CSCE 330 Fall 2022

Quiz 6

Assigned Tuesday, 22-11-01

Give a loop invariant for this program fragment :

```
x := 2;
i := 1;
while (i <= n) do
  begin
    x := x*x;
    i := i+1
  end
```

with precondition $n \geq 1$ and postcondition $x = 2^{2^n}$.

Answer: $x = 2^{2^{i-1}} \wedge i \leq n + 1$.

Also answer the following questions:

1. What is the precondition just before the loop (i.e., at the line with the comment)? **Answer:** $x = 2 \wedge \wedge = 1 \wedge i \leq n + 1$.

2. Let x be the value of the variable `x` before executing the body of the loop and x' be the value of the variable `x` after executing the body of the loop. Write an equation that relates x and x'. **Answer:** $x' = x * x$

3. Let i be the value of the variable `i` before executing the body of the and i' be the value of the variable `i` after executing the body of the loop. Write an equation that relates i and i'. **Answer:** $i' = i + 1$

4. Your invariant should consist of the conjunciton of two formulas. One of them should be very similar to the condition of the why loop. Explain why this formula is needed. **Answer:** To insure that $i = n + 1$, rather than just $i > n$, when the loop is exited.

5. Show that the precondition at the line with the comment implies the invariant. **Answer:** In short: (i) since $i = 1$ and $x = 2$, then $x = 2^{2^{i-1}}$. (2) Since $i = 1$ and $n \geq 1$, then $i \leq n + 1$.

6. Show that the invariant toghether with the negation of the loop condition implies the postcondition. **Answer:** See quiz 7 fall 2007.