Note Title 2/9/2009

HW3 Due Friday, February 13, 2009!

See web site for details!!!

Exercises 2.1(a), 2.1(d) 2.8(b), 2.3(e), 2.5(b),

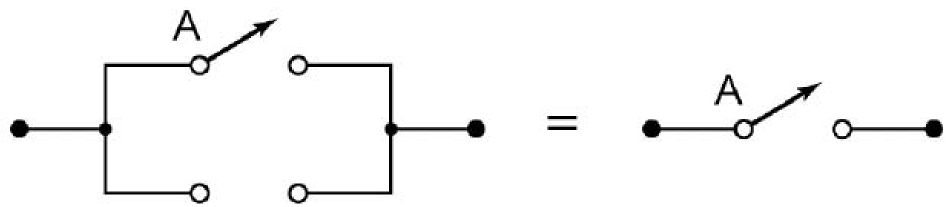
2.6(a), 2.9(a), 2.13(d), 4.21(a), and one more.

See web site for details!!!

HW2 statistics:

90 – 100 : 5
80 – 89 : 3
70 – 79 : 2
60 – 69 : 3

Basic Theorems of Boolean Algebra (Sect. 2.4)



(A + 0 = A)

(2.4)
existence of the null element for OR

0 is the null element for OR.

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

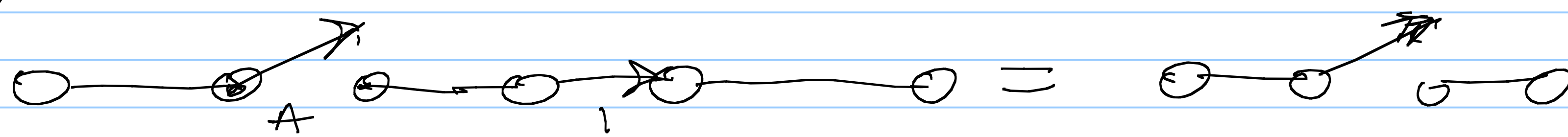
Proof : Consider the two cases

$A=0$; then $A+0 = 0+0=0$

$A=1$ then: $A+1 = 1+1 = 1$

(2.4(D)): Existence of the null element for AND:

$A \cdot 1 = A$ 1 is the null element for AND



A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1

Proof: Consider the two cases

$A=0$: then $A \cdot 1 = 0$

$A=1$: then $A \cdot 1 = A$

D for Dual.

You obtain the Dual formula (expression) of a Boolean expression E by switching $+$ with $\cdot$ and 0 with 1.
Similarly, you define the dual of an equality.
If an equality holds, its dual holds as well!

Identity theorems (laws):

$X+1=1$ (2-5) $\qquad$ $X \cdot 0 = 0$ (2-5D)

Idempotent laws:

$X+X=X$ $\qquad$ $X \cdot X = X$

Laws of complementarity: $X + X' = 1$ (2-8) $\qquad X \cdot X' = 0$ (2-8D)

Involution law (the existence of inverse)

$(X')' = X$ double negation has no effect

X	X'
0	1
1	0

Proof: Analysis of the two possible cases

$X = 0 \Rightarrow X' = 1 \Rightarrow (X')' = 1' = 0$

$X = 1 \Rightarrow X' = 0 \Rightarrow (X')' = 0' = 1$ ✓

$(AB' + D)(AB' + D)' = \ ?$

$(AB' + D)(AB' + D)' = 0$

This is an instance of $XX' = 0$ when $X = (AB' + D)$.

2.5 Commutative, Associative, and Distributive Laws

$XY = YX$ (2-9) $\qquad X + Y = Y + X$ (2-9D)

Commutative laws

$(XY)Z = X(YZ)$ (2-10) $\qquad (X+Y)+Z = X+(Y+Z)$ (2-10D)

Associative laws

$X(Y+Z) = XY + XZ$ (2-11)

Distributive law for product over sum
AND OR

$X+(YZ) = (X+Y)(X+Z)$ (2-11D)

Distributive law for sum over product
OR AND

This does not work for numbers, e.g.
$X=1, \; Y=2, \; Z=3$

$1+(2\cdot3) = (1+2)(1+3)$ false!

$7 = 12$

Table 2-2: Proof of Associative Law for AND

X	Y	Z	XY	YZ	(XY)Z	X(YZ)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

X	Y	Z	YZ	X+YZ	(X+Y)	X+Z	(X+Y)(X+Z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

$=$!

Proof of 2-11D

Alternate proof of the second distributive law (distribution of sum over product (OR over AND))

$X + YZ = (X+Y)(X+Z)$ (p. 41 text)

$(X+Y)(X+Z) = X(X+Y) \cdots$ Look at!

2-6 Simplification theorems

$XY + XY' = X$ (2.12) $(X+Y)(X+Y') = X$ (2-12D)

$\hookrightarrow X(Y+Y') =$ (by 2-8) $= X \cdot 1 =$ (null element for AND) $= X$ ✓

Note that && in Java and C Boolean expressions is <u>not</u> commutative, as the following code fragment illustrates:

if (X != ∅ && Y/X > 5) then ...

Here, Y/X is not evaluated if X = ∅.