Note Title 2/10/2009

HW 3 due Friday, 2009-02-13

1. Text problem 2.1 (a). Use first distributive law and simplify. (10 pts)
2. Text problem 2.1 (d). Use second distributive law and simplify. (10 pts)
3. Text problem 2.3. (d) Use second distributive law (or Theorem 12-14D). (10 pts) 10-14D
4. Text problem 2.3. (e) Let X = {A′•B + D} and use second distributive law. (10 pts)
5. Text problem 2.5 (b). Let X = {A′ + C′} and use second distributive law. (10 pts)
6. Text problem 2.6 (a). First rewrite [A•B] + (C′•D′) = ([A•B] + C′) • ([A•B] + D′). Apply second distributive law to each new term. (10 pts)
7. Text problem 2.9 (a) (10 pts)
8. Text problem 2.13 (d), top of page 50. (10 pts)
9. Text Problem 4.21 (a). Multiply out the expression (first distributive law) and create a truth table. Express the truth table in **Σm ()** notation (rows numbers where the expression = 1). See pages 86 to 88). (10 pts)
10. Express the truth table for Problem 9 above in **ΠM ()** notation (rows where expression = 0). (10 pts)

EXAMPLE 1 Simplify $Z = A'BC + A'$
This expression has the same form as (2-13) if we let $X = A'$ and $Y = BC$
Therefore, the expression simplifies to $Z = X + XY = X = A'$.

EXAMPLE 2 Simplify $Z = [\underbrace{A + B'C}_{} + \underbrace{D + EF}_{}][\underbrace{A + B'C}_{} + \underbrace{(D + EF)'}_{}]$

Substituting: $Z = [\quad X \quad + \quad Y \quad][\quad X \quad + \quad Y' \quad]$

Then, by (2–12D), the expression reduces to

$$Z = X = A + B'C$$

EXAMPLE 3 Simplify $Z = \underbrace{(AB + C)}_{}\underbrace{(B'D + C'E')}_{} + \underbrace{(AB + C)'}_{}$

Substituting: $Z = \quad Y' \quad X \quad + \quad Y$

By (2–14D): $Z = X + Y = B'D + C'E' + (AB + C)'$

Simplify (p. 42-43)

(2-12D) $(X+Y)(X+Y') = X$

(2-14D) $XY' + Y = X + Y$

$\Updownarrow$

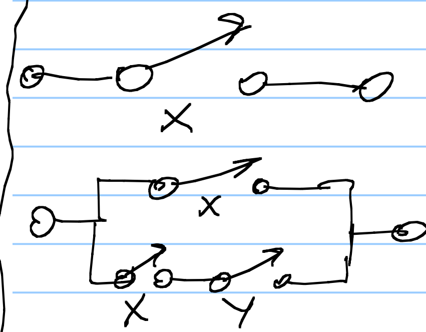
(2-14) $(X+Y')\cdot Y = XY$

absorption

(2-13) $X + XY = X$. Why?

Algebraic manipulation

$X + XY = X(1+Y) = X \cdot 1 = X$



equivalent circuits

XY	XY	X+XY
00	0	0
01	0	0
10	0	1
11	1	1

truth table

Case analysis:

if $X = 0$, then $0 + 0 \cdot Y = 0$ ✓

if $X = 1$, then $1 + 1 \cdot Y = 1$ ✓

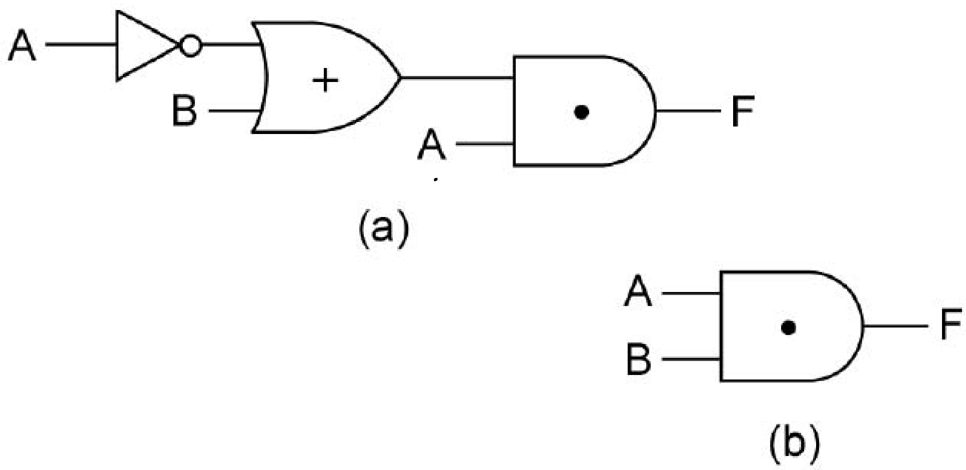
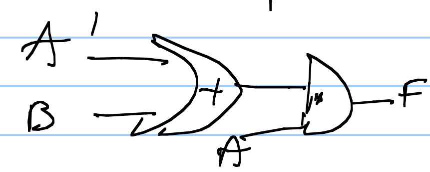
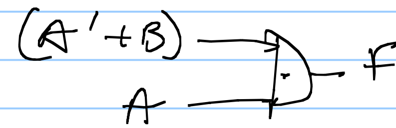


Figure 2-4: **Equivalent Gate Circuits**

Bottom-up conversion of top circuit:





$(A'+B)\cdot A$

Now, use 2-14: $(X'+Y)\cdot X = XY$

Conclude: $F = AB$. So, the two circuits are equivalent.

if $X=0$, $YZ = (0+Y)(0+Z) = YZ$ ✓
if $X=1$, $1 + YZ = (1+Y)(1+Z) = 1$ ✓

$X + YZ = (X+Y)(X+Z)$ (2-11 D)
the second distributive law.

In Boolean logic, OR (+) distributes over AND (·).

This is not true for arithmetic!

EXAMPLE 1: Factor $A + B'CD$. This is of the form $X + YZ$ where $X = A$, $Y = B'$, and $Z = CD$, so

$$A + B'CD = (X + Y)(X + Z) = (A + B')(A + CD)$$

$A + CD$ can be factored again using the second distributive law, so

$$A + B'CD = (A + B')(A + C)(A + D)$$

EXAMPLE 2: Factor $AB' + C'D$

$AB' + C'D = (AB' + C')(AB' + D)$ ← note how $X + YZ = (X + Y)(X + Z)$ was applied here

$= (A + C')(B' + C')(A + D)(B' + D)$ ← the second distributive law was applied again to each term

EXAMPLE 3: Factor $C'D + C'E' + G'H$

$C'D + C'E' + G'H = C'(D + E') + G'H$ ← first apply the ordinary distributive law, $XY + XZ = X(Y + Z)$

The first distributive law (2-11) states that AND (·) distributes over OR (+). This is also true for arithmetic.

$= (C' + G'H)(D + E' + G'H)$ ← then apply the second distributive law

$= (C' + G')(C' + H)(D + E' + G')(D + E' + H)$ ← now identify X, Y, and Z in each expression and complete the factoring

Factor (p. 44-45)

These examples show conversion to product of sums form (also known as conjunctive normal form).
CNF

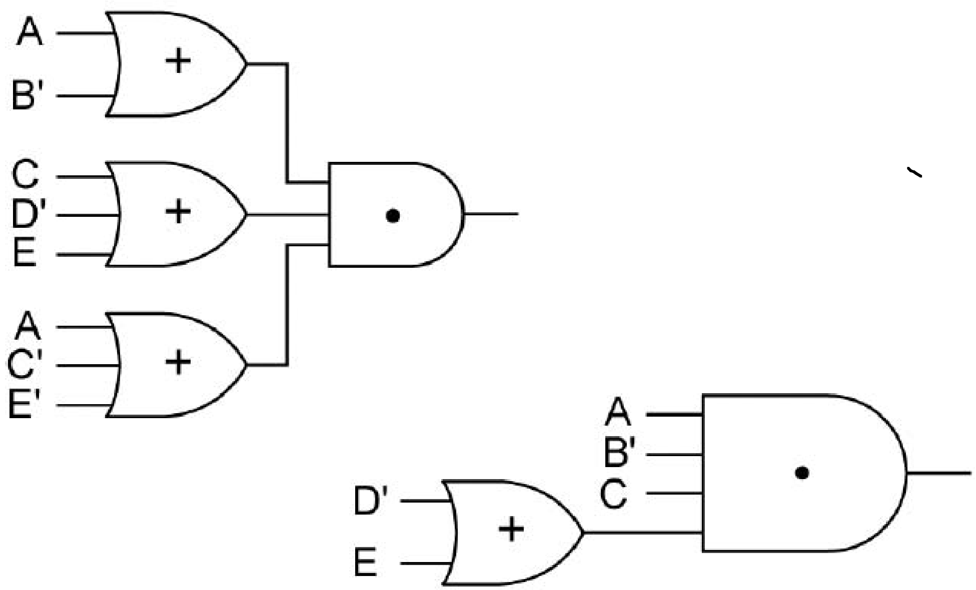


Figure 2-6: **Circuits for Equations (2-18) and (2-20)**

These circuits correspond to expressions in product-of-sums form (conjunctive normal form).
The inverters are not shown

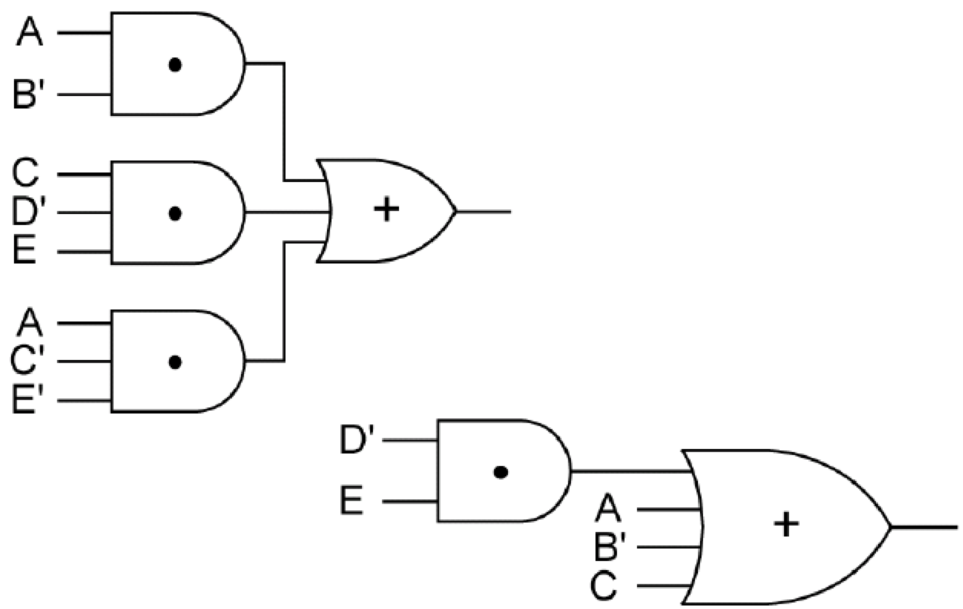


Figure 2-5: **Circuits for Equations (2-15) and (2-17)**

These circuits correspond to expressions in sum-of-product form (disjunctive normal form).

(2-15) $AB' + CD'E + AC'E'$

(2-17) $D'E + A + B' + C = A + B' + C + D'E$

Laws and theorems (p. 52)

Operations with 0 and 1:

1. $X + 0 = X$ — 1D. $X \cdot 1 = X$
2. $X + 1 = 1$ — 2D. $X \cdot 0 = 0$

Idempotent laws:

3. $X + X = X$ — 3D. $X \cdot X = X$

Involution law:

4. $(X')' = X$

Laws of complementarity:

5. $X + X' = 1$ — 5D. $X \cdot X' = 0$

Commutative laws:

6. $X + Y = Y + X$ — 6D. $XY = YX$

Associative laws:

7. $(X + Y) + Z = X + (Y + Z) = X + Y + Z$ — 7D. $(XY)Z = X(YZ) = XYZ$

Distributive laws:

8. $X(Y + Z) = XY + XZ$ — 8D. $X + YZ = (X + Y)(X + Z)$

Simplification theorems:

9. $XY + XY' = X$ — 9D. $(X + Y)(X + Y') = X$
10. $X + XY = X$ — 10D. $X(X + Y) = X$
11. $(X + Y')Y = XY$ — 11D. $XY' + Y = X + Y$

DeMorgan's laws:

12. $(X + Y + Z + \ldots)' = X'Y'Z'\ldots$ — 12D. $(XYZ\ldots)' = X' + Y' + Z' + \ldots$

Duality:

13. $(X + Y + Z + \ldots)^D = XYZ\ldots$ — 13D. $(XYZ\ldots)^D = X + Y + Z + \ldots$

Theorem for multiplying out and factoring:

14. $(X + Y)(X' + Z) = XZ + X'Y$ — 14D. $XY + X'Z = (X + Z)(X' + Y)$

Consensus theorem:

15. $XY + YZ + X'Z = XY + X'Z$ — 15D. $(X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z)$

12 and 12D are very important

13 and 13D define duality

14 and 14D are very useful for factoring & multiplying out, when converting to/from sum-of-products and product-of-sum forms

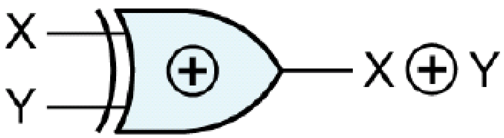
EXAMPLE:

$$(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)$$
$$= (A+B+C'D)(A+B+E)[AC+A'(D'+E)]$$
$$= (A+B+C'DE)(AC+A'D'+A'E)$$
$$= AC + \cancel{ABC} + A'BD' + A'BE + A'C'DE$$

(8D) and (14) used here

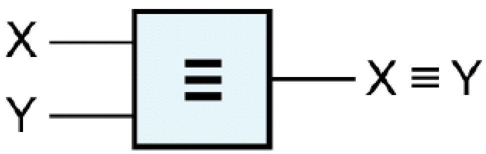
(8D) (second distributive law), again;

Example (3-4), p. 59



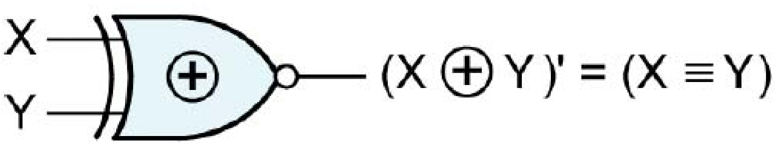
Exclusive OR (XOR)

X	Y	$X \oplus Y$	$X + Y$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	0	0

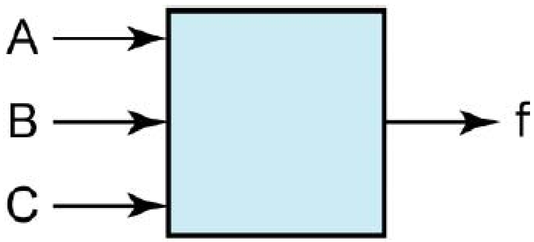


Equivalence (a.k.a. if and only if a.k.a. bi-implication)

X	Y	$X \equiv Y$
0	0	1
0	1	0
1	0	0
1	1	1



Just check the truth tables!



(a)

A B C	f	f'
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	1	0
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

(b)

Figure 4-1: **Combinational Circuit with Truth Table**

$f =$

$A'BC$
$+$
$AB'C'$
$+$
$AB'C$
$+$
ABC'
$+$
ABC

sum-of-product

(minterms)

$f =$

$(A+B+C)$
$(A+B+C')$
$(A+B'+C)$

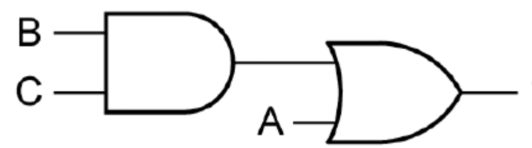
product-of-sum

(maxterms)

Simplify: $A'BC + AB'C' + AB'C + ABC' + ABC =$

$= A'BC + A(B'C' + B'C + BC' + BC) = A'BC + A =$

$= BC + A$. This corresponds to the circuit here →



Section 4.2, p. 86

GIVEN FORM	DESIRED FORM: Minterm Expansion of f	Maxterm Expansion of f	Minterm Expansion of f'	Maxterm Expansion of f'
$f = \sum m(3, 4, 5, 6, 7)$	______	$\prod M(0, 1, 2)$	$\sum m(0, 1, 2)$	$\prod M(3, 4, 5, 6, 7)$
$f = \prod M(0, 1, 2)$	$\sum m(3, 4, 5, 6, 7)$	______	$\sum m(0, 1, 2)$	$\prod M(3, 4, 5, 6, 7)$

Minterm/maxterm notation with

- Σ m (for minterms, sum of products)
- Π M (for maxterms, product of sums.