781 _ 2011-03-29

Note Title 2011-03-29

HW3

Exercises 76, 85, 86 from Schöning's Ch. 2 handout

Herbrand universe $D(F)$ for a closed formula F in Skolem form

Ex. $F = \forall x \forall y \forall z \; P(x, f(y), g(z,x))$

$D(F) = \{ a, f(a), g(a,a), f(g(a,a)),$
$g(a, g(a,a)), g(g(a,a), g(a,a)),$
$f(g(a, g(a,a))), \ldots \}$

Ex. $G = \forall x \forall y \; Q(c, f(x), h(y,b))$

$D(G) = \{ b, c, f(b), f(c), h(b,b), h(c,b),$

$f(f(b)), f(f(c)), f(h(b,b)), \ldots \}$

The Herbrand structure $\mathfrak{A}$

$D(\mathfrak{A}) = \{ a, f(a), g(a,a), \ldots \}$

$f^{\mathfrak{A}} \to f$

$g^{\mathfrak{A}} \to g$

$P^{\mathfrak{A}}$ defined as $(t_1, t_2, t_3) \in P^{\mathfrak{A}}$ iff $g(t_1, t_2) =$

$= g(t_1, f(t_1))$

is not a model of $F = \forall x \forall y \forall z\, P(x, f(y), g(z,x))$

b/c $\mathcal{A}(F) = 0$ for the assignment

$x \to a,\ y \to a,\ z \to a$ and,

$\mathcal{A}(F) = 1$ iff $\mathcal{A}(P(x), f(y), g(z,x)) = 1$

for every assignment to x, y, z.

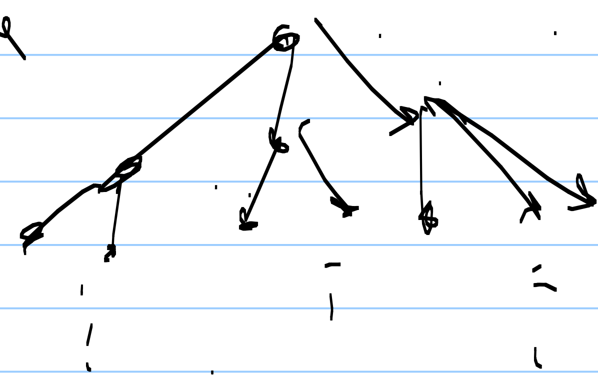
~~Exercise~~ 7.2. Call this structure $\mathcal{B}$

The universe and interpretation of f and g

are set — they are the same for all Herbrand structures

For P, let $P^{\mathcal{B}} = \{(\alpha, \beta, \gamma) \mid \alpha, \beta, \gamma \in D(F)\}$

König's lemma



2011-03-31

Exercise 76

Recall: $A \models B$ iff $A \wedge \sim B$ is unsatisfiable

$\not\models A \wedge \sim B$

Constant $\overset{\text{name}}{\wedge}$ p (the professor)

Predicate (name) : $S(x,y)$, "x is a student of y"

$H(x)$, "x is happy"

$L(x)$, "x likes logic"

(a) $\forall x\,(S(x,p) \to (L(x) \to H(p)))$

(b) $\neg \exists x\; S(x,p) \to H(p)$

or: $\forall x\,((S(x,p) \wedge L(x)) \to H(p))$

$$A \to (B \to C) \equiv (A \wedge B) \to C$$

We want to show that (a) $\wedge \neg$ (b) is a contradiction

Convert (a) into CNF: $\quad (A \to B \equiv \sim A \vee B)$

$$\sim S(x,p) \vee \sim L(x) \vee H(p)$$

$\{ \sim S(x,p), \sim L(x), H(p) \}$. (a)

Convert $\underline{\underline{\sim b}}$ into CNF:

$\sim(\sim \exists x\, (S(x,p) \rightarrow H(p)))$

$\sim(\exists x\, (S(x,p)) \vee H(p))$

$\forall x\, \neg S(x,p) \wedge \neg H(p)$

$\{\neg S(x,p)\}$ $\{\neg H(p)\}$

(b_1) (b_2)

Start from

$$\forall x \, ((S(x,p) \to L(x)) \to H(p))$$

correct per Schöning & Andrew

$$\sim(\sim S(x,p) \vee L(x)) \vee H(p)$$

$$(S(x,p) \wedge \sim L(x)) \vee H(p)$$

$$S(x p) \vee H(p) \wedge (\sim L(x) \vee H(p))$$

$\{S(x,p),\ H(p)\}$ $\quad$ $\{\sim L(x),\ H(p)\}$

a_1 $\quad$ a_2

$a_1 \downarrow [x/p]$ $\quad$ $b_1 \downarrow [x/p]$ $\quad$ $b_2 \downarrow [\,]$

$\{S(p,p),\ H(p)\}$ $\quad$ $\{\sim S(p,p)\}$ $\quad$ $\{\sim H(p)\}$

$H(p)$

$[\,]$