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Schöning, Ch. 1

S. does not use the phrase "propositional variable"; he uses "atomic formula" instead.

<u>Defn.</u> (Syntax of propositional logic)

vee

1. All atomic formulas are formulas
2. If F is a formula, $\neg F$ is also a formula
3. If F and G are formulas, then $(F \vee G)$ and

$(F \wedge G)$ are formulas
wedge

<u>Note:</u> Schöning — Yasuhara (after Hilbert)

Schöning	Yasuhara
$\neg$	$\sim$
$\vee$	$\supset$
$\wedge$	

Schöning, unlike Yasuhara, does not provide a set of axioms and a rule of inference for propositional logic. He is not interested in proof theory, but only in model theory.

Defn (semantics of propositional logic)

$\{0, 1\}$ is the set of truth values

0 — f — false, 1 — t — true

An __assignment__ is a function from a set of atomic formulas to $\{0, 1\}$. An assignment assigns 0 or 1 to one or more atomic formulas.

A __suitable assignment__ assigns 0 or 1 to

every atomic formula in a formula F.
More commonly, a suitable assignment for a formula F is called an interpretation of the formula.

"Tertium non datur."

If $\mathcal{A}$ is a suitable assignment (interpretation) for formula F, and $\mathcal{A}(F) = 1$, then you say that $\mathcal{A}$ is a model for F, or equivalently F holds in $\mathcal{A}$, and indicate this by $\alpha \models F$.

A formula F is <u>satisfiable</u> if it has at least one model; otherwise it is unsatisfiable or contradictory. (We say also that F is a contradiction)

If F holds in every interpretation, then F is a valid formula or a tautology, and we write $\models F$.

Theorem (p. 9) ; A formula F is a tautology iff $\neg F$ is unsatisfiable.

Exercise 3 gives the semantical equivalent ($1 \Leftrightarrow 2$) of (a generalized version of) the deduction theorem and its converse. (The ded. thm corresponds to $1 \to 2$; its converse to $2 \to 1$.)

$F_1 \ldots F_k \models G$ G (logically) follows from $F_1, \ldots F_k$

G follows from $F_1 \ldots F_k$ iff every model of $F_1 \ldots F_k$ is also a model of G.

This is the semantic (model-theoretic) analogue of the syntactic (proof theoretic) notion of proof/derivation from hypothesis

$F_1, \ldots F_k \vdash G$

In the propositional calculus, $\vdash G$ iff $\models G$ (proved in Yasuhara's ch. 9, b/c $\models G$ is a shorthand for: G is a tautology).

Also

$F_1, \ldots, F_k \vdash G$ iff $F_1, \ldots, F_k \models G$

(We do not prove this.)

$$\left(\bigwedge_{i=1}^{k} F_i\right) \to G \equiv (F_1 \to (F_2 \to \dots (F_k \to G)\dots))$$

$$F_1, \dots, F_k \vDash G \overset{1 \to 2}{\Longrightarrow} (F_1 \to (F_2 \to \dots (F_k \to G)\dots))$$

$$(F_1 \to (F_2 \to \dots (F_k \to G)\dots)) \overset{2 \to 1}{\Longrightarrow} F_1, \dots, F_k \vDash G$$

sem. analogue of converse of deduction theorem

sem. analogue of Ded. Thm.

Fact:

$$(A \wedge B) \to C \text{ iff } (A \to (B \to C)).$$