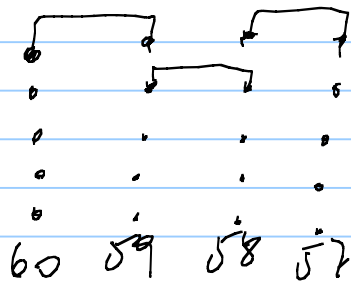


- Circuit 1 - continue
- Design of circuits from incompletely specified functions ("don't-cares") [4.5 text]
- A simple binary adder [4.6]
- Full adders and their use [4.7, first part]
- Karnaugh maps [5] : introduction

Derive check of 60-57:



# Incompletely specified functions (Section 4.5)

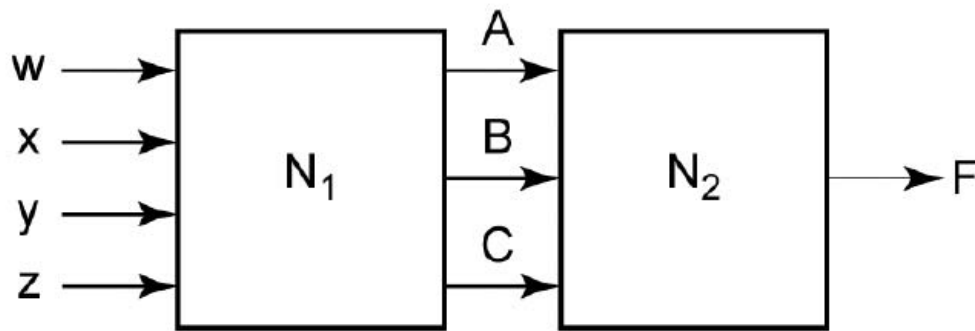


Table 4-5. Truth Table with Don't Cares

A	B	C	F	(1)	(2)	(3)
0	0	0	1			
0	0	1	X	0	1	1
0	1	0	0			
0	1	1	1			
1	0	0	0			
1	0	1	0			
1	1	0	X	0	0	1
1	1	1	1			

Section 4.5, p. 93

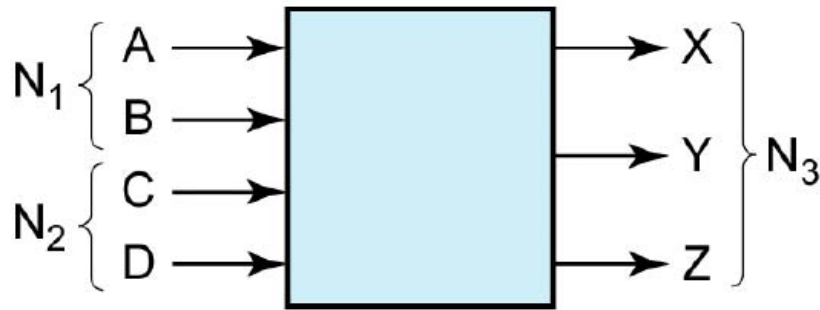
*N1 does not generate these configurations*

(1) all  $x=0$ :  $F = A'B'C' + A'BC + ABC = 1$   
 $= A'B'C' + BC$

(2)  $F = A'B'C' + A'B'C + A'BC + ABC = A'B' + BC$

(3)  $F = A'B'C' + A'B'C + A'BC + ABC' + ABC =$   
 $= A'B' + A'BC + AB = A'[B' + BC] + AB =$   
 $= A'[B' + C] + AB = A'B' + A'C + AB$

# Design of an adder (Example 2 section 4.6)



Section 4.6, p. 95

TRUTH TABLE:

$N_1$		$N_2$		$N_3$		
A	B	C	D	X	Y	Z
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	1	0
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	1	0

TRUTH TABLE:

$N_1$		$N_2$		$N_3$		
A	B	C	D	X	Y	Z
1	0	0	0	0	1	0
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

Section 4.6, p. 95

E.g.,

$$\begin{array}{r}
 N_1 = \text{AB} \\
 + N_2 = \text{CD} \\
 \hline
 N_3 = \text{XYZ}
 \end{array}$$

$$X(A, B, C, D) = \sum m(7, 10, 11, 13, 14, 15)$$

$$Y(A, B, C, D) = \sum m(2, 3, 5, 6, 8, 9, 12, 15)$$

$$Z(A, B, C, D) = \sum m(1, 3, 4, 6, 9, 11, 12, 14)$$

## Design of Binary Adders (4.7)

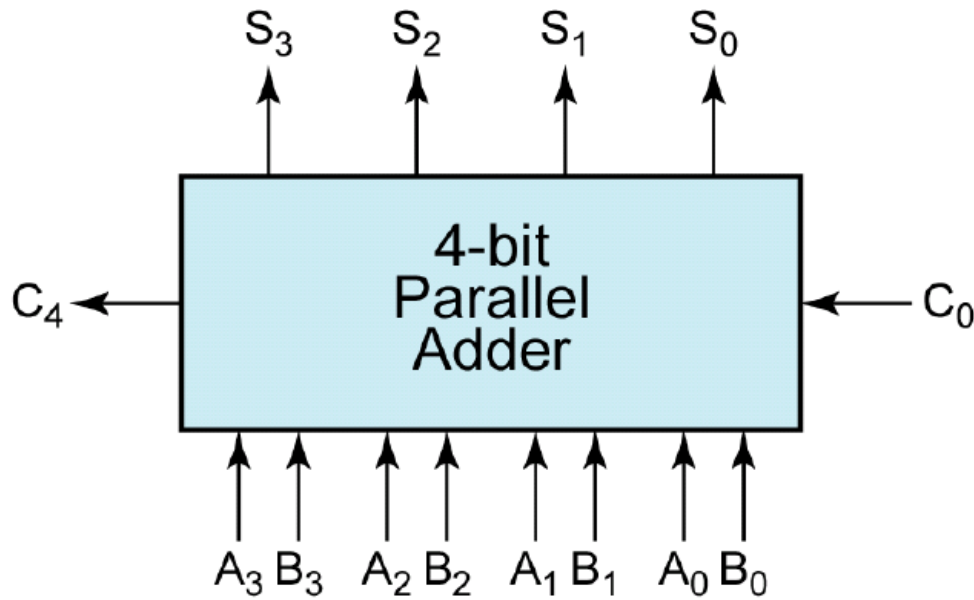


Figure 4-2: Parallel Adder for 4-Bit Binary Numbers

This circuit has  
nine inputs — too big!

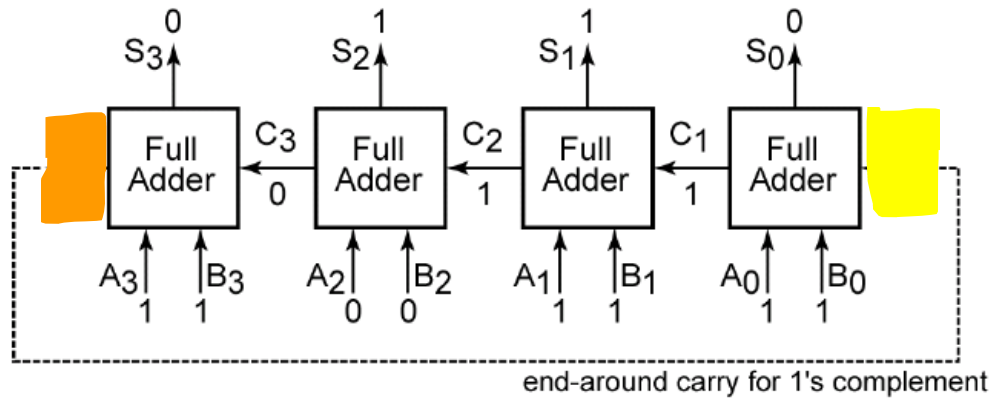
Add two 4-bit unsigned numbers, giving a 4-bit number as result.

Also, allow a 1-bit carry as input and a one-bit carry as output

$$\begin{array}{r}
 \phantom{+} \phantom{A_3} \phantom{A_2} \phantom{A_1} \phantom{A_0} C_0 \\
 + \phantom{+} A_3 \phantom{A_2} \phantom{A_1} \phantom{A_0} \\
 + \phantom{+} B_3 \phantom{B_2} \phantom{B_1} \phantom{B_0} \\
 \hline
 C_4 \phantom{+} S_3 \phantom{+} S_2 \phantom{+} S_1 \phantom{+} S_0
 \end{array}$$

Instead, connect four full adders.

Ex.:



$$\begin{array}{r}
 \phantom{+} 1011 \\
 + 1011 \\
 \hline
 10110
 \end{array}
 \begin{array}{l}
 = A_3 A_2 A_1 A_0 \\
 = B_3 B_2 B_1 B_0 \\
 = S_3 S_2 S_1 S_0
 \end{array}$$

**Figure 4-3: Parallel Adder Composed of Four Full Adders**

Each full adder adds two one-bit numbers and a carry, and produces one one-bit number and a carry.

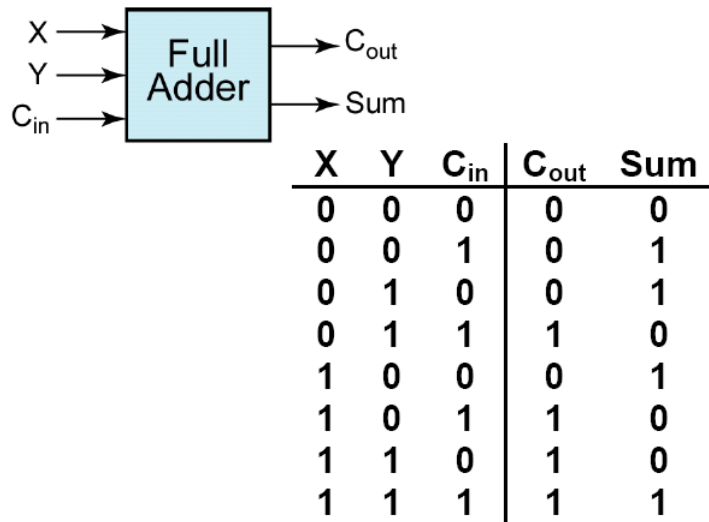


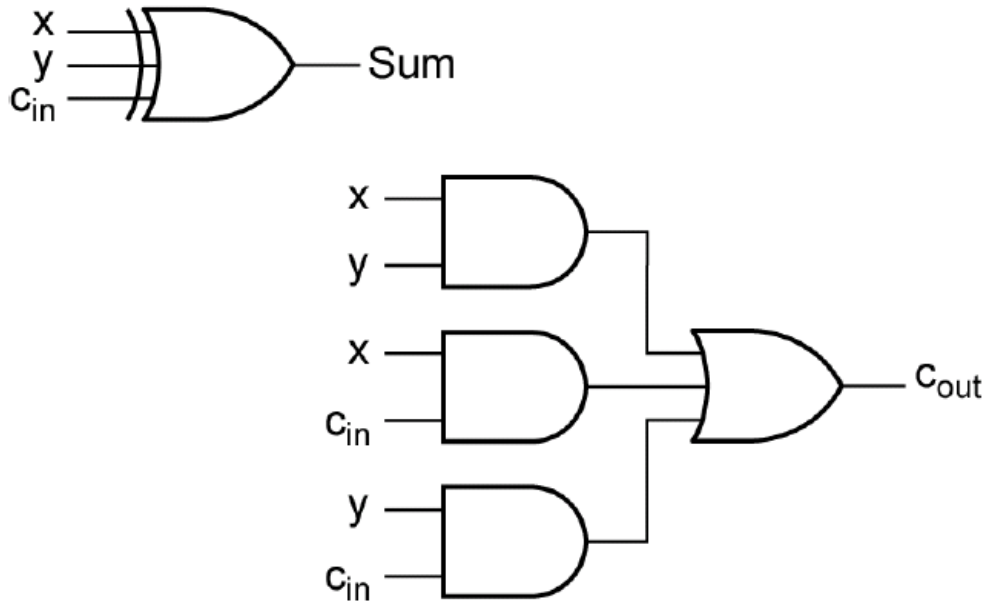
Figure 4-4: Truth Table for a Full Adder

$$\begin{aligned}
 \text{Sum} &= \sum m(1, 2, 4, 8) = \\
 &= X'Y'C_{in} + X'Y C_{in} + XY' C_{in} + XY C_{in} = \\
 &= X' (Y'C_{in} + Y C_{in}) + X (Y' C_{in} + Y C_{in}) = \\
 &= X' (Y \oplus C_{in}) + X (Y \equiv C_{in}) = \\
 &= X' (Y \oplus C_{in}) + X (Y \oplus C_{in})' = \\
 &= X \oplus (Y \oplus C_{in}) = \{3-13\} : \text{associativity}
 \end{aligned}$$

of exclusive-or) =  $X \oplus Y \oplus C_{in}$

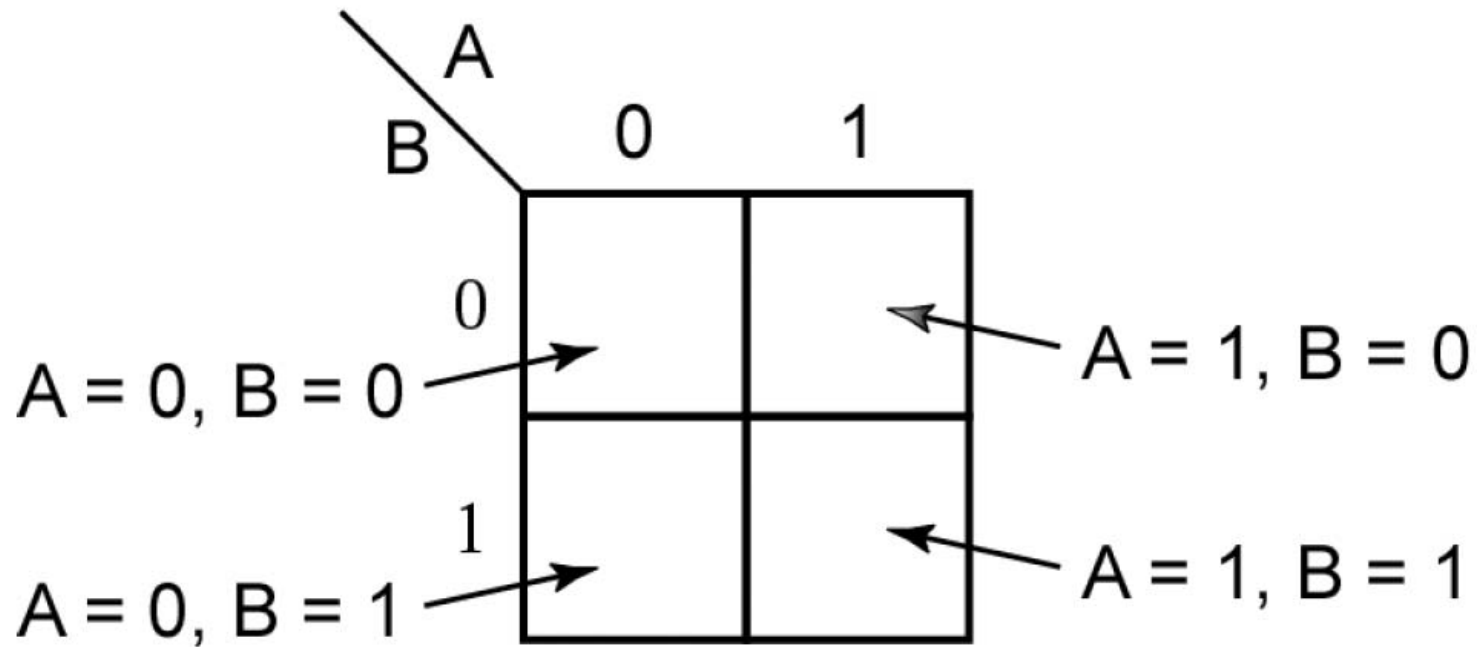
$$\begin{aligned}
 C_{out} &= \sum m(3, 5, 6, 7) = X'Y C_{in} + XY' C_{in} + XY C_{in}' + XY C_{in} = \\
 &= (X'Y C_{in} + XY' C_{in}) + (XY C_{in}' + XY C_{in}) = \\
 &= Y C_{in} + X C_{in} + XY
 \end{aligned}$$

Here are the resulting circuits:



**Figure 4-5: Implementation of Full Adder**

A Karnaugh map for two variables (A and B)



**Section 5.2, p. 121**



(a)

<i>A</i>	<i>B</i>	<i>F</i>
0	0	1
0	1	1
1	0	0
1	1	0

(b)

<i>A</i> \ <i>B</i>	0	1
0	1	0
1	1	0

(c)

<i>A</i> \ <i>B</i>	0	1
0	1	0
1	1	0

$A'B'$  → (row 0, col 0)  
 $A'B$  → (row 1, col 0)

$F = A'B' + A'B$

(d)

<i>A</i> \ <i>B</i>	0	1
0	1	0
1	1	0

$A'B' + A'B = A'$  → (column 0)

$F = A'$

**Figure 5-1a, b, c, and d**