Note Title 1/22/2009

1. (20 points). Calculate (a) X+Y and (b) X-Y for each of the following pair of binary numbers. Simply align the numbers on the radix point and proceed normally. Show carries and borrows clearly.

X = 1011.0101 Y = 110.11

$$\begin{array}{r} 1011.0101 \\ -\ 110.11 \\ \hline \end{array}$$

2. (20 points). Calculate (a) X+Y and (b) X-Y for the following pair of hexadecimal numbers.

X = 2CF3 Y = 2B

3. (10 points). Convert 10101.11_2 (binary) to decimal using positional notation.

4. (10 points). Convert $ABC.04_{16}$ (hexadecimal) to decimal using positional notation.

5. (15 points). Convert 110_{10} (decimal) to binary and hexadecimal by repeated dividing by 2 and 16. Check your work ~~by grouping the base 2 result~~ four bits (to base 16).

6. (15 points). Convert 0.65_{10} (decimal) to binary and hexadecimal by repeated multiplying by 2 and 16. Compute to 9 binary bits and round to 8 bits.

7. (10 points). Convert 10101.11_2 (binary) to hex by grouping.

Q3. Conversion to binary by repeated division

$2 \,\lfloor 51$

$2 \,\lfloor 25$ rem = 1 = a_0

$2 \,\lfloor 12$ rem = 1 = a_1

$2 \,\lfloor 6$ rem = 0 = a_2

$2 \,\lfloor 3$ rem = 0 = a_3

$2 \,\lfloor 1$ rem = 1 = a_4

0 rem = 1 = a_5

$51_{10} = 110011_2$ // $110011.011_2 =$

$= 32+16+2+1 . \left(\frac{1}{4}+\frac{1}{8}\right) = 51.375 \quad \left(51\frac{3}{8}\right)$

$\underbrace{0011}_{3\times 16^1}\underbrace{0011}_{3\times 16^0}{}_2 = 51_{10}$

$= 48+3 = 51_{10}$

like this

$$110011.011_2 = \underbrace{0011}\,\underbrace{0011}.\underbrace{0110}_2 = 33.6_{16} = 3\cdot16+3\cdot16^0\cdot\frac{6}{16} = 51\frac{3}{8}_{10} = 51.375_{10}$$

hex (16)	binary (2)	decimal (10)	octal (8)
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	8	10
9	1001	9	11
A	1010	10	12
B	1011	11	13
C	1100	12	14
D	1101	13	15
E	1110	14	16
F	1111	15	17

$$1101101.101_2 = \underbrace{0110}\,\underbrace{1101}.1010_2 = 6D.A_{16} = 6\cdot16+13\cdot\frac{10}{16} = 109\frac{5}{8}_{10} = 109.625_{10}$$

$$= \underbrace{001}\,\underbrace{101}\,\underbrace{101}.\underbrace{101}_2 = 155.5_8 = 64+5\cdot8+5\cdot\frac{5}{8} = 109\frac{5}{8}_{10} = 109.625_{10}$$

$$a_n 2^n + a_{n-1} 2^{n-1} + a_{n-2} 2^{n-2} + \cdots + \underbrace{a_5 2^5 + a_4 2^4 + a_3 2^3}_{[a_5 \cdot 2^2 + a_4 \cdot 2^1 + a_3 \cdot 2^0]\, 2^3,\ 2^3 = 8} + \underbrace{a_2 2^2 + a_1 2^1 + a_0 \cdot 2^0}_{b \cdot 8^0,\ \text{where } b \text{ is } a_2 2^2 + a_1 2 + a_0}$$

Convert from decimal to hexadecimal:

$101_{10} = 65_{16}$

Check: $6 \times 16^1 + 5 \times 16^0 = 96 + 6 = 101_{10}$ ✓

$16 \,\underline{|\, 101}$

$16 \,\underline{|\, 6}$ rem $5 = a_0$

0 rem $6 = a_1$

Subtraction of binary numbers

$$\begin{array}{r} 1101.01 \\ -\ 111.1 \\ \hline 100.11 \end{array} \qquad \begin{array}{r} {\scriptstyle 1111} \\ 100.11 \\ +\ 111.1 \\ \hline 1100.01 \end{array}$$

Review borrowing in your text!

$$\begin{array}{r} 1100 \\ -\ 111 \end{array}$$

$$1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 1 \cdot 2^3 + 0 \cdot 2^2 + (10)_2 \cdot 2^1 + 0 \cdot 2^0 =$$

$$- \quad 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 0 \cdot 2^3 + 2 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$- \quad 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \qquad\qquad 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$0 \quad 0 \qquad\qquad (1 \quad 0 \quad 0)_2$$