Note Title 1/28/2009

HW1 returned. Some statistics: range was 99 − 67 = 32

avg = 90 median = 93.5 $\sigma = 8.63$

| | |
|---|---|
| 90-100 | 9 |
| 80-89: | 3 |
| 70-79: | 1 |
| 60-69: | 1 |
| | 14 |

Representation of negative numbers (Sect 1.4 of text)

Base 10 first

You need to perform $A - B$.

Assume A and B are one-digit numbers.

You can exploit the identity $A - B = A + (10 - B) - 10$, by representing $-B$ as its <u>10's complement</u>, $10 - B$.

You can exploit the identity $A - B = A + (10 - 1 - B) + 1 - 10$, by representing $-B$ as its <u>9's complement</u>, $9 - B$

You can extend this to numbers of, say, two figures, by doing 100's complement or 99's complement; in general, to numbers of k figures by doing $10^k$'s complement. $\underbrace{\hphantom{xxxx}}_{}$ $(10^k-1)$'s
However it is common to say or write 10's complement / 9's complement however many the figures are.

Here are some examples

| Sign + magnitude | 10's complement | | 9's complement |
|---|---|---|---|
| $\begin{array}{r} +12 \\ -7 \\ \hline +5 \end{array}$ | $\begin{array}{r} 12 \ + \\ 93 \\ \hline (1)05 \end{array}$ | $\begin{aligned} & 12+(100-7)-100 \\ & = 12+93-100 \\ & = 105-100 \\ & = 5 \end{aligned}$ | $\begin{array}{r} 12 \ + \\ 92 \\ \hline (1)04 \ + \\ 1 \\ \hline 5 \end{array}$ |
| $\begin{array}{r} -3 \\ -6 \\ \hline -9 \end{array}$ | $\begin{array}{r} 97 \\ 94 \\ \hline (1)91 \end{array}$ | $\begin{aligned} & (100-3)+(100-6)-200 \\ & = 97+94-200 \\ & = 191-200 \\ & = -9 \end{aligned}$ | $\begin{array}{r} 96 \ + \\ 93 \\ \hline 189 \ + \\ 1 \\ \hline 90 \end{array}$ 9's compl. repr. of $-9$ |

The argument just made holds for bases different from 10, in particular for base 2.

So, we have 2's complement and 1's complement.

Ex., when $k=3$, we do complements w.r.t. 1000 (for 2's complement) and $(1000-1)=111$ for 1's complement.

If you have k bits, you cannot represent both the positive (non negatives) numbers from 0 to $\underbrace{1\cdots1}_{k\text{ times}} = 2^k-1$ and the negative (non positives) numbers from 0 to $-(2^k-1)$. There just are not enough bits!

With sign-magnitude representation, you devote one bit to the sign. So, you have two representations of 0: $+0$, $-0$.

With sign-magnitude representation and k bits, you can represent all negative and positive numbers whose magnitude (i.e., absolute value) is between 0 and $2^{k-1}-1$

with 2's complement representation, you only use one bit string for $\phi$ — so, there is only one representation of zero ($\phi$).

with 1's complement, you still have 2 representations of $\phi$.

The extra bit string gained with 2's complement is used for an extra negative number.

For example, with $k=16$ bits, you can represent from

$-2^{16-1}$ to $+(2^{16-1}-1)$, i.e.,

$-2^{15}$ to $+(2^{15}-1)$, i.e.,

$-32{,}768$ to $32{,}767$

$2^{15} = 2^5 \times 2^{10} =$

$$\begin{array}{r} 1024 \\ \times \ 32 \\ \hline 2048 \\ 3072\phantom{0} \\ \hline 32{,}768 \end{array}$$

For example with $k = 3$, you can represent from $-(2^2-1)$ to $(2^2-1) = -3$ to $3$ with sign magnitude and 1's complement, but from $-2^2$ to $2^2-1 = -4$ to $3$ with 2's complement

| | sign magnitude | 2's complement | 1's complement |
|---|---|---|---|
| $-4$ | ~ | 100 | ~ |
| $-3$ | $-011$ | 101 | 100 |
| $-2$ | $-010$ | 110 | 101 |
| $-1$ | $-001$ | 111 | 110 |
| $-0$ | $-000$ | — | 111 |
| $+0$ | $+000$ | 000 | 000 |
| 1 | $+001$ | 001 | 001 |
| 2 | $+010$ | 010 | 010 |
| 3 | $+011$ | 011 | 011 |

01 1 10 .

$$\begin{array}{rl} 1000 & = 2^3 \\ -\ 011 & = 3 \\ \hline 101 & = 5 \end{array}$$

$$\begin{array}{rl} 111 & = 2^3-1 \\ -011 & = 3 \\ \hline 100 & = 4 \end{array}$$