Note Title 3/1/2009

HW 4 due ~~Friday~~ ~~Monday~~ Friday 2009/03/~~16~~ 20

~~(possible ext. to Wed, to be confirmed on Friday)~~

Instructions

- Show all your steps--answers alone are not sufficient.
- Homework must be done neatly.
- Use straight-edged paper (no notebook tear-outs with ragged edges).
- Please STAPLE papers to a signed cover sheet.

Homework Problems

Problem 5.4 (a). Plot the expression on a 4-variable K-map. (10 points)

Problem 5.4 (b). Simplify the K-map from 5.4 (a) into SOP form. Begin with a fresh map. (10 points)

Problem 5.4 (c). Simplify the K-map from 5.4 (a) into POS form. Begin with a fresh map. (10 points)

Problem 5.6 (a). To work, use guideline summary from class; ignore "essential prime implicants." (20 points)

No! Do not ignore them!

Problem 5.8 (a). (Note that the problem asks for both SOP and POS simplifications.) (20 points)

Problem 5.12 (c). (POS simplification.) (10 points)

Problem 5.21 (b). (Note that POS form is requested even though the problem statement is given in min-terms.) Plot the min-term map, then redraw with 0's, and group the 0's. (20 points)

Section 5.3 Four-variable Karnaugh Maps

A

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

C

D

B

Figure 5-10: **Location of Minterms on Four-Variable Karnaugh Map**

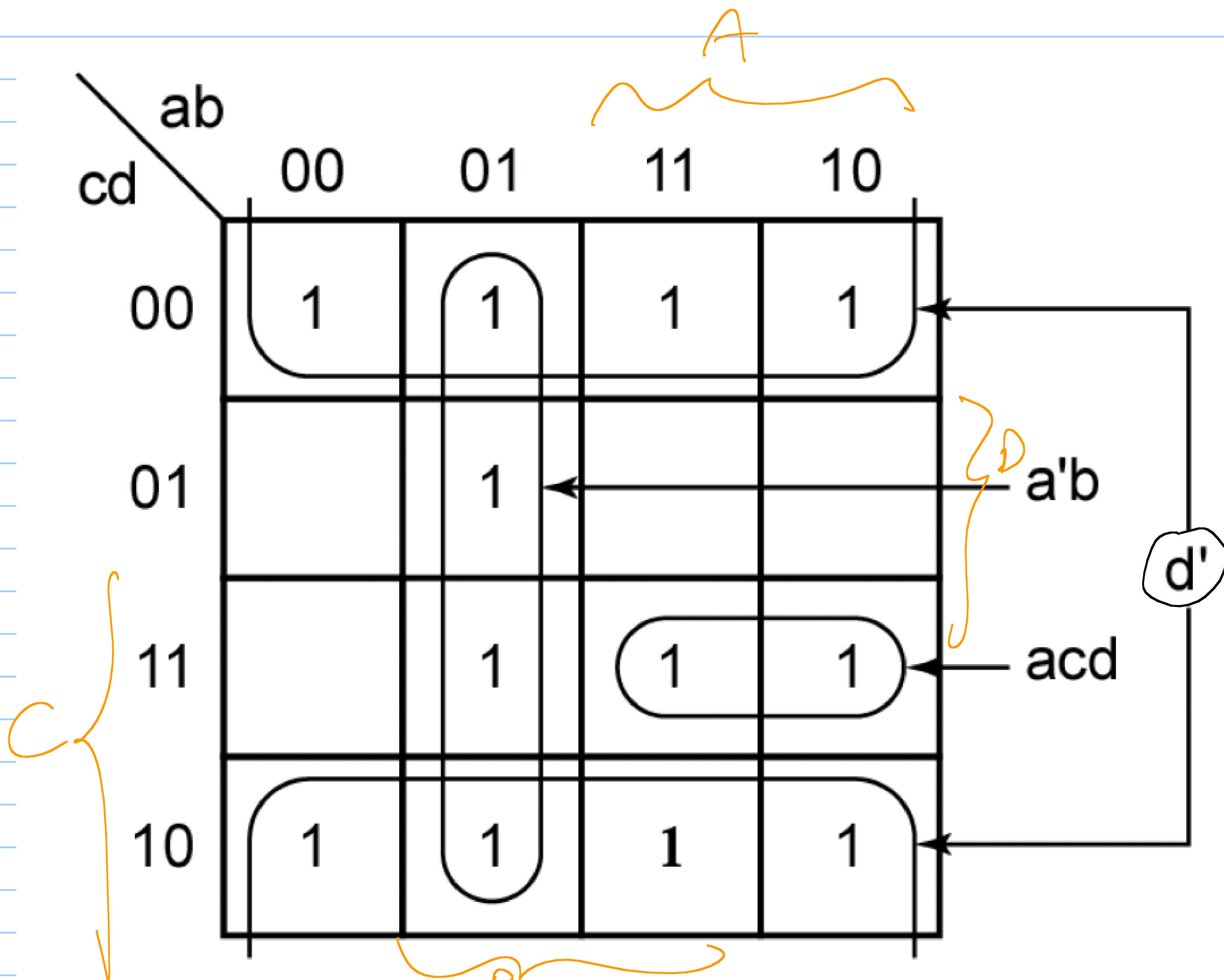
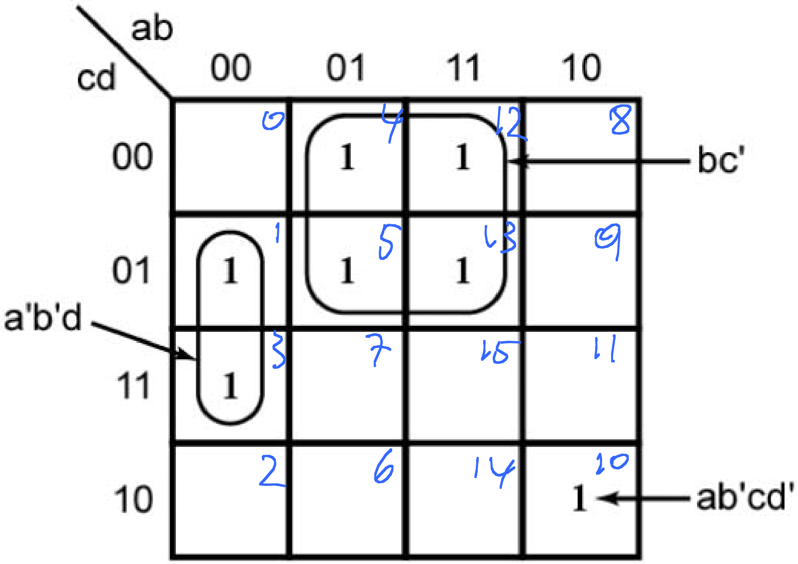
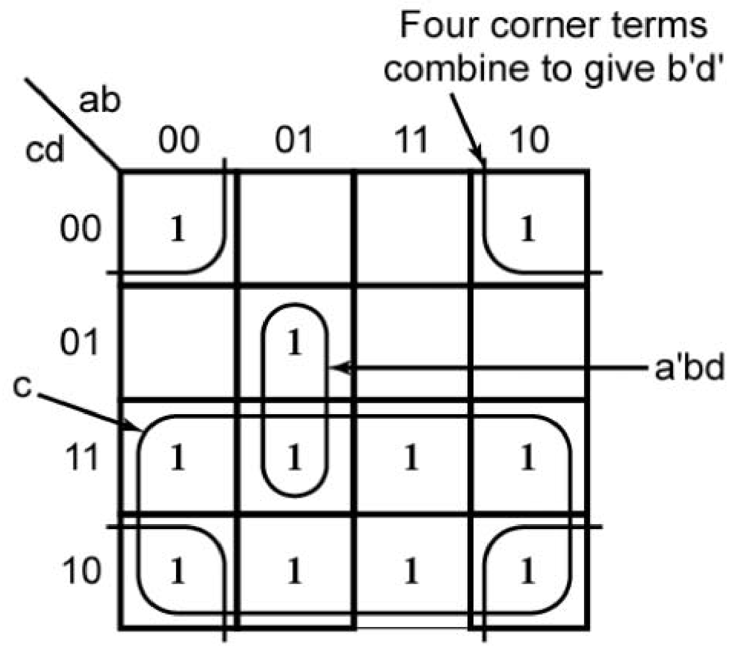


Figure 5-11: **Plot of** ***acd + a'b + d'***



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

(a)

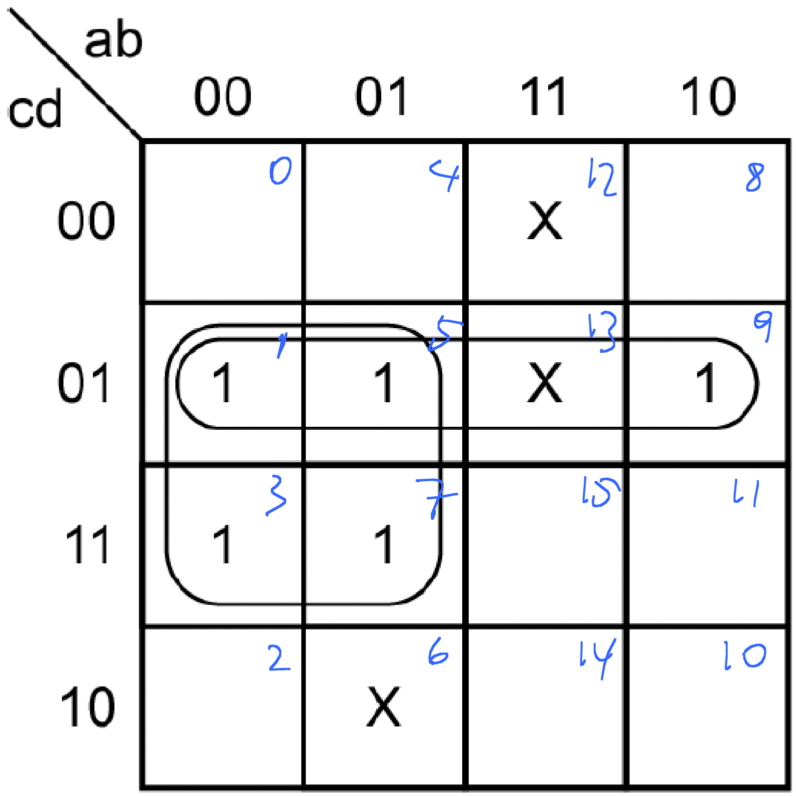


$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

$= c + \underbrace{b'c'd' + b'cd'}_{b'd'(c'+c)} + \cdot$

(b)

Figure 5-12: **Simplification of Four-Variable Functions**



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Figure 5-13: **Simplification of an Incompletely Specified Function**

Important for Circuit 2!

Find the minimum product of sums realization for

$f = x'z' + wyz + w'y'z' + x'y$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

$y'z$ $w'xy$ wxz'

Figure 5-14

Use the Karnaugh map to realize f' and obtain

$f' = y'z + wxz' + w'xy$

Use DeMorgan's law

$f = f'' = (y + z')\cdot(w' + x' + z)\cdot(w + x' + y')$

(a product of sums)

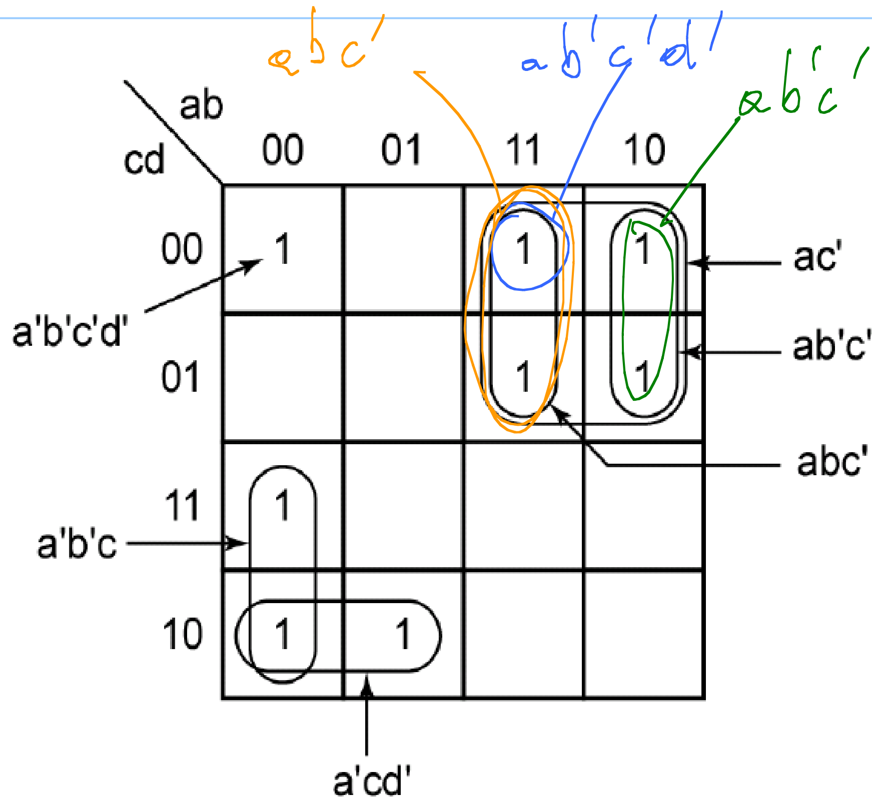


Figure 5-15

p. 129

Define implicant

- a 1, or a group of 1s that can be combined together

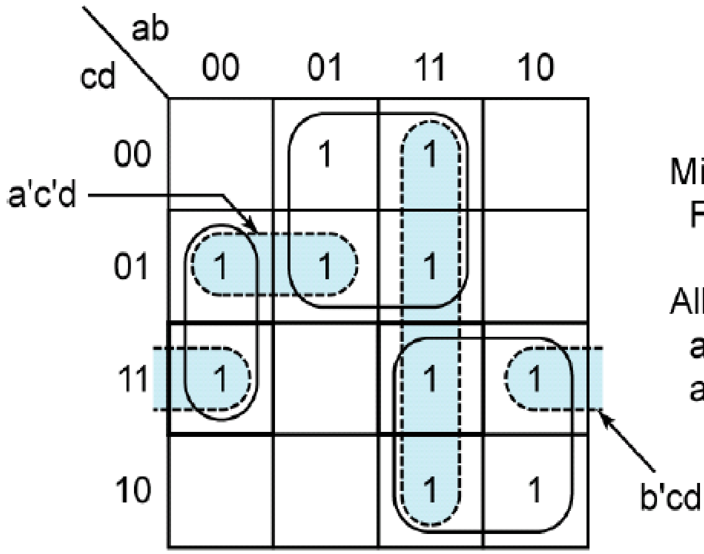
Define prime implicant

- an implicant (product term) that cannot be combined with another implicant

To eliminate a variable.

<u>Examples</u>;

ab'c'd', abc', ab'c', ac' are all implicants; of them, only ac' is a prime implicant

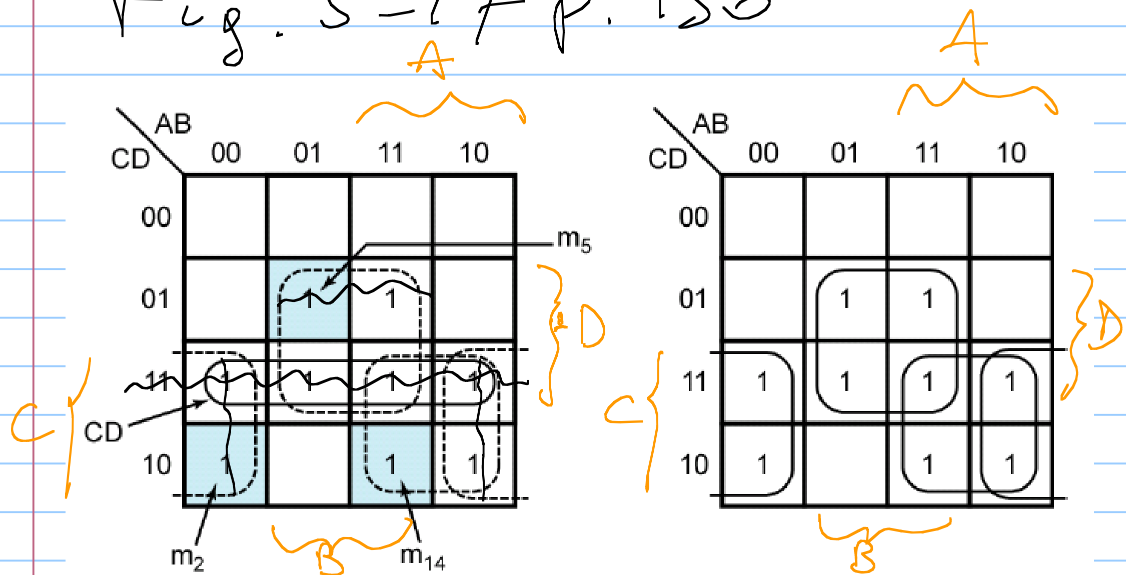


Minimum solution:
F = a'b'd + bc' + ac

All prime implicants:
a'b'd, bc', ac,
a'c'd, ab, b'cd

Figure 5-16: **Determination of All Prime Implicants**

Fig. 5-17 p. 130



f = CD + BD + B'C + AC

(a)

f = BD + B'C + AC

(b)

Here, CD is chosen first

Here, one of the other prime implicants is chosen first

Definition: essential prime implicant.

A prime implicant is essential if it is the only only prime implicant that covers some minterm.

Ex.:
- BD is essential, because no other prime implicant covers m_5
- AC and B'C are also essential
- CD is not essential

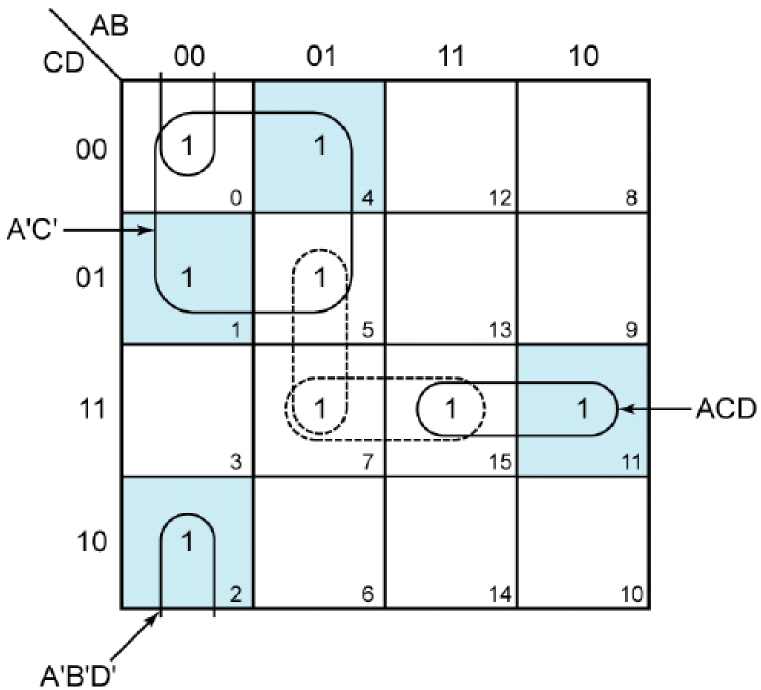


Figure 5-18

Theorem (p.131; p.621)

If a given minterm and all the 1's adjacent to it are covered by a single term, then that term is an essential prime implicant

Example (cf. Figure 5.18):

- $A'C'$ is an essential prime implicant, because minterm 0001_2 ($=1_{10}$) and all the ones adjacent to it (0, 4, 5) are covered by $A'C'$.
- ACD is an essential prime implicant, because minterm 1011_2 ($=11_{10}$) and all the ones adjacent to it (15) are covered by ACD
- $A'B'D'$ is an essential prime implicant, because minterm 1011_2 ($=2_{10}$) and all the ones adjacent to it (0) are covered by it.
- There are no other essential prime implicants

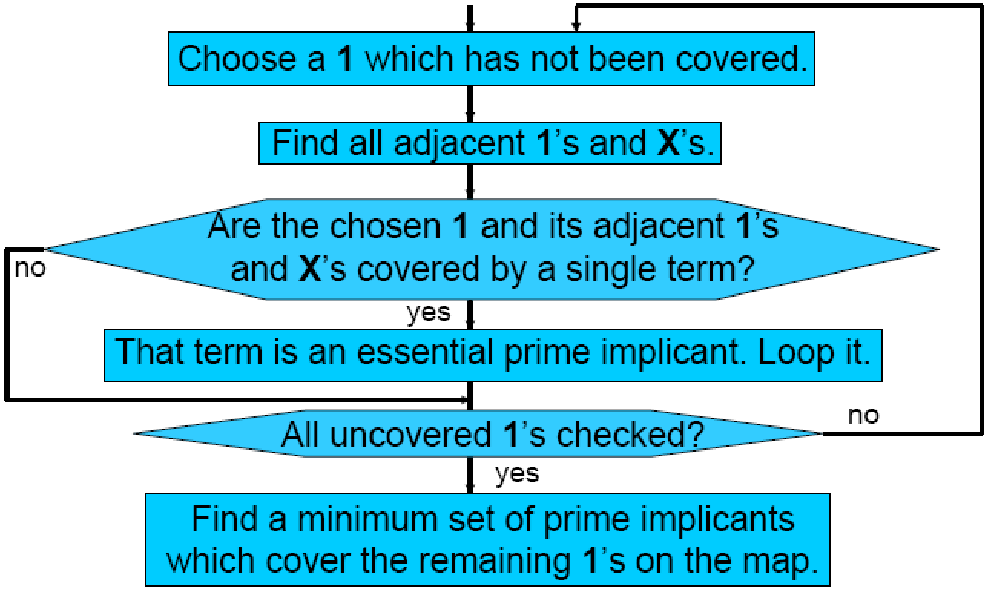


Figure 5-19:
Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

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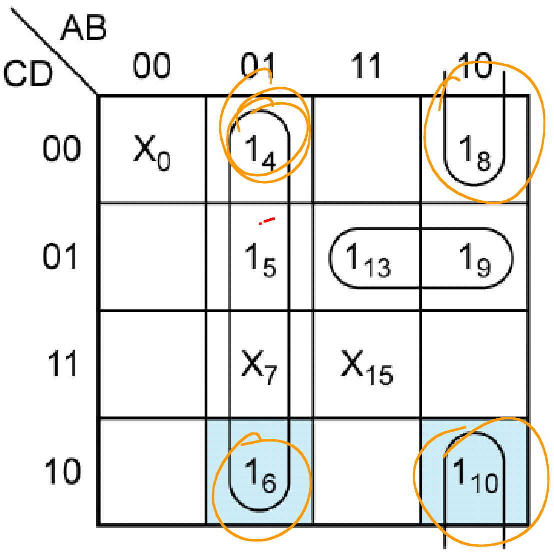
There are five important points to keep in mind when simplifying functions on K-maps:

1. Each square (minterm) on a K-map of two variables has two squares (minterms) that are logically adjacent, each square on a K-map of three variables has three adjacent squares, and so on. In general, each square on a K-map of n variables has n logically adjacent squares, with each pair of adjacent squares differing in exactly one variable.
2. When combining terms (squares) on a K-map we always group squares in powers of 2, that is, two squares, four squares, eight squares, and so on. Grouping two squares eliminates one variable, grouping four squares eliminates two variables, and so on. In general, grouping *2n* squares eliminates n variables.
3. Group as many squares together as possible; the larger the group is, the fewer the number of literals in the resulting product term.
4. Make as few groups as possible to cover all the squares (minterms) of the function. A rninterm is *covered* if it is included in at least one group. The fewer the groups, the fewer the number of product terms in the minimized function. Each minterm may be used as many times as it is needed in steps 4 and 5; however, it must be used at least once. As soon as all minterms are used once, stop. A minterm that has been used in at least one group is said to have been *covered.*
5. In combining squares on the map, always begin with those squares for which there are the fewest number of adjacent squares (the "loneliest" squares on the map). Minterms with multiple adjacent minterms (called *adjacencies)* offer more possible combinations and

on the web site

$F = A'B + AB'D' + AC'D.$



Shaded 1's are covered by only one prime implicant.

Figure 5-20

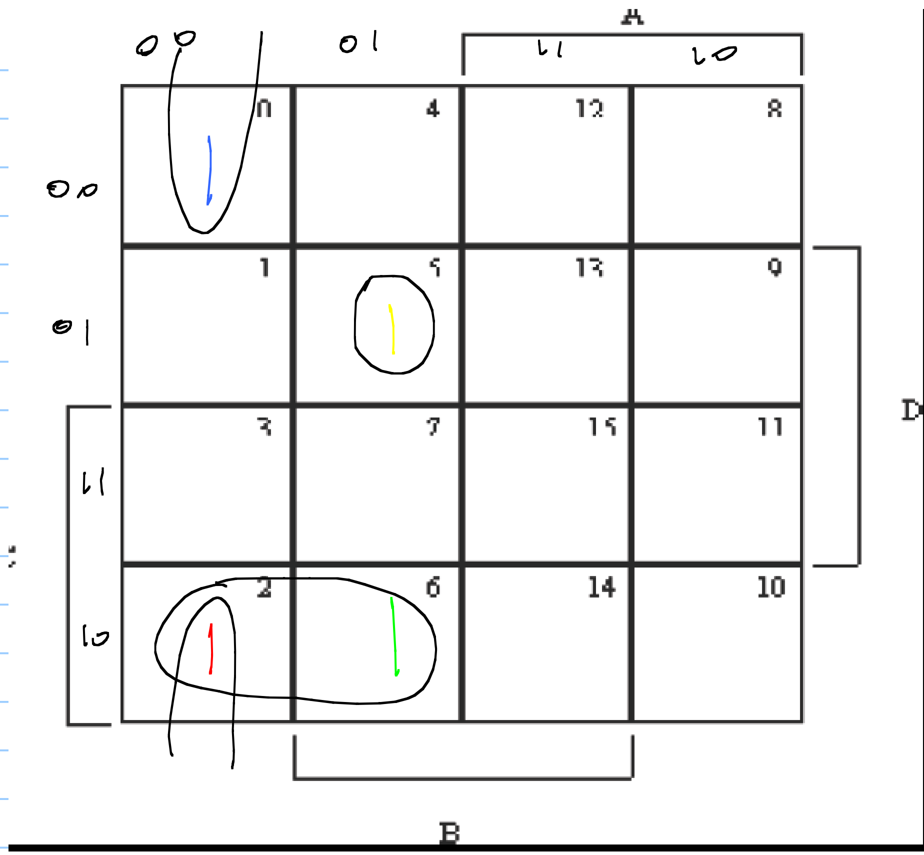
Choose 1_4. Find its $X_0, 1_5, 1_6$ 1 and X neighbors. Can you cover them with a single "loop"? No!

... $(1_8, 1_5, 1_{13}, 1_9)$

Choose 1_6. Its neighbors ~~is~~ are 1_4 and X_7

We can cover 1_6 and 1_4 with a loop, so $A'BD'$ is an essential prime implicant. Choose 1_{10}. Its neighbor is 1_8. $AB'D'$ is an essential prime implicant.

The only ones left are 1_{13} and 1_9 which are covered by the prime implicant $AC'D$



Exercise

Find the minimum sum-of products for

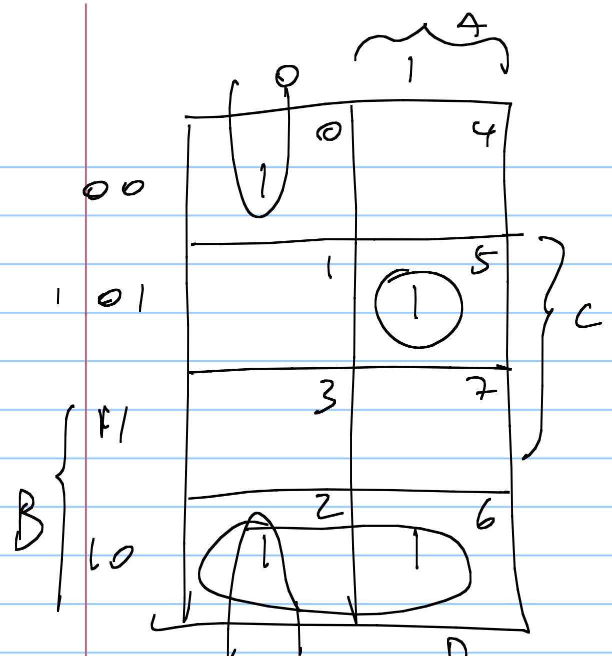
$f(a,b,c,d) = m_0 + m_2 + m_5 + m_6$

$A'BC'D$ is an essential prime implicant because it covers m_5 (and all its adjacent ones).

$A'CD'$ is an essential prime implicant, because it covers m_6 and all its adjacent ones (namely, m_2).

$A'B'D'$ is an essential prime implicant, because it covers m_0 and all its adjacent ones (namely, m_2).

All ones are covered. So, $f = A'BC'D + A'B'D' + A'CD'$



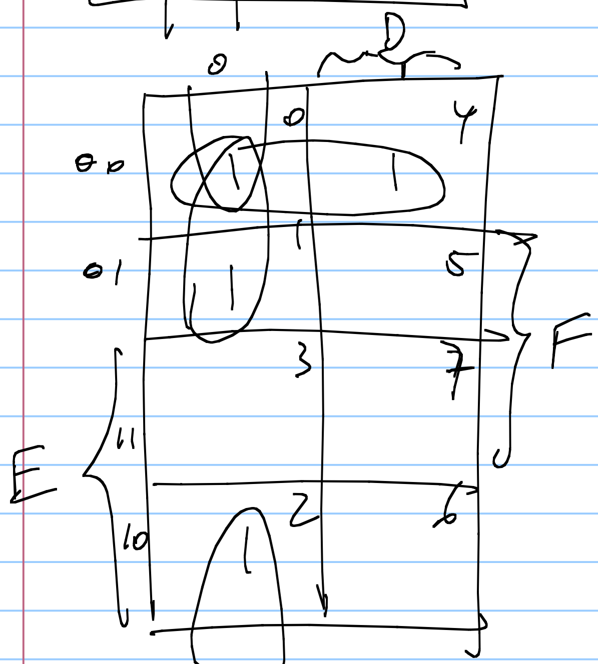
Exercise 5.3

Find the minimum sum-of-products for

(a) $f_1(a,b,c) = m_0 + m_2 + m_5 + m_6$

$AB'C + A'C' + BC'$

(all terms are essential prime implicants)



(b) $f_2(d,e,f) = \sum m(0,1,2,4)$

$E'F' + D'F' + D'E' =$

$=$ (in lexicographical order) $= D'E' + D'F' + E'F'$

(all terms are essential prime implicants)

st \ r	0	1
00	(0) 1	(4) 1
01	(1) 1	(5)
11	(3) 1	(7)
10	(2) 1	(6) 1

$f_3(r,s,t) = rt' + r's' + r's$

r' is an essential prime implicant, because m_1 and all its adjacent ones (namely, m_0 and m_3) are covered by r'

$r' + t'$

yz \ x	0	1
00	(0) 1	(4) 1
01	(1) 1	(5) 1
11	(3) 1	(7) 0
10	(2) 0	(8) 1

$f_4(x,y,z) = M_0 \cdot M_5 = (x'+y'+z')\cdot(x+y'+z)$.

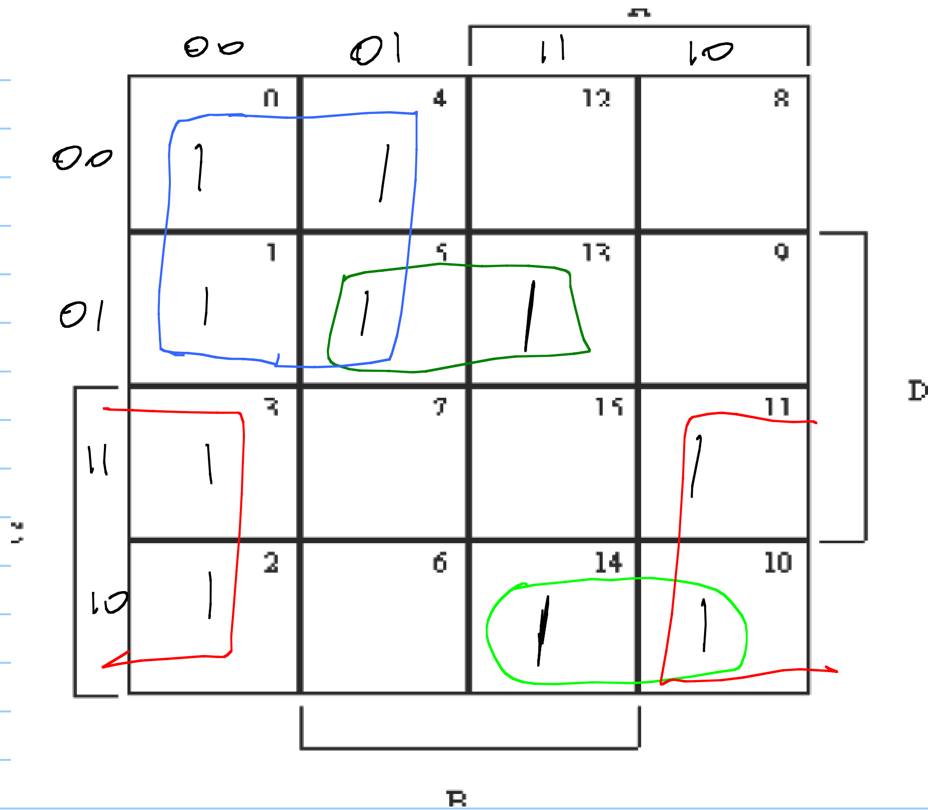
$f_4' = xyz + x'yz'$. Plot f_4', then complement.

1_3 and all its adjacent ones (1_1) are covered by $x'z$

1_8 and all its adjacent ones (1_4) are covered by xz'

The remaining 1s are covered by y. So,

$f_4(x,y,z) = y + x'z + xz'$



Ex 5.13

$F(A,B,C,D) = A'C' + B'C + ACD' + BC'D$

1_4 and all its adjacent ones (1_0 and 1_5) are covered by $A'C'$ (essential)

1_{13} and all its adjacent ones (1_5) are covered by $BC'D$ (essential)

1_2 and all its adjacent ones (1_3, 1_{10}) are covered by $B'C$ (essential)

1_{14} and all its adjacent ones (1_{10}) are covered by ACD'

So, the original expression is minimal.

If m_0 is a don't care, nothing changes, because $A'C'$ is still essential, since it covers 1_4 and all its neighbors (1_1, 1_5, X_0). Similarly for the other minterms, except:

If m_{13} is a don't care, then $BC'D$ is not needed.

If m_{14} is a don't care, then ACD' is not needed

Five-Variable Karnaugh Maps (section 5.5)

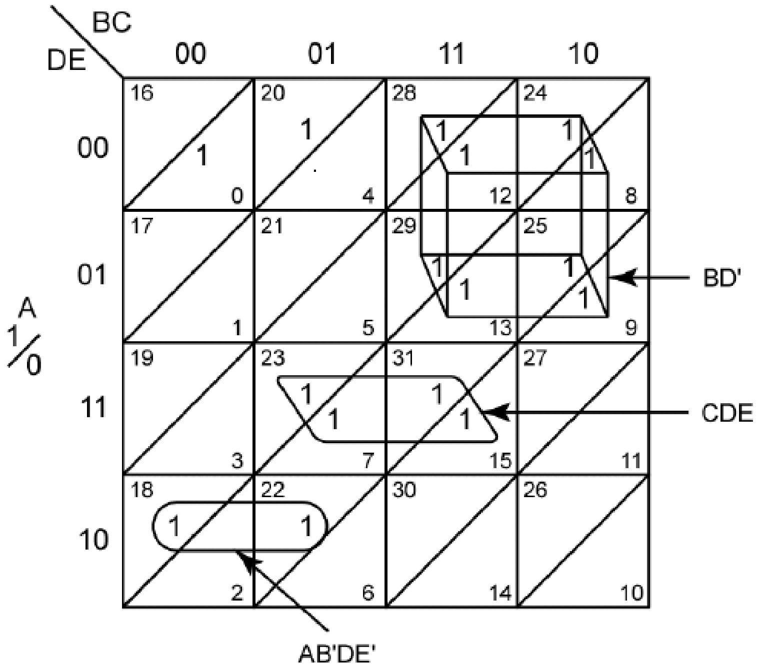


Figure 5-21: **A Five-Variable Karnaugh Map**

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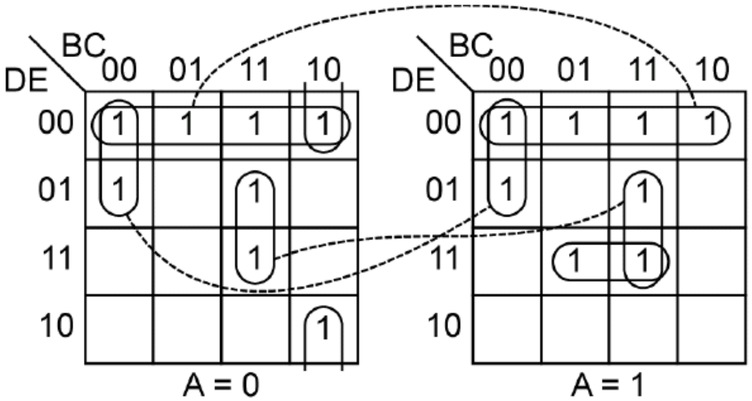
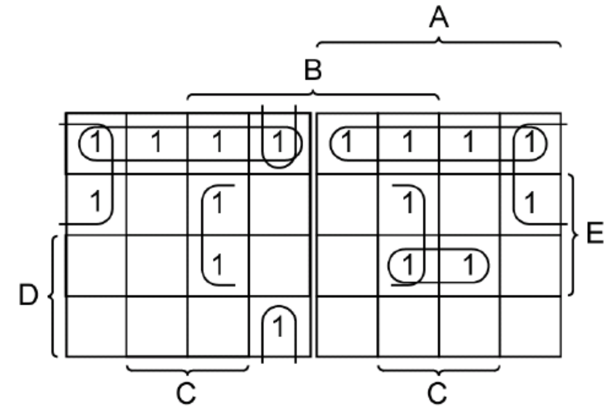


(a)

F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE

Figure 5-28: **Other Forms of Five-Variable Karnaugh Maps**



(b)

F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE

Figure 5-28: **Other Forms of Five-Variable Karnaugh Maps**

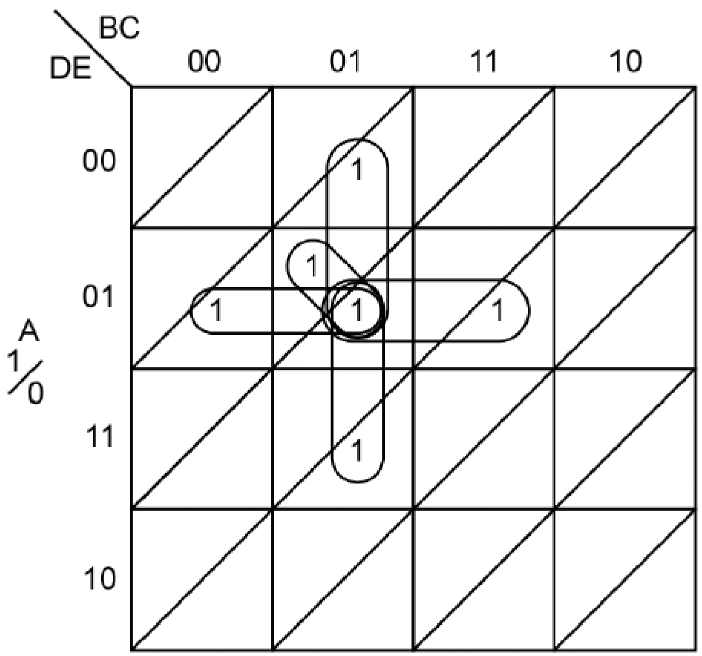


Figure 5-22

Adjacent minterms (product ~~terms~~ terms)

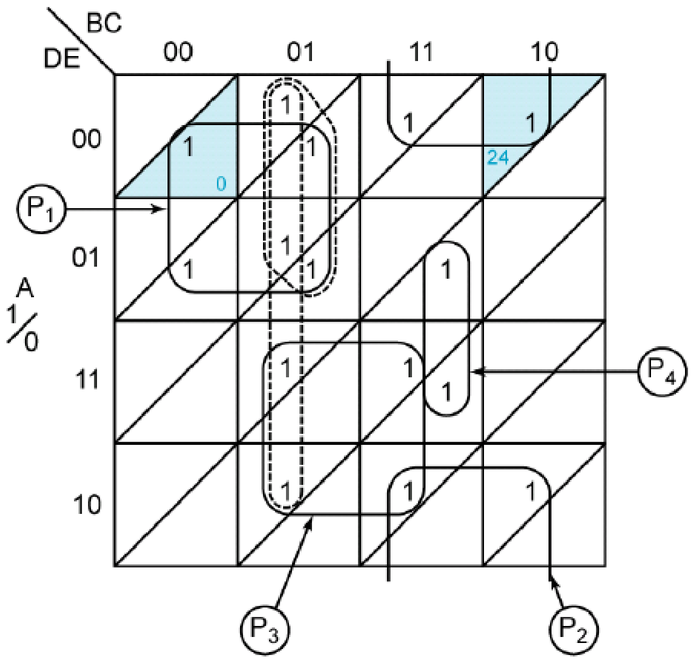


Figure 5-23 (p.134)

$F(A,B,C,D,E) = \sum m(0,1,$

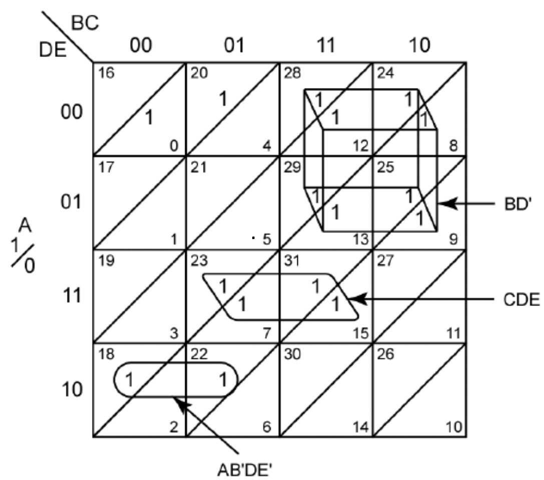


Figure 5-21: **A Five-Variable Karnaugh Map**

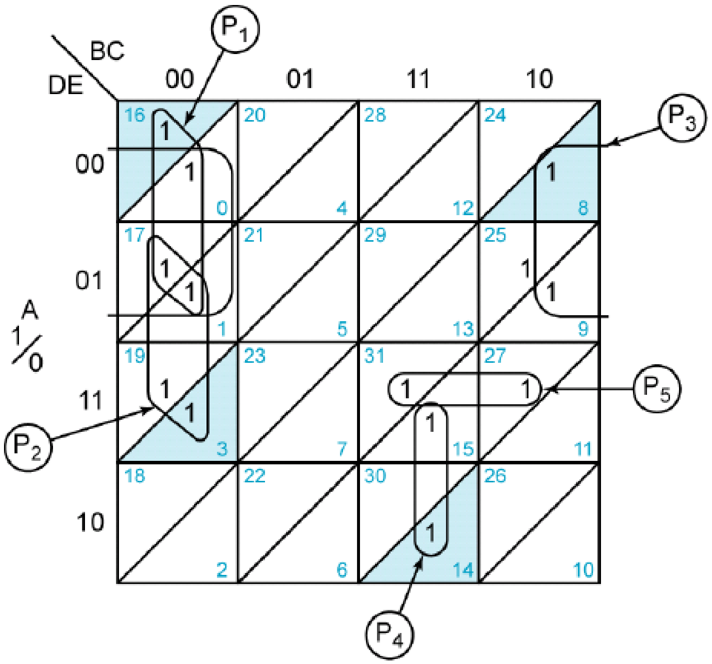


Figure 5-24

$F(A,B,C,D,E)=\sum m($

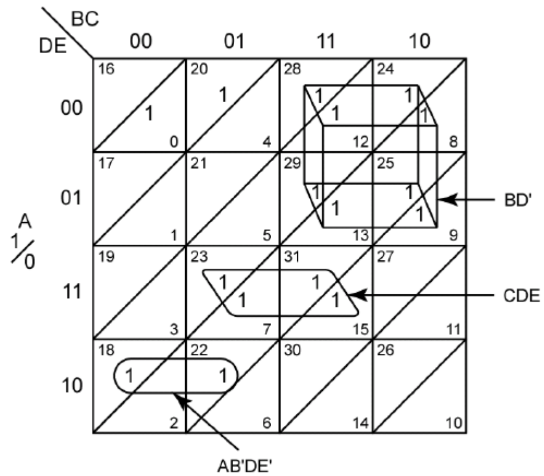
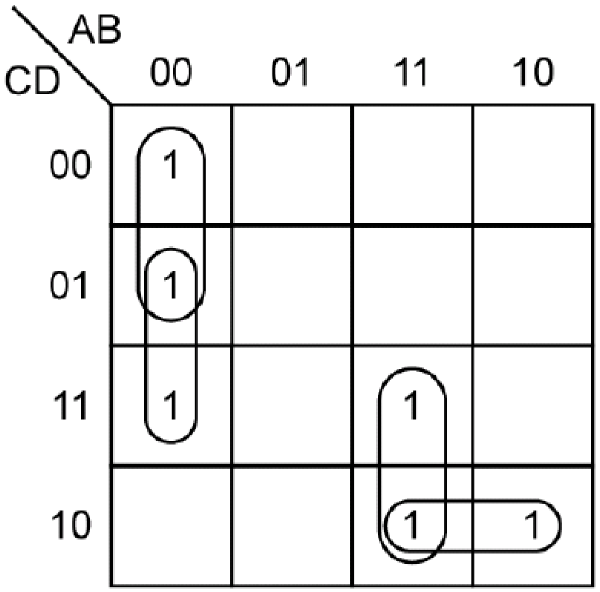


Figure 5-21: **A Five-Variable Karnaugh Map**

Other Uses of Karnaugh Maps (Section 5.6)



F = A'B'(C' + D) + AC(B + D')

Figure 5-25

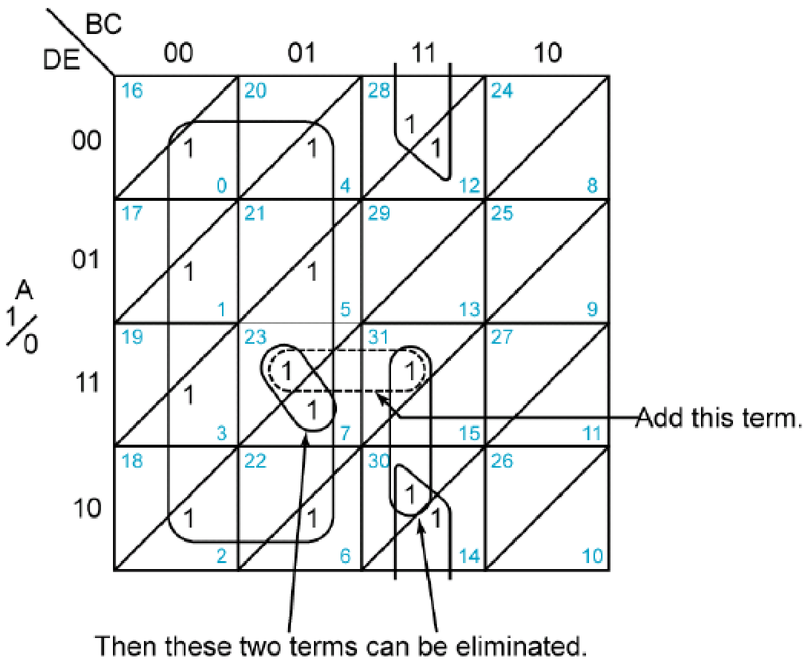

BC
DE
00
01
11
10
A
1/0
Add this term.
Then these two terms can be eliminated.

Figure 5-26