Note Title 2011-12-06

Defn. A BN is a pair (P, G), where

$G = (V, E)$ is a DAG (directed acyclic graph)

and P is a probability distribution over the variables V, such that:

$$P(v \mid \pi(v), W) = P(v \mid \pi(v)),$$ where

[i.e., each variable v is independent on any subsets of its nondescendents given its parents]

$\pi(v)$ is the set of parents of v in G, and

W is a subset of the non-descendants of v in G.

From the definition, one can derive the chain rule for BNs, i.e:

$$P(V) = \prod_{v \in V} P(v \mid \pi(v))$$

Order the vars in V in reverse topological order:

$\langle v_1, v_2, \ldots, v_n \rangle$

$P(v_1, v_2, \ldots, v_n) = P(v_1 \mid v_2, \ldots, v_n) \times$

$P(v_2, \ldots, v_n) =$ (apply the defn of BN) $=$

$P(v_1 \mid \pi(v_1))\, P(v_2, \ldots, v_n) =$

$P(v_1 \mid \pi(v_1))\, P(v_2 \mid v_3, \ldots, v_n)\, P(v_3, \ldots, v_n) =$

$P(v_1 \mid \pi(v_1))\, P(v_2 \mid \pi(v_2))\, P(v_3, \ldots v_n) = \ldots$

$P(v_1 \mid \pi(v_1)) \times P(v_2 \mid \pi(v_2)) \times \ldots \times P(v_{n-1} \mid \pi(v_{n-1})) \times P(v_n)$

So, to specify a BN $= (G, P)$, you provide the DAG $G = (V, E)$

and the conditional probability tables of the form

$P(v \mid \pi(v))$, $\forall v \in V$.

<u>Defn.</u> Let $B = (G, P)$ be a BN. Let $G = (V, E)$.

Let $v, w \in V$. Let $Z \subseteq V$.

The chain between v and w is blocked by Z

either (a) there is a serial or diverging

connection ($\rightarrow (u) \rightarrow$ or $\leftarrow (u) \rightarrow$)

on the chain and $u \in Z$, or

(b) there is a converging connection ($\longrightarrow u \longleftarrow$) and neither u nor any of its descendants is in Z.

__Defn.__ v and w are d-separated given Z if every chain between v and w is blocked by Z.

__Thm__ (w/out proof): $P(v \mid w, Z) = P(v \mid w)$

(i.e., v and w are conditionally independent given Z) [in a BN, etc.] if every chain between v and w is blocked by Z,