Addition of binary numbers.

Carries may be confusing when they are greater than one. So, it is recommended to add binary numbers pairwise (i.e., two at a time). This is especially common when multiplying two numbers in the usual way, i.e, by one-bit multiplication and shift.

$$\begin{array}{r} 1011 \\ \times\ 1101 \\ \hline 1011 \\ 0- \\ 1011-- \\ 1011--- \\ \hline 10001111_2 \end{array} \qquad \begin{array}{r} = 11_{10} \\ = \times 13_{10} \\ \hline 33 \\ 11- \\ \hline 143_{10} \end{array}$$

$1011_2 = 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 = 8+0+2+1 = 11_{10}$

$1101_2 = 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 = 8+4+0+1 = 13_{10}$

$1+1+1 = 3_{10} = 11_2$

$10001111_2 = 1\cdot 2^7 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 =$

$$\begin{array}{r} 128+ \\ 8+ \\ 4+ \\ 2+ \\ 1 \\ \hline 143 \end{array}$$

$1011.11 =$

$1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 + 1\cdot 2^{-1} + 1\cdot 2^{-2} =$

$= 11 + \left(\frac{1}{4} + \frac{1}{2}\right) = 11.75$

$$\begin{array}{r} 1011 \\ \times\ 1101 \\ \hline 1011 \\ 0- \\ \hline 1011 \\ 1011-- \\ \hline 110111 \\ 1011--- \\ \hline 10001111 \\ \hline \end{array}$$

$$\begin{array}{rrl} & 1011.11 & = 11.75_{10} \\ + & 1001.10 & = 9.5_{10} \\ \hline & 10101.01 & \quad 21.25_{10} \\ \end{array}$$

$= 21.25_{10}$

Converting a fractional decimal number to a fractional binary number. (You could rewrite the decimal number using base 2 for each position.)

Ex.

$.5_{10} = 5 \times 10^{-1} = 101 \cdot \frac{1}{10_{10}} = 101 \cdot \frac{1}{1010} = 0.1_2 =$ (check) $= 0 \cdot 2^0 + 1 \cdot 2^{-1} = \frac{1}{2}$ ✓

$$\begin{array}{r} 0.1 \\ 1010 \overline{)\,101\,} \\ 0 \\ \hline 1010 \\ 1010 \\ \hline 0 \end{array}$$

Easier way to convert a decimal fraction (fractional number) to binary is to do repeated multiplication

$$F = (.a_{-1}\, a_{-2}\, a_{-3} \cdots a_{-m})_R = a_{-1} R^{-1} + a_{-2} R^{-2} + a_{-3} R^{-3} + \cdots + a_{-m} R^{-m}$$

Multiplying by R yields:

$$F \cdot R = a_{-1} + a_{-2} R^{-1} + a_{-3} R^{-2} + \cdots + a_{-m} R^{-m+1}$$

Ex, $0.5_{10} = 0.1_2$

$$\begin{array}{r} 0.5 \\ \times \quad 2 \\ \hline 1.0 \end{array}$$

$a_{-1} = 1$

Ex, $.625_{10} = .101_2$

$$\begin{array}{r} .625 \\ \times \quad 2 \\ \hline 1.25 \end{array} \qquad \begin{array}{r} .25 \\ \times \; 2 \\ \hline 0.5 \end{array} \qquad \begin{array}{r} 0.5 \\ \times \; 2 \\ \hline 1.0 \end{array}$$

$a_{-1} = 1 \qquad a_{-2} = 0 \qquad a_{-3} = 1$

Not every decimal fraction has a finite representation as a binary number

$$
\begin{array}{r}
0.7 \\
\times\ 2 \\ \hline
(1).4 \\
\times\ 2 \\ \hline
(0).8 \\
\times\ 2 \\ \hline
(1).6 \\
\times\ 2 \\ \hline
(1).2 \\
\times\ 2 \\ \hline
(0).4 \\
\times\ 2 \\ \hline
(0).8 \\
\times\ 2 \\ \hline
(1).6
\end{array}
$$

$$0.7_{10} = .10\underbrace{\overline{11001}}_{\text{repeats}}$$

All fractions in base b convert to a finite fraction in base B or a repeating fraction in base B.

Do not use binary arithmetic with cents to avoid error due to conversion of fractions from base 10 to base 2.