781 2011-04-05

Note Title 2011-04-05

$KB = \{ P(a), P(b) \}$

$G = \forall x \; P(x)$

(a) $\{ P(a) \}$

(b) $\{ P(b) \}$

$\sim G: \; \sim \forall x \, P(x)$

$\exists x \sim P(x)$

$KB \not\models G$

b/c there are models of KB that are not models of G. E.g.,

$\mathfrak{A} = (U^{\mathfrak{a}}, I^{\mathfrak{a}})$,

$U^{\mathfrak{a}} = \{ 1, 2, 3 \}$.

(c) $\{\sim P(c)\}$ <u>new</u> constant

$\square \notin Res^*(\{P(a)\},$
$\{P(b)\},$
$\{\sim P(c)\})$

$I^a \quad a \to 1,$
$b \to 2$

$P^a = \{1, 2\}$

Example (Schimmling pp 94-95, Group Theory)

Associativity

$y \circ z \qquad x \circ y$

$x \circ y = u \ \& \ y \circ z = v \Rightarrow x \circ v = w$ iff $u \circ z = w$

$\exists x \quad x \circ y = y \quad \& \quad \exists z \; z \circ y = x$

Goal (existance of right-inverse)

$$\exists x \left( \forall y \; (x \circ y = y) \wedge \forall y \; \exists z \; (y \circ z = x) \right)$$

Neg goal after skolemization, etc, is

$$\neg \left( x \circ j(x) = j(x) \wedge k(x) \circ z = x \right)$$

Exercise 85

$Ch(x,y)$ for $x$ is a child of $y$

$Gr(x)$ for $x$ is green

$F(x)$ for $x$ flies

$H(x)$ for $x$ is happy

(a) Every dragon is happy if all its children can fly

$$\forall d \ (\forall x \ (Ch(x, d) \Rightarrow F(x)) \Rightarrow H(d))$$

(b) Green dragons can fly

$$\forall d \ Gr(d) \Rightarrow F(d)$$

(c) A dragon is green if it is the child of at least one green dragon

$$\forall d_1 \ (\exists d_2 \ (Ch(d_1, d_2) \wedge G(d_2)) \Rightarrow Gr(d_1))$$

(d) Show that all green dragons are happy

$$\forall d\,(Gr(d) \supset H(d))$$

$\sim(g)$: $\sim \forall d\,(Gr(d) \supset H(d))$

$$\sim \forall d\,(\sim Gr(d) \vee H(d))$$

$$\exists d \sim(\sim Gr(d) \vee H(d))$$

$$\exists d\,(Gr(d) \wedge \sim H(d))$$

$$Gr(Dragon1) \wedge \sim H(Dragon1)$$

(ng 1) $Gr(Dragon1)$

(ng 2) $\sim H(Dragon1)$

$$\forall d\ (\forall x\ (Ch(x, d) \Rightarrow F(x)) \Rightarrow H(d))$$

$$\forall d\ \{\sim[\forall x\ (Ch(x, d) \Rightarrow F(x))]\ \vee H(d)\}$$

$$\forall d\ \{\sim[\forall x\ (\sim Ch(x, d) \vee F(x))]\ \vee H(d)\}$$

$$\forall d\ \{\exists x \sim(\sim Ch(x, d) \vee F(x))\ \vee H(d)\}$$

$$\forall d\ \{\exists x\ (Ch(x, d) \wedge \sim F(x))\ \vee H(d)\}$$

$$\forall d\ \exists x\ \{(Ch(x, d) \wedge \sim F(x))\ \vee H(d)\}$$

$$\forall d \ \{ Ch(Child(d), d) \wedge \sim F(Child(d)) \vee$$
$$\vee H(d) \}$$

$$\forall d \{ (Ch(Child(d), d) \vee H(d)) \wedge$$
$$(\sim F(Child(d)) \vee H(d)) \}$$

(a1) $Ch(Child(d), d) \vee H(d)$ $\quad (a \wedge b) \vee c$

$\equiv$

(a2) $\sim F(Child(d)) \vee H(d)$ $\quad (a \vee c) \wedge (b \vee c)$

(b) $\sim Gr(d) \vee F(d)$

(c) $\sim Ch(d_1, d_2) \vee \sim Gr(d_2) \vee Gr(d_1)$

(In implicative form (e.g. AILog notation)

green(D1) <- ch(D1, D2) & green(D2).

Prolog: green(D1) :- ch(D1, D2), green(D2).)

(a 2) $\{\sim F(Child(d)), H(d)\}$

(neg 2) $\{\sim H(Dragon1)\}$

(a 1) $\{Ch(Child(d), d), H(d)\}$

$[d \backslash Dragon1]$

$\{\sim F(Child(Dragon1))\}$

$\{Ch(Child(Dragon1), Dragon1)\}$

(b) $\{\sim Gr(e), F(e)\}$

$[e \backslash Child(Dragon1)]$

$\{\sim Gr(Child(Dragon1))\}$

(c) $\{\sim Ch(f_1, f_2), \sim Gr(f_2), Gr(f_1)\}$

$\{\sim Ch(Child(Dragon1), f_2), \sim Gr(f_2)\}$

$[\{ r_2 \setminus \text{Dragon1} \}]$

$\{\neg Gr(\text{Dragon1})\}$

$(\text{Arg } 2)$

$\{Gr(\text{Dragon1})\}$

$\square$