Note Title 1/16/2009

HW 1 due Friday, January 23

Conversion from base b to base B

General technique based on the definition of positional notation for numbers. The conversion formula is the same as the representation formula,

$N = \sum_{i=-m}^{n-1} c_i b^i$, where the number to be converted is

$$c_{n-1} \cdots c_1 c_0 \underset{\uparrow}{\cdot} c_{-1} c_{-2} \cdots c_{-m}$$

radix or fractional point

Express the digits and the original base in the new (target) base B

Ex, Convert 140_{10} into binary. So: $b = 10$, $B = 2$,

$c_2 = 1$, $c_1 = 4$, $c_0 = 0$, $n = 3$, $m = 0$

Express the digits in binary:

$c_1 = 100_2 \quad c_2 = 1_2 \quad c_0 = 0_2$

Express the base in binary:

$b = 10_{10} = 1010_2$

$N = c_0 \cdot b^0 + c_1 b^1 + c_2 b^2 = 0 + 100_2(1010_2) + 1(1010_2)^{10_2} =$

$(1010)^2 =$

$1010 = 10_{10}$

$\times\ 1010 = 10_{10}$

$1\ \ 0$

1010

0

1010

$1100100_2 \overset{?}{=} 100_{10}$

$64\ 32\ \ 4 \quad \checkmark$

$1010 = 10_{10}$

$\times\ 100 = 4_{10}$

0

0

$1010 \rightarrow$

$101000 \overset{?}{=} 40_{10}$

$32+8 = 40 \quad \checkmark$

0

$+\ 1\ 0\ 1\ 0\ 0\ 0$

$+\ 1\ 1\ 0\ 0\ 1\ 0\ 0$

$1\ 0\ 0\ 0\ 1\ 1\ 0\ 0_2 \overset{?}{=} 140_{10}$

$128 + 8 + 4 = 140 \quad \checkmark$

More efficient conversion algorithms.

Converting an integer in base 10 to base 2 by repeated division (page 9 of your text).

$$N = (a_n\ a_{n-1} \cdots a_2\ a_1\ a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R + a_0$$

Divide by R

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R + a_1 = Q_1 \text{, remainder of } a_0$$

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_2 = Q_2 \text{, remainder of } a_1$$

$$\vdots$$

ex: $10 \overline{)345}$ $\quad 34$ remainder 5

Keep this process until Q is 0.

The remainders are the digits of the number in base R.

Ex. Convert 752_{10} to binary

$$= 1011110000_2$$

$512 + 128 + 64 + 32 + 16 =$

512
128
64
32
16
752_{10}

2 | 752		
2 | 376	rem 0	$= a_0$
2 | 188	rem 0	$= a_1$
2 | 94	rem 0	$= a_2$
2 | 47	rem 0	$= a_3$
2 | 23	rem 1	$= a_4$
2 | 11	rem 1	$= a_5$
2 | 5	rem 1	$= a_6$
2 | 2	rem 1	$= a_7$
2 | 1	rem 0	$= a_8$
0	rem 1	$= a_9$

376
2 | 752 rem 0

Quiz 2 assigned