Note Title 1/14/2009

Positional vs. non-positional notation

The Greeks and Romans did not use positional notation to represent numbers

Ex. CCXLIII 243

Each symbol in a Roman numeral stands for the same number regardless of where it is.

E.g., V stands for the number 5. It never stands for the number 50, which is written L.

(tally)
The unary system of numbers makes sum and subtraction very simple. To sum two numbers it is sufficient to juxtapose (concatenate) them.

Ex.

$$1111 + 11 = 111111$$

$$4 + 2 = 6$$

<u>But</u>, multiplication and division are difficult.

Also, the unary system is much less concise than the positional one

In the positional notation, each position of a numeral is associated with a weight, which is a power of the base of the number system used

Ex. (Base 10 or decimal)

4027 is the numeral representing the number
$4 \cdot 10^3 + 0 \cdot 10^2 + 2 \cdot 10^1 + 7 \cdot 10^0$

We are so used to positional notation in base 10 that it is difficult for us to distinguish the numeral 4027 from the number that it represents.

Ex. (fractional number in base 10)
$0.234 = 0 \cdot 10^0 + 2 \cdot 10^{-1} + 3 \cdot 10^{-2} + 4 \cdot 10^{-3}$

The digits of a number in base B are between 0 and B−1.

Base 2 numbers (binary numbers): $B = 2$

Examples $101011_2 =$ (this indicates that this numeral is in base 2)

$= 1\cdot 2^5 + 0\cdot 2^4 + 1\cdot 2^3 + 0\cdot 2^2 \cdot 1\cdot 2^1 + 1\cdot 2^0 =$

$= 32 + \quad 8 + \quad 2 + 1 = 43_{10}$

Example: $0.1011 =$

$= 0\cdot 2^0 + 1\cdot 2^{-1} + 0\cdot 2^{-2} + 1\cdot 2^{-3} + 1\cdot 2^{-4} =$

$= \quad \frac{1}{2} + \quad + \frac{1}{8} + \frac{1}{16} = \frac{11}{16}$ (non-negative)

Now you already know how to convert a binary number to decimal.

Conversion from base b to base B

$$c_{n-1} \cdots c_1 c_0 . c_{-1} \cdots c_{-m}$$

fractional point

$$N = \sum_{i=-m}^{n-1} c_i \cdot b^i$$, where the digits is expressed in base B and b is expressed in base B

Ex, convert 140_{10} to binary $b = 10$ $B = 2$

$$N = \sum_{i=0}^{2} c_i \cdot b^i = 1_2 \cdot 1010_2^{10_2} + 100_2 \cdot 1010_2^{1_2} + 0 \cdot 1010_2^{0_2} =$$

To be completed

Do a table of the first 10 numbers in binary

0	0
1	1
2	10
3	11
4	100
5	101

$100 = 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$

Suggestion:
[Game of NIM]