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HW 1: Exercises 3.2, 3.3, 3.4, 3.5 [H] due on January 19, 2017

a sample space ↓

$\Omega =$

	E_1		E_2	E_2	E_2
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Defn. 3.1 An event, E, is a subset of Ω.

Here, as in most of the book, we adopt the classical interpretation of probability and assume that each outcome is equipossible. So,

$$P(E_1) = \frac{\#\text{ of outcomes in } E_1}{\text{total }\#\text{ of outcomes}} = \frac{6}{36} = \frac{1}{6}$$

$$P(E_2) = \frac{\#\text{ of outcomes in } E_2}{\text{total }\#\text{ of outcomes}} = \frac{3}{36} = \frac{1}{12}$$

$E_1 = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)\}$; $E_2 = \{(1,4),(1,5),(1,6)\}$.

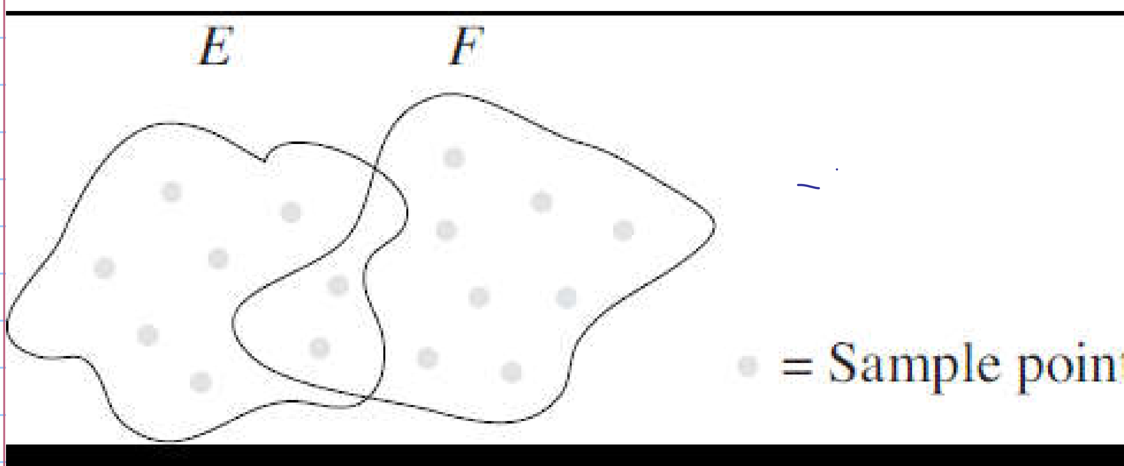
Defn. 3.2 If $E_1 \cap E_2 = \{\}$ then E_1 and E_2 are mutually exclusive.

Defn. 3.3 If $E_1, E_2, \ldots, E_n$ are events s.t. $E_i \cap E_j = \{\}$, $i \neq j$, $i, j \in 1..n$, and s.t. $\bigcup_{i=1}^{n} E_i = \Omega$, then we say that $E_1, E_2, \ldots, E_n$ <u>partition</u> Ω; we also say that they partition $F = \{E_1, \ldots, E_n\}$.

We also say that $E_1, E_2, \ldots E_n$ are <u>mutually exclusive and exhaustive</u>

Thm 3.4:

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$



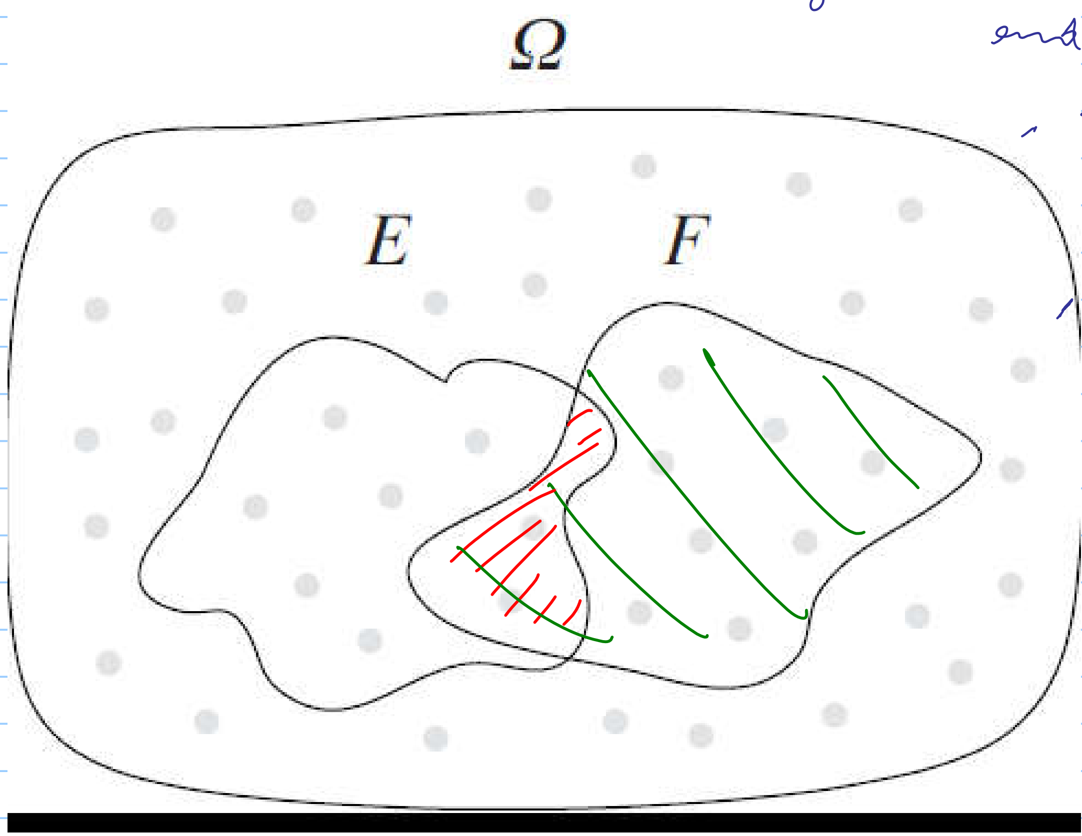
Thm 3.5:

$P(E \cup F) \leq P(E) + P(F)$

When is $P(E \cup F) = P(E) + P(F)$?

When E_1 and E_2 are mutually exclusive.

Defn. 3.6 ; The conditional probability of event E given event F is written as P(E|F) and given by the following, where we assume P(F) > 0 ;



$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{\#\text{ outcomes in } E \cap F}{\#\text{ outcomes in } F}$$

$$= \frac{\;\frac{\#\text{ in } E \cap F}{\#\text{ in } \Omega}\;}{\frac{\#\text{ F}}{\#\text{ in } \Omega}}$$

Table 1 My sandwich choices

Mon	Tues	Wed	Thu	Fri	Sat	Sun
Jelly	Cheese	Turkey	Cheese	Turkey	Cheese	None

P(Cheese | Second Half of the Week) ?

in the second half of the week: 4 outcomes (Thu ... Sun; Cheese ... None)

Cheese occurs twice; there are 4 total outcomes, so $\frac{2}{4}$ $\left(= \frac{1}{2}\right)$.

We could instead use Defn. 3.6 :

$$\frac{P(\text{Cheese \& Second Half})}{P(\text{Second Half})} = \frac{2/7}{4/7} = \frac{2}{4} = \frac{1}{2}$$

Defn. 3.7 Events E and F are independent if $P(E \cap F) = P(E) \cdot P(F)$

Thm If E and F are independent, then $P(E \mid F) = P(E)$.

Assume $P(F) > 0$. $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{P(E) \cdot P(F)}{P(F)} = P(E)$ ✓

The converse also holds: if $P(E \mid F) = P(E)$, then $P(E \cap F) = P(E) \cdot P(F)$

Also, note that independence is symmetric.

Can two mutually exclusive and non-null events be independent?

Let E and F be such events. Then $P(E \cap F) = 0$. Then

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0 \neq P(E)$$

No.

Ignore the grey events on the figure. Let E_1 be "First roll is 6" and E_2 be "Second roll is 6".

		E_1			E_2	
$\Omega =$	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

[redacted]

Then $E_1 = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$E_2 = \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\}$

$P(E_1) = \frac{6}{36} = \frac{1}{6} = P(E_2)$

$P(E_1 \cap E_2) = P(\{(6,6)\}) = \frac{1}{36}$

$P(E_1)P(E_2) \stackrel{?}{=} P(E_1 \cap E_2)$

$\frac{1}{6} \cdot \frac{1}{6} \stackrel{!}{=} \frac{1}{36}$ ✓ Yes

Is E_1 = "Sum of the rolls is 7" independent of E_2 = "Second roll is 4"?

$E_1 = \{16, 25, 34, 43, 52, 61\}$ $E_2 = \{14, 24, 34, 44, 54, 64\}$

$P(E_1) = \frac{6}{36} = \frac{1}{6}$ $P(E_2) = \frac{6}{36} = \frac{1}{6}$. $P(E_1 \cap E_2) = P(\{34\}) = \frac{1}{36} \stackrel{\checkmark}{=} \frac{1}{6} \cdot \frac{1}{6}$

Yes

Is E_1 = "sum of the rolls is 8" independent of E_2 = "second roll is 4"?

$E_1 = \{26, 35, 44, 53, 62\}$ $E_2 = \{14, 24, 34, 44, 54, 64\}$

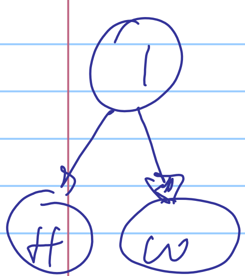
$E_1 \cap E_2 = \{44\}$.

$P(E_1) = \frac{5}{36}$ $P(E_2) = \frac{6}{36} = \frac{1}{6}$ $P(E_1 \cap E_2) = \frac{1}{36} \neq P(E_1)P(E_2)$

No.

Defn 3.8: Two events E and F are said to be conditionally independent given event G if, when $P(G) > 0$,

$P(E \cap F \mid G) = P(E \mid G)\, P(F \mid G)$.



(Bayesian network)

Holmes crashes, Watson crashes

(H) and (W) are not independent, but they are independent given (I) icy roads

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