

2011-02-08

Note Title

2011-02-08

The Compactness Theorem for the propositional calculus (Section 1.4 solving, handout).

The proof below is by Loveland (Theorem 1.2.3)

in:

Loveland, Donald W.

Automated Theorem Proving: A Logical Basis.

North-Holland, 1978.

Thm. A set S of formulas is satisfiable iff every finite subset T of S is satisfiable

Proof.

It is immediate that, if S is a satisfiable set of formulas, then any subset of S is a satisfiable set.

To show the converse, we show that if S is unsatisfiable, then there is a finite subset T of S that is also unsatisfiable.

Assume that S is unsatisfiable.

$\vdash (S \supset (\sim (A \supset A)))$ b/c $(S \supset (\sim (A \supset A)))$ is a
tautology and the prop. calculus is
complete (Thm 9.5 Test here)

$S \vdash (\sim (A \supset A))$ converse of the deduction
theorem

$T \vdash (\sim (A \supset A))$ for some finite $T \subset S$ b/c

proofs/derivations are finite and
so, we can choose as hypotheses only
the formulas in S that are actually
used in the proof.

$\vdash (T \supset (\sim (A \supset A)))$ deduction theorem

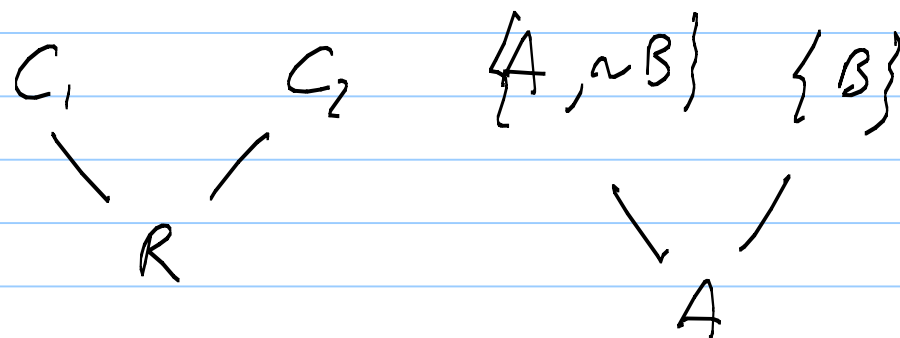
T is unsatisfiable, since $(\sim (A \supset A))$ is a contradiction and by the soundness of the prop. calculus $(T \supset (\sim (A \supset A)))$ is a tautology



Examples of resolvents

$$C_1 = \{A, \sim B\}$$

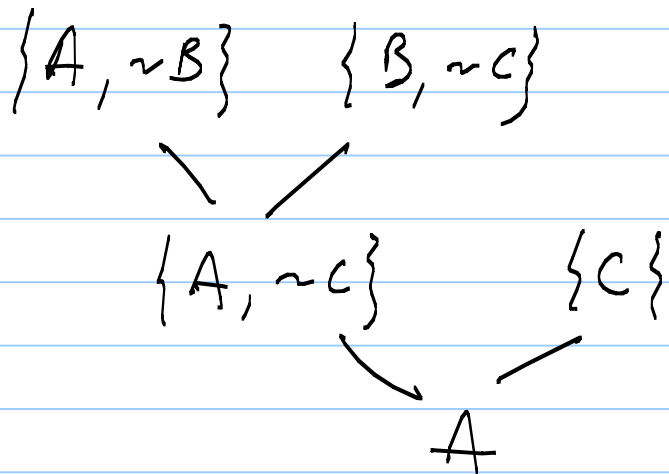
$$C_2 = \{B\}$$



$$C_1 = \{A, \sim B\}$$

$$C_2 = \{B, \sim C\}$$

$$C_3 = \{C\}$$



C_1, C_2, C_3 as just above,

$$C_4 = \{\sim A\}$$

$$\{A, \sim B\} \quad \{B, \sim C\}$$

$$\{A, \sim C\} \quad \{C\}$$

$$\{A\} \quad \{\sim A\}$$



Exercise 30

$$F = \left\{ \{A, \sim B, C\}, \{B, C\}, \{\sim A, C\}, \{B, \sim C\}, \{\sim C\} \right\}$$

$\hat{=} Res.$

$$\{A, \neg B, C\} \cup \{B, C\}$$

$$\text{Res}_1 = \left\{ \{A, C\}, \{\neg B, C\}, \{A, C, \neg C\}, \{A, B, \neg B\}, \{A, \neg B\}, \{B\} \cup \text{Res}_0 \right\}$$

$$\text{Res}_2 = \left\{ \{A, C\}, \{C\}, \{A, B\}, \{A\}, \dots \right\}$$

Recall:

$$K \models G$$

iff

$\neg G \wedge K$ is unsatisfiable
 $\neg G \cup K$ is unsatisfiable

$K \models G$ iff $K \vdash G$ iff $\vdash K \supset G$

Soundness
& completeness
of the p.r.

deduction
theorem
and its
converse

$K \models G$ iff $K \vdash G$ iff $\vdash K \supset G$ iff

iff $\vdash K \supset G$ iff $\vdash \neg K \vee G$ iff
equivalent
formulas

$\sim (\sim K \vee G)$ is a contradiction (i.e. it is unsatisfiable)
iff $K \wedge \sim G$ is unsatisfiable

$\vdash G$: there is a proof (or derivation) of G

$\vDash G$: G is a tautology

$K \vdash G$: there is a proof of G from hypothesis K

$K \models G$: K is a model of G

in every interpretation in which K
holds, G also holds

$\vdash \textcircled{K \supset T}$

There is a proof of $K \supset T$

If $K \vdash G$ then $K \models G$ is soundness
(Thm 9.2 for the P.C.)

If $K \models G$ then $K \vdash G$ is completeness
(Thm 9.5)

Mean trick: $\vdash \equiv \models$

(This is the logo of the Association for

Logic Programming)