Process Systems Engineering: Selected Topics and Supplemental Course Notes

Edward P. Gatzke

Contents

Preface

This is a collection of class notes, handouts, homework assignments, and exam problems developed over the past years teaching courses in Process Systems Engineering at the University of South Carolina. Most of the material relates to ECHE 550, Chemical Process Dynamics and Control. This course covers the basics of dynamic modeling, solution and analysis of ordinary differential equations using Laplace methods, feedback control, and some advanced control topics. Information is also included from other courses, specifically ECHE 589 Intermediate Process Control. The intermediate course includes more advanced topics, such as numerical optimization and discrete time dynamic modeling.

This offering is not provided as a text book for a course. Many important topics are not covered in sufficient detail, while some derivations are provided in excruciating depth. This is expected to provide extra depth and additional examples for topics that may be lacking in other text books. Additionally, practice problems are provided and tutorial materials on Matlab/Simulink are included.

Ed Gatzke
gatzke@sc.edu
Department of Chemical Engineering
University of South Carolina

Chapter 1

Mathematics Review

Objectives

This is a review of various mathematical topics that you probably have seen in previous mathematics courses. Complete the problems where indicated with **"EXERCISE"**. Some topics are just mentioned, without specific review questions.

Function of One Variable

You should understand the basic concept of a mathematical function / algebraic function. In this course, we examine process dynamics. Things change with time, so some value like the pressure in a tank, P could be some function of time, $P(t)$ or $P(t) = f(t)$, or in a specific case $P(t) = 5sin(3t + 2)$. In other cases, you could have a parameter that changes with temperature, like a chemical reaction rate. This could be expressed as $r(T) = ke^{\frac{E}{RT}}$ where E, k, and R are assumed constant.

The mathematical function provides a mapping. A *scalar* function maps one value to another value. $f(x) : \mathbb{R}^1 \to \mathbb{R}^1$. This can be considered a input-output relationship. Some people also use the analogy of a "black-box" you put some number in (the *independent variable*) and another comes out (the *dependent variable*).

Some examples as functions of time:

$f(t) = sin(5\,t)$

$f(t) = 2\,t$

$f(t) = e^{3t}$

Additional examples including constants

$f(t) = c\,e^{\frac{t}{\tau}}$, $\tau > 0$, $c > 0$

Reaction rate r as a function of temperature T:

$r(T) = k_0 e^{\frac{E}{RT}}$

Note that k_o, R, and E are constants.

You should know how to graph functions without the use of a calculator or computer, specifically any function of time (time as independent variable). For some functions, you may want to pick a variety of values of t and evaluate the function values, then graph $f(t)\ vs.\ t$. For a sum of functions $f = f_1 + f_2$ you can plot f_1 and f_2 and add them point by point. When you multiply two functions, $f = f_1 \, f_2$ you can graph f_1 and f_2 then multiply them at each point. In many cases, you need only graph the "interesting" points of the response where something significantly changes. Interesting points could be at $t = 0$, $t = 1$, or $t = \infty$. For trigonometric functions, multiples of $\pi/2$ may be "interesting".

1. ***EXERCISE, graph the following functions by hand:***

   (a) $f(t) = e^{3t}$

   (b) $f(t) = e^{-3t}$

   (c) $f(t) = e^{-0.3t}$

   (d) $f(t) = sin(t) + 2t$

   (e) $f(t) = 2t + t^2$

   (f) $f(t) = e^t - t^3 + sin(t) - 1$

   (g) $f(t) = t\,(sin(t))$

Function of Two or More Variables

Sometimes, a value will be a function of multiple different values. Again, the mathematical function provides a mapping. A function can also map one value to another value. $f(x) : \mathbb{R}^n \to \mathbb{R}^1$, $n > 0$.

Example, in a topographic (elevation) map, elevation is a function of map position :
$ELEVATION = z = f(x, y)$ or $z = -(x^2 + y^2)$.

Example, reaction rate expression as a function of concentrations and temperature:
$r = f(C_A, \, C_B, \, T) = 3.0 \, e^{\frac{3}{8.14\,T}} \, C_A^2 \, C_B$

Example, distance from the point $(4, 2)$
$d = f(x, y) = \sqrt{(x-4)^2 + (y-2)^2}$

2. **Exercise:**

   (a) What is the function describing points on a circle of radius r as a function of x and y?

   (b) What is the function describing points on a sphere of radius r as a function of x, y, and z?

   (c) What is the function that determines the distance from the point $(3, -1, 2)$?

   (d) Assuming an ideal gas, what is the function for pressure of a gas as a function of volume, temperature, and moles of gas?

   (e) Given that you have a function of only two variables with points in $x \times y$ and a specified function of x and y, you should realize that $f(x, y)$ gives values that can be plotted in 3 dimensions. This surface (manifold) specifies the function. Try to sketch $z = f(x, y) = x^2 + y^2$ in 3D.

Solving Equations of One Variable

If you have a function of one variable, you may be able to find a *solution* to the equation $f(x) = 0$. This means you find a value of x that *satisfies* the equation. The values of x that satisfy the equation are also called the *roots* of the equation.

Sometimes you can easily solve the equation *analytically*. This means that you get a closed-form expression for the solution that satisfies the equation. For the function $f(x) = x^3$, $x = 0$ is the solution to the equation. For the quadratic equation, $ax^2 + bx + c$, the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. **YOU SHOULD KNOW THIS EQUATION.** Note, imaginary roots do not mean that something is incorrect. In many process systems engineering problems, roots of a polynomial should have imaginary components.

In many cases, you may have a simple polynomial function that requires the roots to be found. This means, given $f(x)$, what are values of x to make $f(x) = 0$?

- In the general case, $(x - r_1)(x - r_2)...(x - r_n) = 0$, roots = r_1, r_2, ... r_n that satisfy $f(x)$
- In the specific second order case, quadratic equation for: $ax^2 + bx + c = 0$ with 2 roots at $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- There are analytical expressions for roots of polynomials up to fifth order, but they are in general very, very complex.

There are a variety of numerical methods to find roots. You may have a very complex nonlinear expression, $f(x)$, that is not easily factored into roots or solved directly. To solve $f(x) = 0$ you can graph the expression, then examine the graph to locate *zero crossings* at the values of x that satisfy the function. Using the bisection method, you can evaluate the function at two points, x_L and x_R, $x_R \geq x_L$. Assuming that $f(x_L) \leq 0$ and $f(x_U) \geq 0$, you know a root must lie in the region $x_L \leq x \leq x_R$. Bisect the region to find $x_M = x_L + \frac{x_R - x_L}{2}$ and evaluate the function at x_M. Update bounds, keeping the region that must contain a solution.

3. ***EXERCISE***: Find analytically solutions to the following equations:

   (a) $f(x) = (x - 3)(x^2 - x + 12)$

   (b) $f(x) = (x^2 + 6x + 8)\,(x - 4)\,x$

   (c) $f(x) = 2x^2 + 3x + 5$

   (d) $f(x) = x^2 + x + 10$

4. ***EXERCISE***: Find numerical solutions that satisfy the following equations.

   (a) $f(x) = e^x - x^3 + sin(x) - 1$, multiple different solutions, x in radians

   (b) $f(\omega) = \pi + \tan^{-1}(20\omega) - 2\omega$, ω in radians

Solving Equations of Multiple Variables

In some cases, you have multiple unknown values. Using a degree of freedom analysis, you must have as many independent equations as unknown values in order to find a solution. For example, given the following equations:

$$\begin{aligned} 3 &= x + e^y \\ 2 &= yx \end{aligned}$$

You can say that $x = 1 - e^y$ using equation 1, then $y = \frac{2}{x}$ or $y = \frac{2}{1-e^y}$. Now, the second equation is $f(y) = \frac{2}{1-e^y} - y$ which can be satisfied if $f(y) = 0$, so try to find y such that $f(y) = 0$, if it exists. Once you find a value for y that satisfies the function, you can determine values for x from equation 1. Alternatively, for a 2D nonlinear case, you can plot $f_1(x, y) = 0$ and $f_2(x, y) = 0$ and determine the points where the two lines intercept.

5. ***EXERCISE***: Find solutions to the following equations:

$$\begin{aligned} 3 &= x^2 y \\ 4 &= x + \frac{1}{y} \end{aligned}$$

Check your solution to make sure your values for x and y satisfy both equations.

6. ***EXERCISE***: Find solutions to the following equations:

$$\begin{aligned} 4 &= x^2 + y^2 \\ 0 &= x^2 - y \end{aligned}$$

Slope of a Line

You must be able to find the slope (derivative) of a function, $\frac{df}{dt}(t)$ or $\frac{df}{dx}(x)$ given the function and know the derivative of simple functions. Note that the derivative of a function of time is also a function of time! You should also remember how to use the chain rule!

7. ***EXERCISE, calculate the derivative of the following functions:***

   (a) $f(t) = 3t^3 + 2t + 7$
   (b) $f(t) = \sqrt{t}$
   (c) $f(t) = e^t$
   (d) $f(t) = sin(at) + 3t^2$
   (e) $f(x) = (e^{ax})^2$
   (f) $f(t) = sin(3t^4)$

The derivative evaluated at a point, $\frac{df}{dt}(t)\Big|_{t=ts}$ is the slope of the function $f(t)$ at time $t = ts$. This also defines the slope of the line tangent to $f(t)$ at time $t = ts$.

8. ***EXERCISE*** Graph $f(t) = t^2 + t$ and find the value of function and the value of the slope for $f(t) = t^2 + t$ at $t = 0$, $t = -1$, $t = 1$

Basic Algebra Properties

You should know how to solve basic equations using algebraic properties, such as the distributive property, $ax + bx = (a + b)\,x$.

In some cases, you will have to solve an equation that includes a variety of constants. To solve for x in the equation $ax = by + cx$ with constants a, b, c. First, get terms with x on one side: $ax - cx = by$, then use distributive property: $(a - c)x = by$, finally divide to solve for x in terms of y and some constants: $x = \frac{by}{(a-c)}$

9. Solve the following equation for $f(x) = 0$:

$$f(x) = (2x) + 5x + 6x) - x(2 + 3x)$$

10. Solve the following equation for x.

$$ax = xy + d + 3$$

11. Evaluate the follow fractions with different denominators just to make sure you know what a common denominator is.

   (a) $\frac{2}{3} + \frac{5}{7}$

Partial Fraction Expansion

Partial Fraction Expansion of fractions with polynomials in the numerator and denominator allows for simplification of complex polynomials. Use your preferred method simplify complex fractions involving polynomials.

12. **EXERCISE** Find A and B in the following expression

$$\frac{5x + 2}{(2x + 1)(3x + 2)} = \frac{A}{2x + 1} + \frac{B}{3x + 2}$$

Determinant of a Matrix

13. ***EXERCISE:*** find the determinant of the following matrices:

   (a)
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

Multiple Linear Equations

You can find the solution of multiple linear equations by row reduction / Gaussian elimination. Linear equations are simple coefficients and variables (no x^2 terms, no e^x terms, just $ax = b$ with a and b constant coefficients.) You have learned a variety of ways to solve systems of linear equations, but a standard method is often called row reduction or Gaussian elimination.

14. ***EXERCISE:*** Solve the following set of linear equations by hand:

$$\begin{aligned} 1x + 1y + 1z &= 0 \\ 1x + 2y + 3z &= 1 \\ 3x + 3y + 1z &= 2 \end{aligned}$$

Scalar values

Numbers can be a *constant scalar* (3, -0.1, e, π) or a *variable scalar* (x, y, z). These are just basic real numbers.

Vector Values

There are many examples of vectors in 3 dimensions, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ or $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Note that you are not limited to 3 dimensions, you could specify all four concentrations in a reactor at a given time:

$$\begin{bmatrix} C_A \\ C_B \\ C_C \\ C_D \end{bmatrix}$$

Or you could specify all flow rates in a process at some time:

$$[F_1\ F_2\ F_3\ F_4\ F_5\ F_6\ F_7\ F_8\,]^T$$

Partial Derivative / Gradient of a Multivariable Function

Partial derivative as slope of the tangent surface in direction of one variable.

15. **EXERCISES**

    (a) What is $\frac{\delta f}{\delta x}$ of $f(x, y) = x^2 + y^2$?

    (b) What is $\frac{\delta f}{\delta x}$ of $f(x, y) = x^2 + y^2$ evaluated at $x = 1$, $y = 1$?

(c) What is the gradient of $f(x, y) = x^2 + y^2$?
$\begin{bmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \end{bmatrix}$

(d) What is the equation of the plane tangent to $f(x, y) = x^2 + y^2$ at $x = 2$, $y = 1$?

Integration of a Function

16. **EXERCISE** Integrate the following basic functions:

    (a) $f(x) = x^2$
    (b) $f(x) = e^{ax}$
    (c) $f(x) = sin(ax)$
    (d) $f(x) = cos(bx)$
    (e) $f(x) = ln(x)$ (Integration by parts)
    (f) $f(x) = x^2 e^x$ (Integration by parts)

Differential Equations

Basic differential equations such as:

$$\frac{dy}{dx} = y$$

This can be solved by separation of variables,

$$\frac{dy}{y} = dx$$

Integrating to get $ln(y) = x + c$. Assuming $c = 0$, $y = e^x$ where y is a function of x. The same differential equation can be put in the form

$$\frac{df}{dt}(t) = f(t)$$

with the solution $f(t) = ce^t$. Obviously, given that you know $f(t) = ce^t$, $\frac{df}{dt} = ce^t$, so $f(t) = ce^t$ is the differential equation solution, where the constant c can be found from initial conditions for $f(t)$ or $\frac{df}{dt}(t)$

17. **EXERCISE:** Go to http://www.ncsu.edu/felder-public/ILSdir/ilsweb.html and take the learning style test. **Record your four results.**
    Go to http://www.ncsu.edu/felder-public/ILSdir/styles.htm and read about your learning style.

18. ***EXERCISE: Send Dr. Gatzke an email:*** **gatzke@sc.edu Please include:**

    (a) **Your learning style test results from the previous exercise.**
    (b) **Your preferred email address. You may include more than one.**
    (c) **Your permanent home address and phone number for future survey information.**

Chapter 2

Linear Algebra

Objective

Demonstrate solution methods for systems of linear equations. Show that a system of equations can be represented in matrix-vector form.

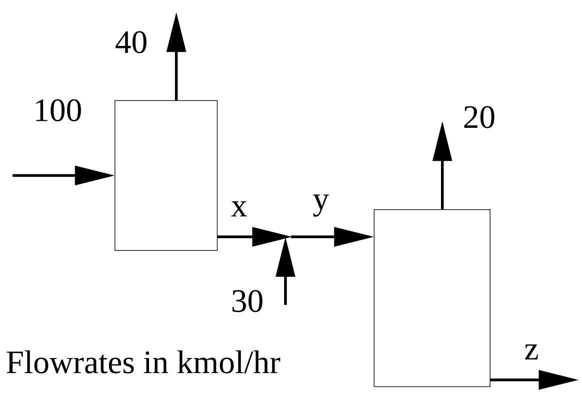


Figure 2.1: Two distillation columns in series.

2.1 Example System

Two distillation columns in series with a additional feed stream mixing in with the bottoms stream of the first column. The flow rate of three streams are unknown. As indicated in the Figure 2.1, the flow rate of streams x, y, and z are unknown. No reaction is taking place. The steadystate flow rates must be calculated.

Basic Mass Balance:

$$accumulation = in - out + created - destroyed$$

Mass Balance on first column (In this case, assume steady state: *accumulation* = 0):

$$0 = 100 - 40 - x$$

Mass balance on mixing point:

$$0 = x + 30 - y$$

Mass balance on second column:

$$0 = y - 20 - z$$

Three linear equations:

$$\begin{aligned} 0 &= 100 - 40 - x \\ 0 &= x + 30 - y \\ 0 &= y - 20 - z \end{aligned}$$

Note that you could write too many equations. You could write an overall balance:

$$0 = 100 - 40 - 20 - z$$

Ending up with an overspecified system of equations, 4 equations, 3 unknowns. Stick with the three equations from above for now.

Note that these are linear equations. The unknown variables have constant linear coefficients, nonlinear terms do not appear (no x^2, no $\sqrt{x}$, no e^x).

You can rearrange the set of three equations (without the overall balance equation) to get all the variable terms on the left side and the constants on the right. After some The set of equations can be written as:

$$\begin{aligned} 1x + 0y + 0z &= 60 \\ -1x + 1y + 0z &= 30 \\ 0x - 1y + 1z &= -20 \end{aligned} \tag{2.1}$$

As we will see later, this can be more compactly written as:

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

You may already realize that the solution to this problem is $x = 60$, $y = 90$, and $z = 70$. For more complex systems, this is not quite so easy. To solve the three linear equations simultaneously in a general manner, you can perform row reduction using three possible row operations:

RULES

1. Add (or subtract) one row to (or from) another
2. Multiply or divide a row by a scalar value (any real scalar$\neq 0$)
3. Swap position of rows

Typically you would perform these operations until you have a triangular representation (all 0's above or below the diagonal). The triangular form allows for quick solution.

The set of linear equations in Equation 2.1 can be compactly written using only the coefficients as:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ -1 & 1 & 0 & 30 \\ 0 & -1 & 1 & -20 \end{array}$$

We need to perform steps 1-3 to get the system of equations in triangular form with ones on the diagonal and zeros below the diagonal, like

$$\begin{array}{ccc|c} 1 & a & b & d \\ 0 & 1 & c & e \\ 0 & 0 & 1 & f \end{array}$$

We can look at the original system of equations and realize that we must get zeros in position 2,1 (row 2, column 1) and position 3,2 (row 3, column 2). You can multiply row 2 by -1 using Rule 2:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ 1 & -1 & 0 & -30 \\ 0 & -1 & 1 & -20 \end{array}$$

Next, swap position of rows 2 and 3 using Rule 3 to get:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ 0 & -1 & 1 & -20 \\ 1 & -1 & 0 & -30 \end{array}$$

Then, subtract row 1 from row 3 using Rule 1 to get:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ 0 & -1 & 1 & -20 \\ 0 & -1 & 0 & -90 \end{array}$$

Then, multiply rows 2 and 3 by -1 using Rule 2:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ 0 & 1 & -1 & 20 \\ 0 & 1 & 0 & 90 \end{array}$$

Subtract row 2 from row 3 using Rule 1 again to get:

$$\begin{array}{rrr|r} 1 & 0 & 0 & 60 \\ 0 & 1 & -1 & 20 \\ 0 & 0 & 1 & 70 \end{array}$$

Now, all coefficients below the diagonal are 0. The solution can be found quickly. From equation 3 (row 3), $z = 70$. Using equation 2 (row 2) $y - z = 20$, but you know that $z = 70$

so $y = 90$. Equation 1 (row 1) gives $x = 60$, so the overall solution is $x = 60$, $y = 90$, and $z = 70$.

CHECK SOLUTIONS: You can plug your solution back into the original three equations and verify that the equations are satisfied. **THIS WILL HELP YOU ON EXAMS.**

Note that the general Gaussian elimination or row reduction method specifies that you start with column 1 and perform operations until all coefficients below the diagonal are 0, then move to column 2 and perform operations until all coefficients below the diagonal are zero, etc.

2.2 Linear Equations - Special Cases

In general, there are three possibilities for a "square" set of linear equations.

2.2.1 Case A - One solution

Consider a simpler system: $x + y = 1$ and $x - y = 1$. Graphically, you can plot the two lines and look for the intersection of two lines which occurs at $x = 1$, $y = 0$. The system of equations is:

$$\begin{array}{rr|r} 1 & 1 & 1 \\ 1 & -1 & 1 \end{array}$$

Subtracting row 1 from row 2 gives:

$$\begin{array}{rr|r} 1 & 1 & 1 \\ 0 & -2 & 0 \end{array}$$

This implies $-2y = 0$ or $y = 0$ and $x + y = 1$ or $x = 1$ as you already realized.

In 3 dimensions (3 unknowns) each row represents a plane. Two equations can intersect to give a line, and a line can intersect with a third plane to give a point, the single solution (in a single solution case).

2.2.2 Case B - No solution

Consider the system $x + y = 1$ and $x + y = 2$. Graphically, this represents two lines that never intersect.

$$\begin{array}{rr|r} 1 & 1 & 1 \\ 1 & 1 & 2 \end{array}$$

Note that column 1 and column 2 are identical. Subtracting row 1 from row 2 gives:

$$\begin{array}{rr|r} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array}$$

You know that $0x + 0y = 1$ cannot be true. For a "square" system, if Gaussian elimination results in a 0 on the diagonal, this may be the case.

2.2.3 Case C - Many solutions

Consider the system $x + y = 1$ and $2x + 2y = 2$. Graphically, this represents two lines that are coincident.

$$\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \end{array}$$

Subtracting twice the value of row 1 from row 2 gives:

$$\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}$$

These equations are consistent. $0x + 0y = 0$ and $x + y = 1$ are consistent. There is no single solution, as many solutions make the equation $x + y + 1$ consistent.

2.3 Nonsquare Systems

The original example was for a "square" system with 3 unknowns and 3 equations. You may often end up with more (or fewer) equations than unknowns.

Consider the original set of equations:

$$\begin{aligned} 1x + 0y + 0z &= 60 \\ -1x + 1y + 0z &= 30 \\ 0x - 1y + 1z &= -20 \end{aligned}$$

One additional equation can be specified using a mass balance on the entire system, $0 = 100 + 30 - 40 - 20 - z$.

$$\begin{aligned} 1x + 0y + 0z &= 60 \\ -1x + 1y + 0z &= 30 \\ 0x - 1y + 1z &= -20 \\ 0x + 0y + 1z &= 70 \end{aligned} \tag{2.2}$$

These four linear equations are not "linearly independent." You can test this by using row operations to make two rows identical. Simultaneously adding row 1 and row 3 to row 2 will make row 2 the same as row 4.

$$\begin{aligned} 1x + 0y + 0z &= 60 \\ 0x + 0y + 1z &= 70 \\ 0x - 1y + 1z &= -20 \\ 0x + 0y + 1z &= 70 \end{aligned} \tag{2.3}$$

This set of equations can still be satisfied using the original solution $x = 60$, $y = 90$, and $z = 70$. In other cases, having more equations than unknowns may complicate the solution process a bit.

2.3.1 Reconciliation and Nonsquare Systems

For curve fitting, parameters that appear linearly can be formulated as a nonsquare solution to a linear algebraic system of equations. Given that you have some (scalar valued) measured value, y, that depends on a process parameter, x. Assume the model takes the form:

$$y = mx + b \tag{2.4}$$

Technically, you only need two data points to find m and b, the model parameters. Assuming that you have more than two data points, we often desire to determine the "best-fit" for the line. These parameters minimize the sum of the square of the model error. For an experiment with four data points:

$$\begin{aligned}
y(1) &= m\,x(1) + b \\
y(2) &= m\,x(2) + b \\
y(2) &= m\,x(3) + b \\
y(4) &= m\,x(4) + b
\end{aligned} \tag{2.5}$$

Here, you know values of y and x but m and b are your unknown values. This can be written as a set of equations:

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix} = \begin{bmatrix} x(1) & 1 \\ x(2) & 1 \\ x(3) & 1 \\ x(4) & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix}$$

You can get the "best-fit" solution to this overspecified set of equations using the psuedo-inverse of the matrix:

$$x = (A^T A)^{-1} A^T b$$

2.4 Vectors

A group of unknown (or known) values can be "stacked" to form a vector. In the example problem, the unknowns x, y, and z can be described by the vector $\underline{x}$:

$$\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The solution to the problem has a known value and can be written as a vector $\underline{x}_{soln}$:

$$\underline{x}_{soln} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

Note that the underbar is used to distinguish between $\underline{x}$ (the vector) and x the unknown. A vector is NOT limited to 2 or 3 unknowns (dimension of the vector).

2.5 The Matrix

A matrix is similar to a vector, having 2 dimensions. One may think of it as a group of vectors augmented together. A Matrix has a size, $m \times n$ representing m rows and n columns. The values for m and n are sometimes written as subscripts for the matrix. For example, the 2x3 matrix $\underline{\underline{A}}_{2\times 3}$ with two rows and three columns may have values:

$$\underline{\underline{A}}_{2\times 3} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}$$

Note that each of the six elements has two indices. The first index is the row, the second is the column. For the applications in this class, a matrix will have constant coefficient values. Some example matrices:

$$\underline{\underline{A}}_{2\times 3} = \begin{bmatrix} 0 & -2 & 1 \\ 5 & 1 & 0.2 \end{bmatrix} \quad \underline{\underline{B}}_{3\times 3} = \begin{bmatrix} 6 & 0 & 0 \\ -2 & 0 & -1 \\ 3 & -1 & 5 \end{bmatrix}$$

Square Matrix - A matrix with indices equal ($m = n$).

Note: A vector can be seen as a special matrix having only 1 column.

Transpose - The transpose operator swaps the indices of a matrix (or vector). For example, for $\underline{\underline{A}}_{2\times 3}$ as before:

$$\left(\underline{\underline{A}}_{2\times 3}\right)^T = \begin{bmatrix} a_{1,1} & a_{2,1} \\ a_{1,2} & a_{2,2} \\ a_{1,3} & a_{2,3} \end{bmatrix}$$

Example. For the matrix $\underline{\underline{A}}$

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\underline{\underline{A}}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Finally, one can take the transpose of a vector. For $\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\underline{x}^T = [x\, y\, z] = \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T$$

Row Vector - The transpose of a vector is also known as a row vector.

Dot Product - The dot product of two vectors is the sum of the product of the elements taken individually. Examples:

$$\underline{x} \cdot \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x^2 + y^2 + z^2$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1x + 2y + 3z$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32$$

Matrix Multiplication - Two matrices can be multiplied together. For example $\underline{\underline{A}}_{m \times n}$ can be multiplied by $\underline{\underline{B}}_{n \times j}$. Matrix $\underline{\underline{A}}$ has m rows and n columns, while $\underline{\underline{B}}$ has n rows and j columns.

$$\underline{\underline{A}}_{m \times n} = \begin{bmatrix} \dots & r_1 & \dots \\ \dots & r_2 & \dots \\ & \vdots & \\ \dots & r_m & \dots \end{bmatrix}$$

Here, each row up to r_m is a row vector with n elements.

$$\underline{\underline{B}}_{n \times j} = \begin{bmatrix} \vdots & \vdots & & \vdots \\ c_1 & c_2 & \dots & c_j \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

Here, each column up to column c_j is a vector (column vector) with n elements. To compute $\underline{\underline{A}}_{m \times n}\underline{\underline{B}}_{n \times j}$ or simply $\underline{\underline{A}} \times \underline{\underline{B}}$ or just $\underline{\underline{A}}\ \underline{\underline{B}}$

$$\underline{\underline{A}}_{m \times n}\underline{\underline{B}}_{n \times j} = \begin{bmatrix} r_1^T \cdot c_1 & r_1^T \cdot c_2 & \dots & r_1^T \cdot c_j \\ r_2^T \cdot c_1 & r_2^T \cdot c_2 & \dots & r_2^T \cdot c_j \\ \vdots & \vdots & & \vdots \\ r_m^T \cdot c_1 & r_m^T \cdot c_2 & \dots & r_m^T \cdot c_j \end{bmatrix}$$

Method - To compute $\underline{\underline{A}}_{m \times n}\underline{\underline{B}}_{n \times j}$, the result will have j columns. The first column of the result is computed by taking the dot product of $\underline{\underline{B}}_{1 \times j}$ (first column of $\underline{\underline{B}}$) with the transpose of all the rows of $\underline{\underline{A}}$. The second column of the result is computed by taking the dot product of $\underline{\underline{B}}_{2 \times j}$ (second column of $\underline{\underline{B}}$) with the transpose of all the rows of $\underline{\underline{A}}$. Repeat up to the j^{th} column of $\underline{\underline{B}}$ which produces the j^{th} column of the result.

Note: In general, $\underline{\underline{A}}\ \underline{\underline{B}} \neq \underline{\underline{B}}\ \underline{\underline{A}}$.

Conformable - In order to multiply $\underline{\underline{A}}_{m \times n}\underline{\underline{B}}_{n \times j}$ the "inner" dimensions must be equal. In $\underline{\underline{A}}_{m \times n}\underline{\underline{B}}_{n \times j}$, if the first matrix has n columns and the second matrix must n rows.

Matrix Multiplication Examples:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -4+10 \\ 4+5 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x+2y \\ x+y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 4-6 & 3 \\ 2+2 & -1 \\ 10+0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 4 & -1 \\ 10 & 0 \end{bmatrix}$$

2.6 Column Example

Consider again the equations from the original distillation column example:

$$\begin{aligned} 1x + 0y + 0z &= 60 \\ -1x + 1y + 0z &= 30 \\ 0x - 1y + 1z &= -20 \end{aligned}$$

Notice that the variables (with constant coefficients) are on the left side and constant values are on the right hand side. This set of linear equations can be represented in the compact notation $\underline{\underline{A}}\,\underline{x} = \underline{b}$ where

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 60 \\ 30 \\ -20 \end{bmatrix}$$

Identity Matrix - The identity matrix has values of one on the diagonal and zeros elsewhere. It is defined as $\underline{\underline{I}}$ and for a square matrix $\underline{\underline{A}}\;\underline{\underline{I}} = \underline{\underline{A}}$ and $\underline{\underline{I}}\;\underline{\underline{A}} = \underline{\underline{A}}$.

$$\underline{\underline{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.6.1 How to solve sets of linear equations

We need a solution to the matrix equation $\underline{\underline{A}}\,\underline{x} = \underline{b}$. You cannot "divide" by a matrix:

$$\underline{x} \neq \underline{b}/\underline{\underline{A}}$$

There is no "division" operator for a matrix. Instead, an inverse is defined for some square matrices such that

$$\underline{\underline{A}}\ \left(\underline{\underline{A}}\right)^{-1} = \underline{\underline{I}}$$

Also,

$$\left(\underline{\underline{A}}\right)^{-1}\ \underline{\underline{A}} = \underline{\underline{I}}$$

Now, to solve $\underline{\underline{A}}\,\underline{x} = \underline{b}$ for $\underline{x}$

First, multiply on the left by $\left(\underline{\underline{A}}\right)^{-1}$

$$\left(\underline{\underline{A}}\right)^{-1}\underline{\underline{A}}\,\underline{x} = \left(\underline{\underline{A}}\right)^{-1}\underline{b}$$

Realizing that $\left(\underline{\underline{A}}\right)^{-1}\ \underline{\underline{A}} = \underline{\underline{I}}$ replace $\left(\underline{\underline{A}}\right)^{-1}\ \underline{\underline{A}}$ with $\underline{\underline{I}}$.

$$\underline{\underline{I}}\,\underline{x} = \left(\underline{\underline{A}}\right)^{-1}\underline{b}$$

Now, realizing $\underline{\underline{I}}\,\underline{x}$ is $\underline{x}$, the solution is

$$\underline{x} = \left(\underline{\underline{A}}\right)^{-1}\underline{b}$$

Note that multiplying on the right will not lead to a solution.

$$\underline{\underline{A}}\,\underline{x}\ \left(\underline{\underline{A}}\right)^{-1} = \underline{b}\ \left(\underline{\underline{A}}\right)^{-1}$$

2.6.2 How determine a matrix inverse

To solve $\underline{\underline{A}}\,\underline{x} = \underline{b}$, you need to know $\left(\underline{\underline{A}}\right)^{-1}$. We are going to use row reduction to calculate $\left(\underline{\underline{A}}\right)^{-1}$. Start with $\underline{\underline{A}} \mid \underline{\underline{I}}$. use row reduction techniques until $\underline{\underline{A}}$ is $\underline{\underline{I}}$. $\left(\underline{\underline{A}}\right)^{-1}$ if it exists will be on the right where $\underline{\underline{I}}$ was originally.

Inverse Example

Solve the following for $\underline{x}$ using $\left(\underline{\underline{A}}\right)^{-1}$:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \underline{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

For this procedure, one must first calculate $\left(\underline{\underline{A}}\right)^{-1}$. Set up $\underline{\underline{A}} \mid \underline{\underline{I}}$ as:

$$\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}$$

Use row reduction to get

$$\begin{array}{cc|cc} 1 & 0 & ? & ? \\ 0 & 1 & ? & ? \end{array}$$

Then verify that $\underline{\underline{A}}\ \left(\underline{\underline{A}}\right)^{-1} = \underline{\underline{I}}$. Use $\left(\underline{\underline{A}}\right)^{-1}$ to calculate $\underline{x}$ using $\underline{x} = \left(\underline{\underline{A}}\right)^{-1}\underline{b}$. Verify solution again to be safe.

START

Start by using row reduction on

$$\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}$$

Multiply row 2 by $1/3$ to get :

$$\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & \frac{4}{3} & 0 & \frac{1}{3} \end{array}$$

Then subtract row 1 from row 2 to get:

$$\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -\frac{2}{3} & -1 & \frac{1}{3} \end{array}$$

Now, multiply row 2 by -3/2 to get:

$$\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}$$

To get the left side looking like the identity matrix, subtract 2 times row 2 from row 1. Note that this is a compound use of row reduction rules.

$$\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}$$

You now have $\left(\underline{\underline{A}}\right)^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Now verify that $\underline{\underline{A}} \ \left(\underline{\underline{A}}\right)^{-1} = \underline{\underline{I}}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1(-2) + 2(\frac{3}{2}) & 1(1) + 2(-\frac{1}{2}) \\ 3(-2) + 4(\frac{3}{2}) & 3(1) + 4(-\frac{1}{2}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

You may also verify that $\left(\underline{\underline{A}}\right)^{-1} \ \underline{\underline{A}} = \underline{\underline{I}}$

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2+3 & -4+4 \\ \frac{3}{2} - \frac{3}{2} & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, compute the solution, $\underline{x} = \left(\underline{\underline{A}}\right)^{-1} \underline{b}$.

$$\underline{x} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -10+6 \\ \frac{15}{2} - 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4\frac{1}{2} \end{bmatrix}$$

Again, verify the solution is the solution to the original equations:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \underline{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -4 \\ 4\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -4+9 \\ -12+18 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Just as expected...

2.6.3 Steady State Control Example

Two pumps are used to fill two tanks. The pumps usually operate at 50%, keeping the tanks at levels of 75 inches and 80 inches respectively.It is known that a 1% increase in pump 1 increases the height of tank 1 by 5 inches and the height of tank 2 by 3 inches. For a 1% change in pump 2, the height of tank 2 increases by 4 inches. It is desired to change the operating levels of the tanks to 110 inches and 89 inches.

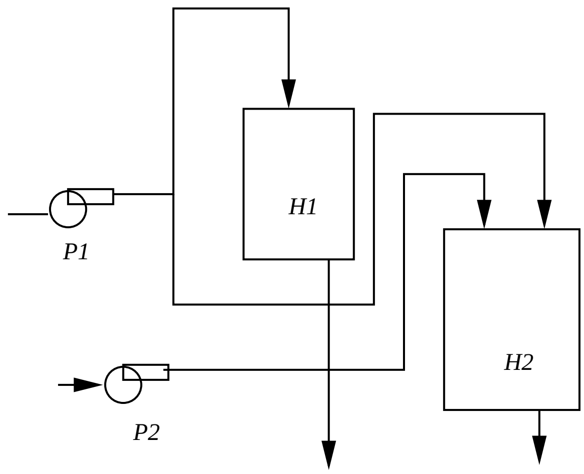


Figure 2.2: Pump / Tank example

What do you know:

$$\begin{aligned} 5\,\Delta P_1(\%) &= \Delta H_1(inches) \\ 3\,\Delta P_1(\%) + 4\,\Delta P_1(\%) &= \Delta H_2(inches) \end{aligned}$$

You know the target (reference, setpoint) for H_1 and H_2 as 110 and 89. This translates into $\Delta H_1 = 110 - 75 = 35$ and $\Delta H_2 = 89 - 80 = 9$. You need to increase tank 1 by 35 inches and increase tank 2 by 9 inches. You do not know the final values of the pump speeds. You do know the original steadystate values, 50% and 50%, realizing that:

$$P_{final} = P_{ss} + \Delta P$$

You can now set up linear equations to solve for ΔP_1 and ΔP_2, then calculate the final values for the pump speeds.

$$\begin{bmatrix} 5 & 0 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix} = \begin{bmatrix} \Delta H_1 \\ \Delta H_2 \end{bmatrix}$$

2.7 Visualization

Each row in $\underline{\underline{A}}\,\underline{x} = \underline{b}$ is a single linear equation. For a 2D problem ($\underline{x}$ with 2 elements / unknowns) the equation defines a line in the (x, y) plane. Two equations define two lines, and the unique solution to $\underline{\underline{A}}\,\underline{x} = \underline{b}$ is the point $\underline{x}$ where the lines intersect. In some cases, there may be many solutions to $\underline{\underline{A}}\,\underline{x} = \underline{b}$ and in some cases there may be no solutions to $\underline{\underline{A}}\,\underline{x} = \underline{b}$.

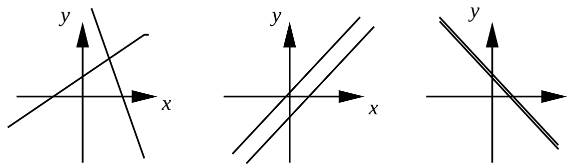


Figure 2.3: Three 2D examples with two equations. Each equation (row) represents a line. The first case has one solution, the second case has no solution, and the third case has many solutions.

For a 3D problem, each row defines the equation for a plane in 3 space. The intersection of 2 non-parallel planes is a line in 3 space, and the intersection of a line and a plane in 3 space is a point. Again, in some cases there may be a single solution, many solutions, or no solutions.

For higher dimensions, each equation defines a ***hyperplane*** in a n dimensional space, $\mathbb{R}^n$.

2.7.1 Linear Transform

A vector in $\mathbb{R}^n$ means x has n elements. Matrix multiplication of a matrix of size $m \times n$ times a vector of size $n \times 1$ "maps" the vector from $\mathbb{R}^n$ to $\mathbb{R}^m$.

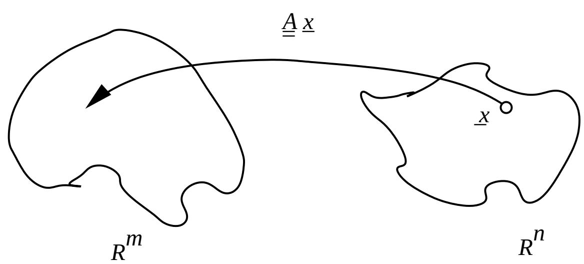


Figure 2.4: Matrix multiplication as a mapping from $\mathbb{R}^n$ to $\mathbb{R}^m$.

2.7.2 Range

The range of a matrix is the space of all possible points that may be mapped to in a matrix multiplication of that matrix times an unknown vector.

Range Example 1

For example, the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

can only map to points on the line $x + y$ in 3D as follows.

$$\underline{\underline{A}}\,\underline{x} = 2x + 2y + 0z$$

The columns of the matrix define possible directions for the matrix to transform a vector. In this example, columns 1 and 2 are the same, and column 3 is the zero vector. $\underline{\underline{A}}\,\underline{x}$ where $\underline{x}$ takes any real value will always be on the line defined by the direction $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

Range Example 2

In another example, the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

can only map to a variety of points in 3D as follows.

$$\underline{\underline{A}}\,\underline{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} z$$

Again, the columns of the matrix define possible directions for the matrix to transform a vector. In this example, only points in the directions of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ can be reached when multiplying $\underline{\underline{A}}\,\underline{x}$. These two directions form a plane in 3 dimensional space.

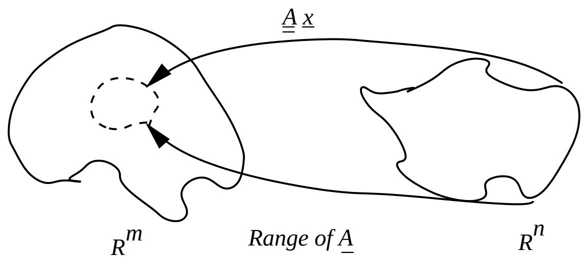


Figure 2.5: Range of $\underline{\underline{A}}$ as space in $\mathbb{R}^m$ of all possible mappings from $\mathbb{R}^n$ using matrix multiplication.

Range Example 3

In another example, the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

can only map to a variety of points in 3D as follows.

$$\underline{\underline{A}}\,\underline{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} z$$

Here, column 3 is linearly dependent upon columns 1 and 2. This means that you can find some combination of columns 1 and 2 that give column 3. Column 3 lies in the plane defined by columns 1 and column 2.

Underlying point: For $\underline{\underline{A}}\,\underline{x} = \underline{b}$ to have a solution, the $\underline{b}$ must be in the range of $\underline{\underline{A}}$.

For the last examples, if $\underline{b} = \begin{bmatrix} ? \\ ? \\ 1 \end{bmatrix}$ (if $\underline{b}$ has element in the z position) there will not be a solution to $\underline{\underline{A}}\,\underline{x} = \underline{b}$. In such a case, the possible range of $\underline{\underline{A}}$ does not include $\underline{b}$.

Range Example 4

In another example, the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

can map to all of the points in 3D as follows.

$$\underline{\underline{A}}\,\underline{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} z$$

Here, column 3 is NOT linearly dependent upon columns 1 and 2. This means that you can find some combination of columns 1, 2, and 3 that give any point in 3 dimensions.

Rank - The rank of a matrix is the number of linearly independent columns. For a square matrix of size $n \times n$, there is a unique solution if there are n independent columns. The matrix would have rank n.

Chapter 3

Laplace Transforms / Deviation Variables

3.1 Simple System Example

Consider a tank draining from an initial height of h_o at time $t = 0$. With no flow into the tank ($F_{in} = 0$) and $F_{out} = \alpha h(t)$ the mass balance can be written:

$$A \frac{dh}{dt}(t) = 0 - \alpha\, h(t)$$

Moving $\alpha\, h(t)$ to the left half side and dividing by α gives:

$$\frac{A}{\alpha}\,\frac{dh}{dt}(t) + h(t) = 0$$

A is the tank area (constant) and α is the proportionality constant for flow out of the tank. These parameters can be replaced by $\tau = A/\alpha$ to give the following differential equation:

$$\tau\,\frac{dy}{dt}(t) + y(t) = 0 \tag{3.1}$$

The initial tank height at time $t = 0$ **can be assumed to be** $y(t)|_{t=0} = y_o$. Take the Laplace transform of Equation 3.1:

$$L\left\{\tau\,\frac{dy}{dt}(t)\right\} + L\left\{y(t)\right\} = 0$$

$L\{y(t)\}$ is easy, $L\{y(t)\} = y(s)$ so we have:

$$L\left\{\tau\,\frac{dy}{dt}(t)\right\} + y(s) = 0$$

$L\left\{\tau\,\frac{dy}{dt}(t)\right\}$ is a bit more complex. First, you can realize that τ is constant. Convince yourself of this! The L operator on a constant times a function is the same as a constant times the Laplace of the function:

$$L\left\{c\,f(t)\right\} = \int_0^\infty c\,e^{-st}\,f(t) = c\int_0^\infty e^{-st}\,f(t) = c\,L\left\{f(t)\right\}$$

So you can take the constant value outside the L operator:

$$\tau\, L\left\{\frac{dy}{dt}(t)\right\} + y(s) = 0$$

Now, you must remember that $L\left\{\frac{df}{dt}(t)\right\}$ is just $s\, f(s) - f(t)|_{t=0}$.

$$\tau\ (sy(s) - y(t)|_{t=0}) + y(s) = 0$$

And we have initial conditions for the height of the tank, $y(t)|_{t=0} = y_o$

$$\tau\ (sy(s) - y_o) + y(s) = 0$$

Solving for $y(s)$:

$$\tau\, sy(s) - \tau\, y_o + y(s) = 0$$
$$\tau\, sy(s) + y(s) = \tau\, y_o$$
$$(\tau s + 1)\, y(s) = \tau\, y_o$$
$$y(s) = \frac{\tau\, y_o}{(\tau s + 1)}$$

Now rearrange a little bit

$$y(s) = \tau\, y_o\, \frac{1}{(\tau s + 1)}$$
$$y(s) = \tau\, y_o\, \frac{1}{(\tau s + 1)}\, \frac{1/\tau}{1/\tau}$$
$$y(s) = y_o\, \frac{1}{(s + \frac{1}{\tau})}$$

This you realize is a constant y_o times the term $\frac{1}{s+\frac{1}{\tau}}$. To get $y(t)$ you must use the inverse Laplace transform, L^{-1} for the $\frac{1}{s+1/\tau}$ part.

$$L^{-1}\left\{y(s)\right\} = L^{-1}\left\{y_o\, \frac{1}{(s + \frac{1}{\tau})}\right\}$$

Again, y_o is a constant and can be factored out

$$L^{-1}\left\{y(s)\right\} = y_o\, L^{-1}\left\{\frac{1}{(s + \frac{1}{\tau})}\right\}$$

And we know from lecture that $L\left\{e^{-at}\right\} = \frac{1}{s+a}$, so in our case, $a = \frac{1}{\tau}$.

$$y(t) = y_o\, e^{-(\frac{1}{\tau})t}$$

This is the solution to the original differential equation! Now check your result. At time $t = 0$ your solution for $y(t)$ is $y_o\, e^{-(\frac{1}{\tau})0} = y_o\, 1 = y_o$. This matches the initial conditions. The derivative of your result can also be found

$$\frac{dy}{dt}(t) = \frac{d}{dt}\left\{y_o\, e^{-(\frac{1}{\tau})t}\right\} = y_o\ -(\frac{1}{\tau})\, e^{-(\frac{1}{\tau})t}$$

$$\frac{dy}{dt}(t) = -\frac{y_o}{\tau}\, e^{-(\frac{1}{\tau})t}$$

Plug that back in the original differential EQ, along with your solution for $y(t)$:

$$\tau\,\frac{dy}{dt}(t) + y(t) = 0$$

$$\tau\,(-\frac{y_o}{\tau})\, e^{-(\frac{1}{\tau})t} + y_o\, e^{-(\frac{1}{\tau})t} = 0$$

And we know we have the solution!

3.2 First-Order System Modeling

The first order system model is:

$$\tau\,\frac{dy}{dt}(t) + y(t) = K\,u(t)$$

Taking the Laplace transform:

$$\tau\, sy(s) - \tau y(t)|_{t=0} + y(s) = K\,u(s)$$

If we assume that $y(t)|_{t=0} = 0$ this simplifies the equation to

$$\tau\, sy(s) + y(s) = K\,u(s)$$

We can then solve for $y(s)$

$$(\tau\, s + 1)y(s) = K\,u(s)$$

$$y(s) = \frac{K}{(\tau\, s + 1)}\, u(s)$$

Here, $\frac{K}{(\tau\, s+1)}$ is the process model relating $u(s)$ and $y(s)$. This is sometimes called $g(s) = \frac{K}{(\tau\, s+1)}$. Given $u(t)$ you can find $u(s)$, and given a model of your system you can find $g(s)$. Realizing that $y(s) = g(s)u(s)$ you can then find $y(t)$.

From a ***process reaction curve*** (the data for $y(t)$ and $u(t)$ given a step in the input $u(t)$) you can find the PROCESS GAIN K from the equation:

$$K = \frac{y_{fin} - y_{init}}{u_{fin} - u_{init}} = \frac{\Delta y}{\Delta u}$$

The time constant is a bit trickier. First, lets assume $u(t)$ is a step at time $t = 0$ from a value of 0 to a new value of A. The Laplace transform of the step function is:

$$u(s) = \frac{A}{s}$$

Now, we have enough information to get $y(s)$ and $y(t)$

$$y(s) = \frac{K}{(\tau\, s + 1)}\, u(s)$$

$$y(s) = \frac{K}{(\tau\, s + 1)}\, \frac{A}{s}$$

To solve this easily, we need the partial fraction expansion:

$$y(s) = \frac{K}{(\tau\, s + 1)}\, \frac{A}{s} = \frac{Z_1}{(\tau\, s + 1)} + \frac{Z_2}{s}$$

One way to get the partial fraction expansion is: first multiply each term by the denominator of term and set that term to zero:

$$(\tau\, s + 1)\, \frac{K}{(\tau\, s + 1)}\, \frac{A}{s} = (\tau\, s + 1)\, \frac{Z_1}{(\tau\, s + 1)} + (\tau\, s + 1)\, \frac{Z_2}{s}$$

$$(\tau\, s + 1)|_{s=-\frac{1}{\tau}}\, \frac{K}{(\tau\, s + 1)}\, \frac{A}{s} = (\tau\, s + 1)|_{s=-\frac{1}{\tau}}\, \frac{Z_1}{(\tau\, s + 1)} + (\tau\, s + 1)|_{s=-\frac{1}{\tau}}\, \frac{Z_2}{s}$$

Some terms cancel, others don't:

$$\frac{KA}{s}|_{s=-1/\tau} = Z_1 + 0$$

$$\frac{KA}{-1/\tau} = Z_1 + 0$$

$$-KA\tau = Z_1$$

Do this for the second term, Z_2/s

$$s\frac{K}{(\tau\, s + 1)}\, \frac{A}{s} = s\frac{Z_1}{(\tau\, s + 1)} + s\frac{Z_2}{s}$$

Cancel similar terms and evaluate at $s = 0$

$$\frac{K}{(\tau\, s + 1)}\, A = s\frac{Z_1}{(\tau\, s + 1)} + Z_2$$

$$\frac{K}{(\tau\, (0) + 1)}\, A = 0 + Z_2$$

$$KA = Z_2$$

The result can be written:

$$y(s) = \frac{K}{(\tau\, s + 1)}\, \frac{A}{s} = \frac{Z_1}{(\tau\, s + 1)} + \frac{Z_2}{s}$$

Substitute in Z_1 and Z_2

$$y(s) = \frac{-KA\tau}{(\tau\, s + 1)} + \frac{KA}{s}$$

Simplify terms:

$$y(s) = -KA\,\frac{\tau}{(\tau\,s+1)} + KA\,\frac{1}{s}$$

$$y(s) = -KA\,\frac{\tau}{(\tau\,s+\frac{1}{\tau})}\,\frac{1/\tau}{1/\tau} + KA\,\frac{1}{s}$$

$$y(s) = -KA\,\frac{1}{(s+\frac{1}{\tau})} + KA\,\frac{1}{s}$$

$$y(s) = -KA\,\frac{1}{(s+\frac{1}{\tau})} + KA\,\frac{1}{s}$$

We can invert each term in this expression. $L\{e^{-at}\}$ is $\frac{1}{s+a}$, so $L^{-1}\{\frac{1}{(s+\frac{1}{\tau})}\}$ is just $e^{-(\frac{1}{\tau})\,t}$. We know for the step function from 0 to 1 at time 0 the Laplace transform is $\frac{1}{s}$. The resulting solution $y(t)$ is composed of two different functions, $e^{-(\frac{1}{\tau})\,t}$ and a step at time 0.

$$y(s) = -KA\,\frac{1}{(s+\frac{1}{\tau})} + KA\,\frac{1}{s}$$

$$L^{-1}\left\{y(s)\right\} = L^{-1}\left\{-KA\,\frac{1}{(s+\frac{1}{\tau})}\right\} + L^{-1}\left\{KA\,\frac{1}{s}\right\}$$

Again, using the argument about constants times a function, we can pull out the KA terms.

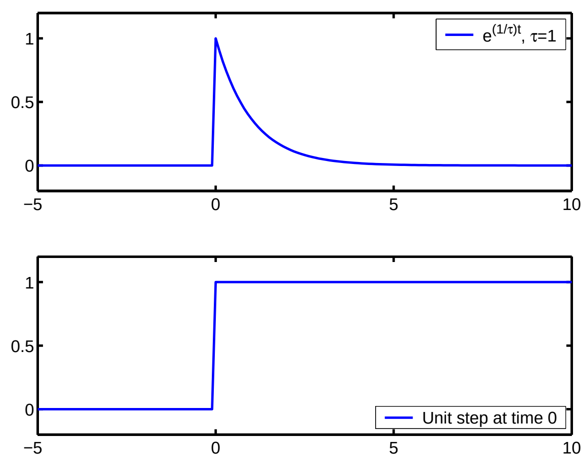
$$L^{-1}\left\{y(s)\right\} = -KA\,L^{-1}\left\{\frac{1}{(s+\frac{1}{\tau})}\right\} + KA\,L^{-1}\left\{\frac{1}{s}\right\}$$

$$y(t) = -KA\,e^{-(\frac{1}{\tau})\,t} + KA$$

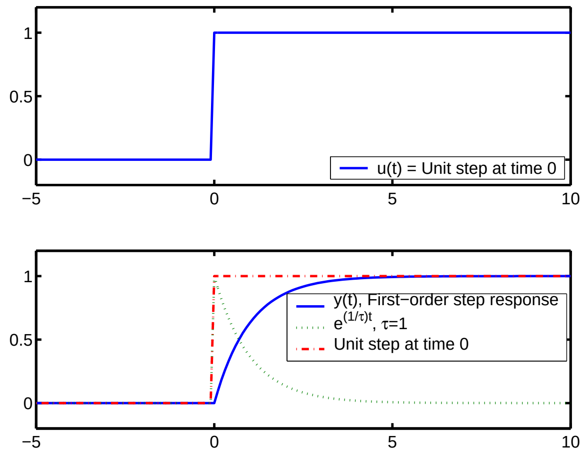
$$y(t) = KA(-\,e^{-(\frac{1}{\tau})\,t} + 1)$$

$$y(t) = KA(1 - e^{-(\frac{1}{\tau})\,t})$$

Laplace transforms assume everything is 0 before time 0. This function $y(t)$ only is defined for $t \geq 0$. The two separate functions that comprise $y(t)$ are shown in the following graph, $e^{-(\frac{1}{\tau})\,t}$ and a unit step at time zero:



Graphing the actual system (the sum of the two functions):



3.3 Deviation Variables

Lets examine a realistic First-Order system, the tank system.

$$A\frac{dh}{dt}(t) = F_i(t) - \alpha\, h(t)$$

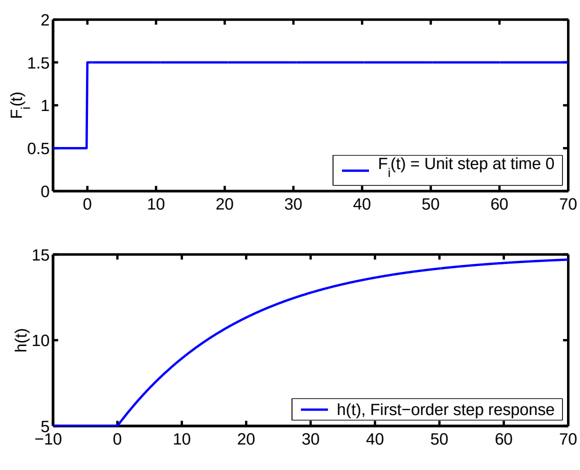
Assume the flow manipulated and has units of $\frac{m^3}{s}$. The height of the tank will be measured, and the height of the tank is given in units of m. The area of the tank is $2\, m^2$. For the outlet term to be consistent with the units of other terms ($\frac{m^3}{s}$), α must have units of $\frac{m^2}{s}$. Assume α has a value of $0.1\, \frac{m^2}{s}$. The mass balance can be written as:

$$2\frac{dh}{dt}(t) = F_i(t) - 0.1\, h(t)$$

Now, assume that you normally operate this tank at a flow rate of entering the tank of $0.5\, \frac{m^3}{s}$. This means we know the steady state flow rate into the tank, $F_{iss} = 0.5\, \frac{m^3}{s}$. This also means we can figure out the steady state height of the tank from the mass balance. At steady state, $\frac{dh}{dt}(t) = 0$

$$\begin{aligned}
2\frac{dh}{dt}(t) &= F_i(t) - 0.1\, h(t) \\
\frac{dh}{dt}|_{ss} &= F_{iss} - 0.1\, h_{ss} \\
0 &= 0.5\, \frac{m^3}{s} - 0.1\, \frac{m^2}{s}\, h_{ss} \\
-0.5\, \frac{m^3}{s} &= -0.1\, \frac{m^2}{s}\, h_{ss} \\
5\, m &= h_{ss}
\end{aligned}$$

So now we know h_{ss}, the steady state height of the tank. Now to make our life easier when taking Laplace transform, we put everything in **Deviation Variables**. This means we subtract the steady state from the normal functions of time. The purpose of this is to make the functions all start at a value of 0. Currently, a step response for the tank system looks like:



Using the variables in deviation form, assume $y(t) = h(t) - h_{ss}$. This means that if we start at steady state at time 0, $y(t)$ will equal 0 at the initial steady state value, $y(t)|_{t=0} = 0$. The other deviation variable can be written $u(t) = F_i(t) - F_{iss}$. This means the input $u(t)$ equals 0 at the initial starting point, $u(t)|_{t=0} = 0$. Also, taking the derivative WRT time of $y(t) = h(t) - h_{ss}$ yields

$$\frac{dy}{dt}(t) = \frac{dh}{dt}(t) - \frac{dh_{ss}}{dt}(t)$$

But h_{ss} does not change with time.

$$\frac{dy}{dt}(t) = \frac{dh}{dt}(t) - 0$$

$$\frac{dy}{dt}(t) = \frac{dh}{dt}$$

The dynamic mass balance is written as:

$$2\frac{dh}{dt}(t) = F_i(t) - 0.1\,h(t)$$

The steady state mass balance is written as:

$$0 = F_{iss} - h_{ss}$$

Subtracting the steady state mass balance from the dynamic mass balance gives:

$$2\frac{dh}{dt}(t) - 0 = F_i(t) - F_{iss} - 0.1\,h(t) - (-h_{ss})$$

$$2\frac{dh}{dt}(t) = (F_i(t) - F_{iss}) - (0.1\,h(t) - h_{ss})$$

And replacing what we can with deviation variables:

$$2\frac{dy}{dt}(t) = u(t) - 0.1\,y(t)$$

To put this in the "traditional" $\tau\frac{dy}{dt} + y = Ku$ form, divide by 0.1.

$$\frac{2}{0.1}\frac{dy}{dt}(t) = \frac{1}{0.1}u(t) - 1\,y(t)$$

$$20\frac{dy}{dt}(t) + y(t) = 10\,u(t)$$

So we know that $\tau = 20$ and $K = 10$ for this process.

Now, you can easily take the Laplace transform of this dynamic model.

$$L\left\{20\,\frac{dy}{dt}(t)\right\} + L\left\{y(t)\right\} = L\left\{10\,u(t)\right\}$$

$$20\,L\left\{\frac{dy}{dt}(t)\right\} + L\left\{y(t)\right\} = 10\,L\left\{u(t)\right\}$$

$$20\ (sy(s) - y(t)|_{t=0}) + y(s) = 10\, u(s)$$

Since we put everything in deviation variables, $y(t)|_{t=0}$ is now 0.

$$20\ (sy(s) - 0) + y(s) = 10\, u(s)$$

$$20\, sy(s) + y(s) = 10\, u(s)$$

Solving for $y(s)$:

$$20\, sy(s) + y(s) = 10\, u(s)$$

$$(20\, s + 1)\, y(s) = 10\, u(s)$$

$$y(s) = \frac{10}{(20\, s + 1)}\, u(s)$$

Again, you see this in the form $\frac{K}{\tau s+1}$. We want to get the expression for $y(s)$ as a function of s, not a function of s and $u(s)$. We know the value for $u(t)$. In the original variables, $F_i(t)$ changed from 0.5 to 1.5 at time t=0. We do not know the Laplace transform for a step from 0.5 to 1.5 at time $t = 0$. In deviation variables, $u(t)$ changes from a value of 0 to a value of 1 at time $t = 0$. We know the Laplace transform of a step function from 0 to 1 at time $t = 0$. This value is $u(s) = \frac{1}{s}$

$$y(s) = \frac{10}{(20\, s + 1)}\, \frac{1}{s}$$

Using our partial fraction expansion:

$$y(s) = \frac{-200}{(20\, s + 1)} + \frac{10}{s}$$

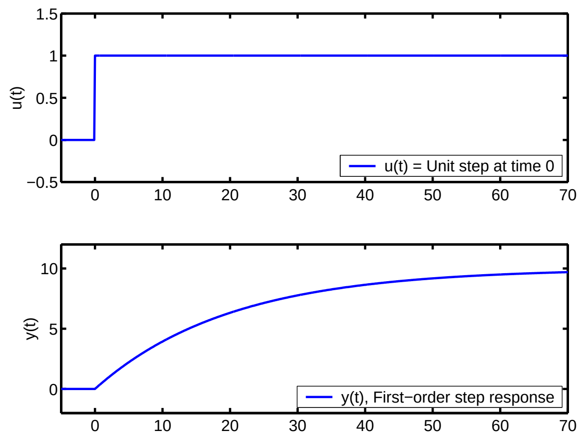
$$y(s) = \frac{-200}{(20\, s + 1)} \frac{1/20}{1/20} + \frac{10}{s}$$

$$y(s) = \frac{-10}{(s + \frac{1}{20})} + \frac{10}{s}$$

$$y(s) = 10 \left(\frac{1}{s} - \frac{1}{(s + \frac{1}{20})} \right)$$

$$y(t) = 10 \left(1 - e^{-(\frac{1}{20})t} \right)$$

This expression for $y(t)$ can be plotted. Note that $y(t)$ and $u(t)$ start at zero.



What value does the response take when $t = \tau$? In this case, $\tau = 20$.

$$y(t|_{t=20}) = 10\left(1 - e^{-(\frac{1}{20})20}\right)$$

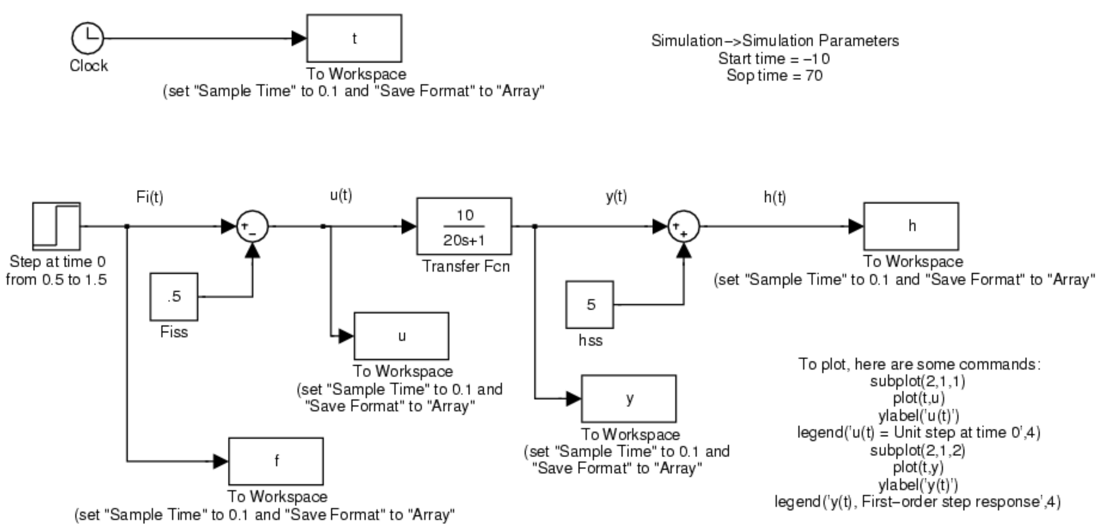
$$y(t|_{t=20}) = 10\left(1 - e^{-1}\right)$$

$$y(t|_{t=20}) = 10\left(1 - 0.3678\right)$$

$$y(t|_{t=20}) = 10\left(0.6321\right)$$

$$y(|_{t=20}) = 6.32$$

So at time $t = \tau$ the response is 6.32, or 63% of the final value of 10.
This can also be simulated in Simulink:



After running the simulation, the results will be put in vectors in the Matlab workspace. These vectors are named (in this example) t, y, u, h, and f. Note that the step occurs at time

$t = 0$, so you should start the simulation at time $t = -10$. Also note that the "To Workspace" blocks must have the "Save Format" set to "Array".

The following plotting command will let you plot $u(t)$ and $y(t)$ on the same figure:

```
subplot(2,1,1)
plot(t,u)
ylabel('u(t)')
legend('u(t) = Unit step at time 0',4)
subplot(2,1,2) plot(t,y)
ylabel('y(t)')
legend('y(t), First-order step response',4)
```

Chapter 4

Basic Procedures for Common Problems

4.1 Steady State Multivariable Modeling and Control

1. Determine what variables are available to manipulate (inputs, Δu) and what variables are available to measure (outputs, Δy)

2. Note how many input and output variables you have.

3. Start to write equations for the **output variables**. This means write something in the form:

$$\begin{aligned} \Delta y_1 &= ??? \\ \Delta y_2 &= ??? \\ \vdots & \quad \vdots \\ \Delta y_n &= ??? \end{aligned}$$

4. Read through the problem and establish relationships between individual inputs (Δu_i) and individual outputs (Δy_j). The relationships generally represent the **gain** of the individual input output relationship, for example $\Delta y_j = K\Delta u_i$. For example: "Changing input 1 by 2% decreases output 1 by 5" means $\Delta u = 2\%$ and $\Delta y = -5$ and

$$-5 = K2$$

Or $K = -5/2$ and $\Delta y_1 = -2.5\Delta u_1$.

5. Put all of the relationships into the equations. Keep reading through the word expression until you relate all specified inputs and outputs:

$$\begin{aligned} \Delta y_1 &= -2.5\Delta u_1 + ??? \\ \Delta y_2 &= 4??? \\ \vdots & \quad \vdots \\ \Delta y_n &= ??? \end{aligned}$$

6. Write out the equations with all input variable in every equation, even if they have a 0 coefficient.

$$\begin{aligned}
\Delta y_1 &= -2.5\Delta u_1 + 0\Delta u_2 + 3\Delta u_3 \\
\Delta y_2 &= 0\Delta u_1 + 4\Delta u_2 + 1\Delta u_3 \\
\Delta y_3 &= 5\Delta u_1 + 10\Delta u_2 + 2\Delta u_3
\end{aligned}$$

7. Realize that this can be put in the form:

$$\underline{\Delta y} = \underline{\underline{K}}\,\underline{\Delta u}$$

4.2 Dynamic Modeling

1. Try to figure out what is changing with time. Try to figure out what are manipulated inputs ($u_i(t)$), what are disturbances ($d_i(t)$) and what are measurements ($y_i(t)$).
2. Start to write dynamic mass and energy balances for the items that are changing.
3. Note the accumulation term
   (a) Changing volume: $V(t) = Ah(t) \;\rightarrow\; A\frac{dh}{dt}(t)$
   (b) Changing amount of species in a tank: $VC_A(t) \;\rightarrow\; V\frac{dC_A}{dt}(t)$
   (c) Changing temperature in a tank: $V\rho C_p(T(t) - T^*) \;\rightarrow\; V\rho C_p\frac{dT}{dt}(t)$
4. Don't forget reaction terms for reacting systems. $Vr(t)$ where $r(t)$ is the reaction rate, usually in the form $r(t) = kC_A(t)$ (or more complex).
5. Write your equations and check units.

4.3 State Space

1. Identify $\underline{x}$ as the values that are changing with time in your accumulation term.
2. Identify your manipulated inputs $\underline{u}$.
3. Identify your measurement equations. Your measurements should be expressed as functions of the states and inputs.
4. Write your dynamic equations, including terms for every state and input (with 0 coefficient if necessary).
5. Reorder the terms in you dynamic equations such that states come first in order, then inputs. For example:

$$\frac{dx_1}{dt} = 2x_1 + 3x_2 + 0x_3 + 2u_1 + 5u_2$$

6. Put the dynamic equations in the form

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

7. Write your measurement equations, including terms for every state and input (with 0 coefficient if necessary).

8. Put your measurement equations in the form:

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

4.4 Laplace Transform of Dynamic Equations

1. If your steady state values are not all = 0, take your dynamic model equations and establish the steady state values for you inputs, states, and outputs. This is accomplished by solving for unknowns with the accumulation terms = 0.

2. If your equations are nonlinear, **linearize** your equations.

$$A\frac{dh}{dt}(t) = F_{in}(t) - \sqrt{h(t)}$$

   Here, $\sqrt{h(t)}$ is nonlinear. Near steady state, it can be approximated as

$$\sqrt{h(t)} \simeq \sqrt{h_{ss}} + \frac{1}{2}h_{ss}^{-\frac{1}{2}}\,(h(t) - h_{ss})$$

   such that

$$A\frac{dh}{dt}(t) = F_{in}(t) - \left(\sqrt{h_{ss}} + \frac{1}{2}h_{ss}^{-\frac{1}{2}}\,(h(t) - h_{ss})\right)$$

3. Subtract the steady state model equations from the dynamic model equations to put everything in **deviation variables**. For example, $y(t) = h(t) - h_{ss}$ and $u(t) = F_{in}(t) - F_{inss}$.

   (a) Remember to express the accumulation term with your deviation variables. For $y(t) = h(t) - h_{ss}$, taking the derivative, $\frac{dy}{dt}(t) = \frac{dh}{dt}(t)$ because h_{ss} is constant.

4. Express your dynamic problem using deviation variables $u(t)$, $y(t)$, $d(t)$. These functions of time should = 0 at time $t = 0$.

5. Take the Laplace transform of your system.

6. Solve algebraically to get in the form

$$y(s) = g(s)\,u(s)$$

   or

$$\frac{y(s)}{u(s)} = g(s)$$

7. If you have disturbances and inputs, your model can look like

$$y(s) = g(s)\,u(s) + g_d(s)\,d(s)$$

Note that to get $g(s)$ you can assume $d(s) = 0$ then solve for $g(s)$. To get $g_d(s)$ you assume $u(s) = 0$ and solve for $g_d(s)$.

8. If you multiple inputs inputs, your model can look like

$$y(s) = g_1(s)\,u_1(s) + g_2(s)\,u_2(s)$$

9. If you have multiple inputs and multiple measurements, your model can look like

$$\begin{aligned} y_1(s) &= g_{11}(s)\,u_1(s) + g_{12}(s)\,u_2(s) \\ y_2(s) &= g_{21}(s)\,u_1(s) + g_{22}(s)\,u_2(s) \end{aligned}$$

10. Given the input as a function of time $u(t)$ (or input and disturbances) you can determine $u(s)$ (or $u(s)$ and $d(s)$).

11. Plug in to get an expression for $y(s)$ in terms of the variable s

4.5 Laplace of Complex Functions

1. You should be familiar with basic functions of time (step, impulse, ramp, exponential decay, sinusoid).

2. If the function is not 0 for $t < 0$ you should put the function in deviation variables. For example, a step in $F_{in}(t)$ at time 0 from 2 to 3 can be expressed as a unit step in $u(t)$ at time 0 with $u(t) = F_{in}(t) - F_{inss}$

3. You should be able to express the complex function as a single function of time. Multiply by the Heaviside function if needed. For a function that ramps from 0 with a slope of 2 until time 10 settling out at a value of 20, this can be expressed as

$$f(t) = 2\,t\,H(t) \;+\; (-2)\,(t-10)\,H(t-20)$$

4. Sketch the individual terms in your function as functions of time, then add them together to check your formulation. You can plug in numbers to check your function.

5. For each term, shift it in time such that the "event" occurs at time zero and determine the Laplace transform. Use the time shift operator if necessary to express the function as some $f(s)$. For the example:

$$f(s) = \frac{2}{s^2} + \frac{-2}{s^2}e^{-20s}$$

4.6 Solving for $y(t)$

1. Establish $y(s)$ as a function of s. (Develop dynamic model, take Laplace of model, and determine $u(s)$ and $d(s)$ if needed)

2. Your response may be in the form

$$y(s) = \frac{N_1(s)}{D_1(s)} + \frac{N_2(s)}{D_2(s)} e^{-\alpha s} + ... + \frac{N_3(s)}{D_3(s)} e^{-\beta s}$$

   This expression with multiple terms will be treated as multiple different responses, each shifted in time.

3. If you have a time delay, $e^{-\alpha s}$, ignore it for now.

4. Take a term from $y(s)$ and determine the **poles,** the roots of $D_i(s)$.

5. Perform a **Partial Fraction Expansion** on the term. For expressions with unique poles p_i the result looks like:

$$\frac{N_1(s)}{D_1(s)} = \frac{Z_1}{(s-p_1)} + \frac{Z_2}{(s-p_2)} + ... + \frac{Z_n}{(s-p_n)}$$

   For non-unique poles or imaginary roots, check the Appendix. Non-unique Poles will result in

$$\frac{Z_1}{(s-p_1)} + \frac{Z_2 s}{(s-p_1)} + \frac{Z_3 s^2}{(s-p_1)}$$

   while imaginary roots result in sin or cosine in your $y(t)$

6. Now you should be able to determine the inverse Laplace transform of each expression to yield a function of time, $y_1(t)$.

$$y_1(t) = Z_1 e^{-p_1 t} + Z_2 e^{-p_2 t} + ... + Z_n e^{-p_n t}$$

7. If you had a time delay in your term, shift the response by the time delay:

$$y_1(t) = \left(Z_1 e^{-p_1(t-\alpha)} + Z_2 e^{-p_2(t-\alpha)} + ... + Z_n e^{-p_n(t-\alpha)}\right) \; H(t-\alpha)$$

8. Do this procedure for all your terms in the original $y(s)$

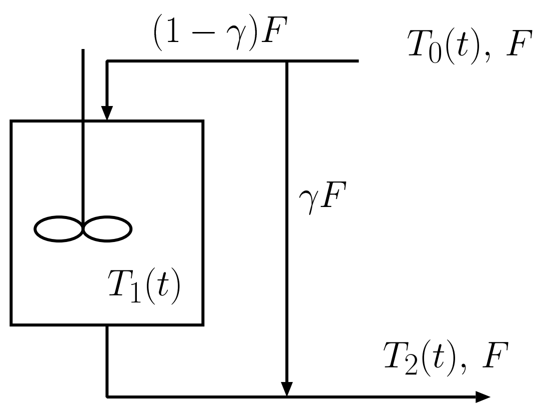
9. Add up all $y_i(t)$ to get $y(t)$

Chapter 5

Lead-Lag

Objective:

A constant volume, constant flowrate mixer is used in the configuration below. Determine the unit step response for the outlet temperature $T_2(t)$.



5.1 Modeling Example System

1. Dynamic Model - Develop an energy balance for the mixing tank:

$$\frac{d\left(\rho V C_p (T_1(t) - T^*)\right)}{dt} = \gamma F \rho C_p \left(T_o(t) - T^*\right) - \gamma F \rho C_p \left(T_1(t) - T^*\right)$$

$$\rho V C_p \, \frac{d\left(T_1(t) - T^*\right)}{dt} = \gamma F \rho C_p \left(T_o(t) - T^*\right) - \gamma F \rho C_p \left(T_1(t) - T^*\right)$$

$$\frac{V}{\gamma F} \frac{d\left(T_1(t) - T^*\right)}{dt} = \left(T_o(t) - T^*\right) - \left(T_1(t) - T^*\right)$$

2. Determine an energy balance on the mixing point:

$$0 = (1-\gamma) F\rho C_p \left(T_o(t) - T^*\right) + \gamma F\rho C_p \left(T_1(t) - T^*\right) - F\rho C_p \left(T_2(t) - T^*\right)$$

$$0 = (1-\gamma)\,\left(T_o(t) - T^*\right) + \gamma \left(T_1(t) - T^*\right) - \left(T_2(t) - T^*\right)$$

3. Put your dynamic (and steady state) equations into deviation variables. In this case, we will use the following deviation variables: $u(t) = T_o(t) - T^*$, $x(t) = T_1(t) - T^*$, and $y(t) = T_2(t) - T^*$.

$$\frac{V}{\gamma F}\,\frac{dx(t)}{dt} = u(t) - x(t)$$

$$0 = (1-\gamma)\,u(t) + \gamma x(t) - y(t)$$

4. Take the Laplace transform of the equations:

$$\frac{V}{\gamma F}\,\left(sx(s) - x(t=0)\right) = u(s) - x(s)$$

$$0 = (1-\gamma)\,u(s) + \gamma x(s) - y(s)$$

Because of the deviation variables, $x(t=0) = 0$

$$\frac{V}{\gamma F}\,\left(sx(s)\right) = u(s) - x(s)$$

Rearranging the mixing tank equation:

$$x(s) = \frac{1}{\frac{V}{\gamma F}\,s + 1} u(s)$$

5. We need the relationship between $u(t)$ and $y(t)$. Substitute the mixing tank equation into the mixing point equation:

$$0 = (1-\gamma)\,u(s) + \frac{\gamma}{\frac{V}{\gamma F}\,s + 1} u(s) - y(s)$$

Rearrange to get in the form $y(s) = g(s)u(s)$

$$0 = (1-\gamma)\,u(s) + \frac{\gamma}{\frac{V}{\gamma F}\,s + 1} u(s) - y(s)$$

$$y(s) = (1-\gamma)\,u(s) + \frac{\gamma}{\frac{V}{\gamma F}\,s + 1} u(s)$$

$$y(s) = (1-\gamma)\,u(s) + \frac{\gamma}{\frac{V}{\gamma F}\,s + 1} u(s)$$

$$y(s) = (1-\gamma)\,u(s)\left(\frac{\frac{V}{\gamma F}\,s + 1}{\frac{V}{\gamma F}\,s + 1}\right) + \frac{\gamma}{\frac{V}{\gamma F}\,s + 1} u(s)$$

$$y(s)=\frac{(1-\gamma)\,u(s)\left(\frac{V}{\gamma F}\,s+1\right)+\gamma u(s)}{\frac{V}{\gamma F}\,s+1}$$

$$y(s)=\frac{u(s)\left((1-\gamma)\frac{V}{\gamma F}\,s+(1-\gamma)\right)+\gamma u(s)}{\frac{V}{\gamma F}\,s+1}$$

$$y(s)=\frac{u(s)\left((1-\gamma)\frac{V}{\gamma F}\,s+(1-\gamma+\gamma)\right)}{\frac{V}{\gamma F}\,s+1}$$

$$y(s)=\frac{u(s)\left((1-\gamma)\frac{V}{\gamma F}\,s+1\right)}{\frac{V}{\gamma F}\,s+1}$$

$$\frac{y(s)}{u(s)}=\frac{\left(\frac{V(1-\gamma)}{\gamma F}\,s+1\right)}{\frac{V}{\gamma F}\,s+1}$$

For $\gamma=1$ this reduces to (a first order system):

$$\frac{y(s)}{u(s)}=\frac{1}{\frac{V}{\gamma F}\,s+1}$$

For $\gamma=0$ this reduces to a pure gain system. The original equation

$$\frac{y(s)}{u(s)}=\frac{\left(\frac{V(1-\gamma)}{\gamma F}\,s+1\right)}{\frac{V}{\gamma F}\,s+1}$$

is is in the form

$$\frac{y(s)}{u(s)}=\frac{K\,(\xi\,s+1)}{\tau\,s+1}$$

with $K=1$, $\tau=\frac{V}{\gamma F}$, and $\xi=\frac{V(1-\gamma)}{\gamma F}$.

If we want this in the form:

$$\frac{K\,(\xi\,s+1)}{\tau\,s+1}=A_0+\frac{A_1}{\tau s+1}$$

$$\frac{K\,(\xi\,s+1)}{\tau\,s+1}=\frac{A_o\tau s+A_0}{\tau s+1}+\frac{A_1}{\tau s+1}$$

$$\frac{K\,(\xi\,s+1)}{\tau\,s+1}=\frac{A_o\tau s+A_0+A_1}{\tau s+1}$$

$$\frac{K\xi\,s+K}{\tau\,s+1}=\frac{A_o\tau s+(A_0+A_1)}{\tau s+1}$$

$$\begin{aligned}K\xi &= A_0\tau\\ K &= A_0+A_1\end{aligned}$$

$$
\begin{aligned}
\frac{K\xi}{\tau} &= A_0 \\
K &= \frac{K\xi}{\tau} + A_1
\end{aligned}
$$

$$A_1 = K - \frac{K\xi}{\tau}$$

$$A_1 = K\left(1 - \frac{\xi}{\tau}\right)$$

$$A_0 = K\frac{\xi}{\tau}$$

Now, letting $\rho = \frac{\xi}{\tau}$

$$A_0 = K\rho$$

$$A_1 = K(1-\rho)$$

$$\frac{K\,(\xi\,s+1)}{\tau\,s+1} = K\rho + \frac{K(1-\rho)}{\tau s+1}$$

This means the lead lag transfer function is really just two systems in parallel, a pure gain system and a first order system. The value ρ can be seen as a weighting value.

Back to the problem, we wanted step response. This means that $u(s) = \frac{1}{s}$

$$y(s) = \frac{K\,(\xi\,s+1)}{\tau\,s+1}\frac{1}{s}$$

Using partial fraction expansion, we need to break this down to

$$y(s) = \frac{K\,(\xi\,s+1)}{\tau\,s+1}\frac{1}{s} = \frac{Z_1}{\tau s+1} + \frac{Z_2}{s}$$

Multiply by $\tau s + 1$ and set $s = -\frac{1}{\tau}$ to get Z_1.

$$Z_1 = \frac{K\,(\xi\,s+1)}{s}|_{s=-\frac{1}{\tau}}$$

$$Z_1 = \frac{K\left(\xi\ -\frac{1}{\tau}+1\right)}{-\frac{1}{\tau}}$$

$$Z_1 = -K\tau\left(\xi\,(-\frac{1}{\tau})+1\right)$$

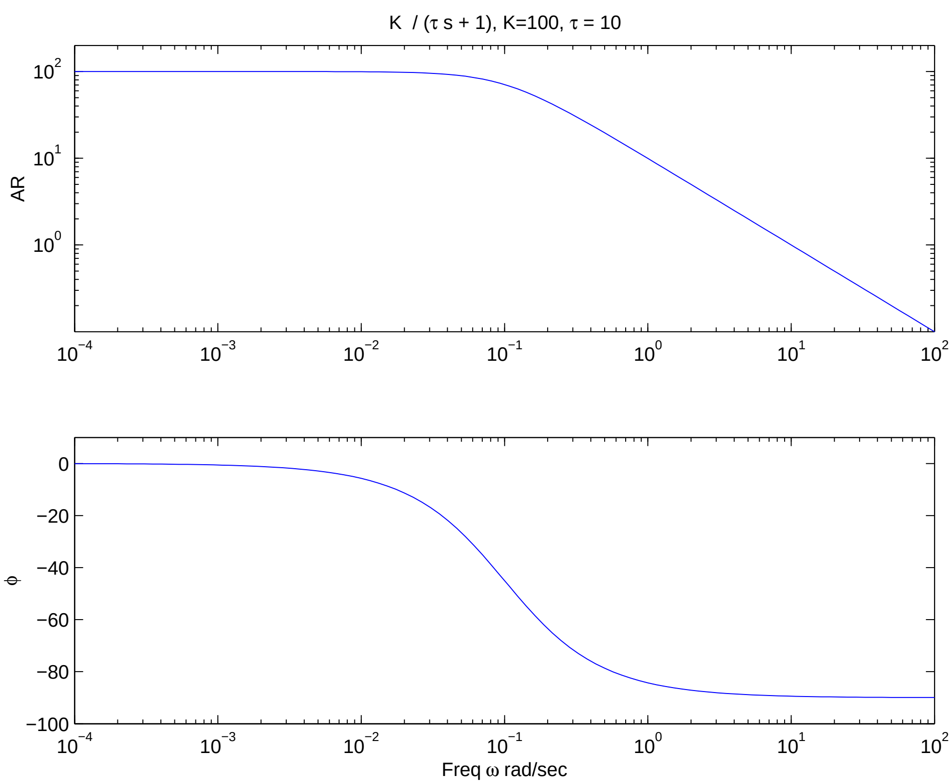
To get Z_2, multiply by s and set $s = 0$

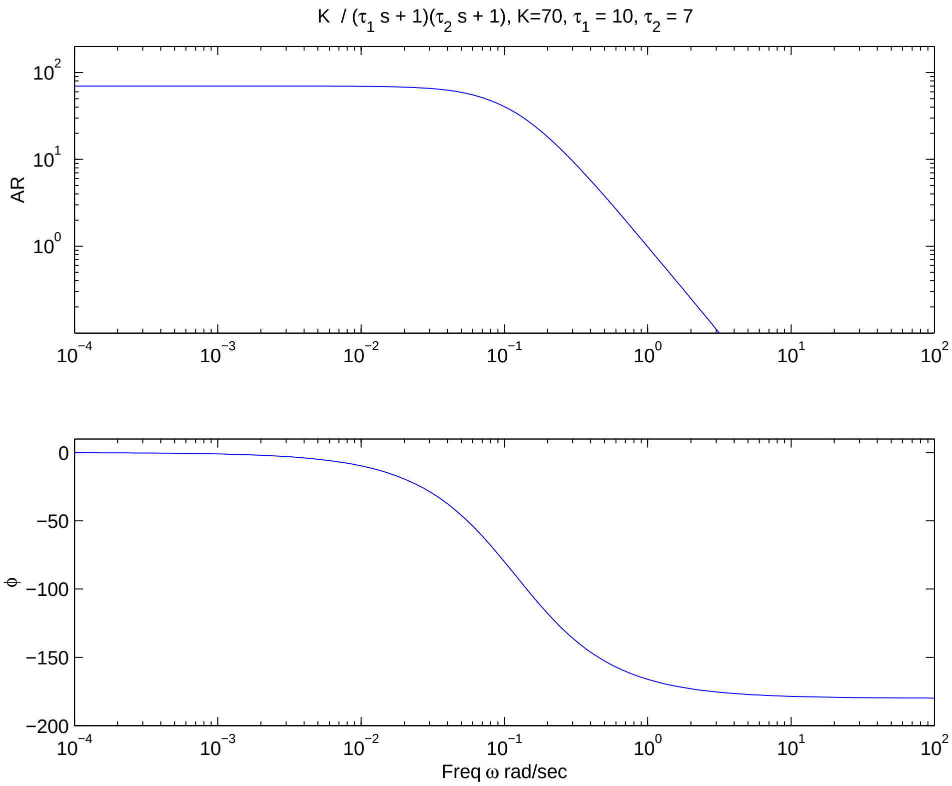
$$Z_2 = \frac{1}{1} = 1$$

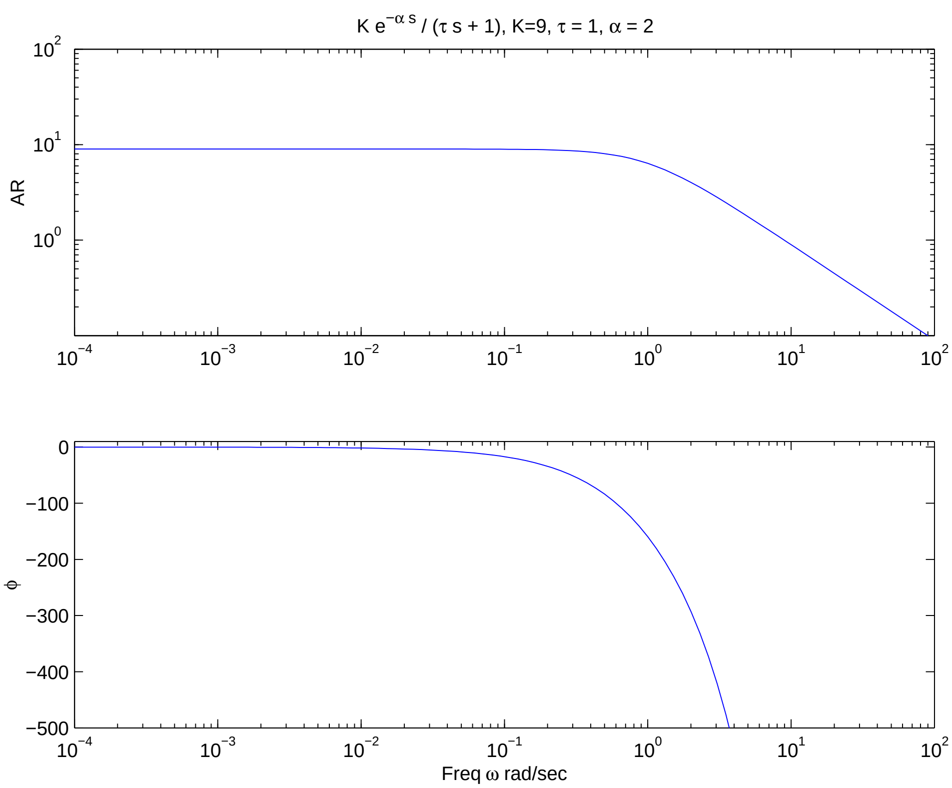
Chapter 6

Frequency Analysis

6.1 Bode Plots of Simple Systems




K / (τ1 s + 1)(τ2 s + 1), K=70, τ1 = 10, τ2 = 7
AR
φ
Freq ω rad/sec


K e−α s / (τ s + 1), K=9, τ = 1, α = 2
AR
φ
Freq ω rad/sec

6.2 Derivations of Frequency Response for Simple Systems

6.2.1 First-Order System

$$g(s) = \frac{K}{\tau s + 1}$$

$$g(j\omega) = \frac{K}{\tau j\omega + 1}$$

$$g(j\omega) = \frac{K}{1 + \tau\omega j}$$

$$g(j\omega) = \frac{K}{1 + \tau\omega j}\,\frac{1 - \tau\omega j}{1 - \tau\omega j}$$

$$g(j\omega) = \frac{K(1 - \tau\omega j)}{(1 + \tau\omega j)(1 - \tau\omega j)}$$

$$g(j\omega) = \frac{K - K\tau\omega j}{1 + \tau^2\omega^2 j^2}$$

$$g(j\omega) = \frac{K - K\tau\omega j}{1 + \tau^2\omega^2(-1)}$$

$$g(j\omega) = \frac{K - K\tau\omega j}{1 - \tau^2\omega^2}$$

$$g(j\omega) = \frac{K}{1 - \tau^2\omega^2} - \frac{K\tau\omega}{1 - \tau^2\omega^2} j$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\left(\frac{K}{1 - \tau^2\omega^2}\right)^2 + \left(\frac{-K\tau\omega}{1 - \tau^2\omega^2}\right)^2}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2 + (-K\tau\omega)^2}{(1 - \tau^2\omega^2)^2}}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2 + K^2\tau^2\omega^2}{(1 - \tau^2\omega^2)^2}}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2(1 + \tau^2\omega^2)}{(1 - \tau^2\omega^2)^2}}$$

$$AR(\omega) = |g(j\omega)| = K\sqrt{\frac{(1 + \tau^2\omega^2)}{(1 - \tau^2\omega^2)^2}}$$

$$AR(\omega) = |g(j\omega)| = K\sqrt{\frac{1}{(1 - \tau^2\omega^2)}}$$

For phase angle as a function of frequency ω

$$\phi(\omega) = \angle g(j\omega) = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{\left(\frac{-K\tau\omega}{1-\tau^2\omega^2}\right)}{\left(\frac{K}{1-\tau^2\omega^2}\right)}\right)$$

$$\phi(\omega) = \angle g(j\omega) = \arctan(-\tau\omega)$$

6.2.2 Second-Order System

$$g(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1}$$

$$g(j\omega) = \frac{K}{\tau^2(j\omega)^2 + 2\tau\zeta(j\omega) + 1}$$

$$g(j\omega) = \frac{K}{\tau^2(-1)\omega^2 + 2\tau\zeta j\omega + 1}$$

$$g(j\omega) = \frac{K}{1 - \tau^2\omega^2 + 2\tau\zeta\omega j}$$

$$g(j\omega) = \frac{K}{(1-\tau^2\omega^2) + 2\tau\zeta\omega j}\ \frac{(1-\tau^2\omega^2) - 2\tau\zeta\omega j}{(1-\tau^2\omega^2) - 2\tau\zeta\omega j}$$

$$g(j\omega) = \frac{K\left((1-\tau^2\omega^2) - 2\tau\zeta\omega j\right)}{(1-\tau^2\omega^2)^2 - (2\tau\zeta\omega j)^2}$$

$$g(j\omega) = \frac{K\left((1-\tau^2\omega^2) - 2\tau\zeta\omega j\right)}{(1-\tau^2\omega^2)^2 - (-1)(2\tau\zeta\omega)^2}$$

$$g(j\omega) = \frac{K\left((1-\tau^2\omega^2) - 2\tau\zeta\omega j\right)}{(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2}$$

$$g(j\omega) = \frac{K(1-\tau^2\omega^2)}{(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2} - \frac{K2\tau\zeta\omega}{(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2}\ j$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\left(\frac{K(1-\tau^2\omega^2)}{(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2}\right)^2 + \left(\frac{-K2\tau\zeta\omega}{(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2}\right)^2}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2(1-\tau^2\omega^2)^2 + (-K2\tau\zeta\omega)^2}{\left((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2\right)^2}}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2(1-\tau^2\omega^2)^2 + K^2(2\tau\zeta\omega)^2}{\left((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2\right)^2}}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\frac{K^2\left((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2\right)}{\left((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2\right)^2}}$$

$$AR(\omega) = |g(j\omega)| = K\sqrt{\frac{((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2)}{((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2)^2}}$$

$$AR(\omega) = |g(j\omega)| = K\sqrt{\frac{1}{((1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2)}}$$

For phase angle as a function of frequency ω

$$\phi(\omega) = \angle g(j\omega) = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{-\frac{K2\tau\zeta\omega}{(1-\tau^2\omega^2)^2+(2\tau\zeta\omega)^2}}{\frac{K(1-\tau^2\omega^2)}{(1-\tau^2\omega^2)^2+(2\tau\zeta\omega)^2}}\right)$$

$$\phi(\omega) = \angle g(j\omega) = \arctan\left(\frac{-2\tau\zeta\omega}{(1-\tau^2\omega^2)}\right)$$

$$\phi(\omega) = \angle g(j\omega) = \arctan\left(-\tau\omega\right)$$

6.2.3 Time Delay System

$$g(s) = e^{-\alpha s}$$

$$g(j\omega) = e^{-\alpha j\omega}$$

$$g(j\omega) = e^{-\alpha\omega\, j}$$

Using the Euler Identity:

$$e^{j\,\theta} = \cos(\theta) + j\,\sin(\theta)$$

$$g(j\omega) = e^{(-\alpha\omega)\,j} = \cos(-\alpha\omega) + j\,\sin(-\alpha\omega)$$

$$g(j\omega) = \cos(-\alpha\omega) + \sin(-\alpha\omega)\,j$$

$$AR(\omega) = |g(j\omega)| = \sqrt{\left(\cos(-\alpha\omega)\right)^2 + \left(\sin(-\alpha\omega)\right)^2}$$

$$AR(\omega) = |g(j\omega)| = \sqrt{1} = 1$$

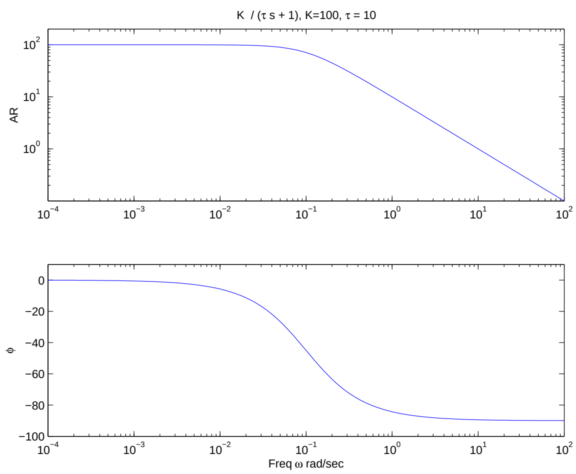
$$\phi(\omega) = \angle g(j\omega) = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{\sin(-\alpha\omega)}{\cos(-\alpha\omega)}\right)$$

$$\phi(\omega) = \angle g(j\omega) = \arctan\left(\tan(-\alpha\omega)\right)$$

$$\phi(\omega) = \angle g(j\omega) = -\alpha\omega$$

6.3 Frequency Response Questions

1. The Bode Plot for a first order system is given below. Identify the transfer function for the system.



2. Sketch the Bode plot for the following transfer function. Label any distinguishing characteristics.

$$g(s) = \frac{100e^{-2s}}{(10s+1)}$$

3. You are in charge of operating the sludge furnace at the local Ideal Gas company plant. You must design a holding tank with limited level variation, given that the supply flow of sludge varies beyond your control. The flow rate from the upstream process varies with a period of 45 min and an amplitude of $\pm 1 \frac{m^3}{hr}$. Your goal is to calculate the cross sectional area of a buffer tank that will vary in height by $\pm 0.1\,m$. The flow rate from the tank is given as $F = kh$ where $k = 1\frac{m^2}{hr}$.

a. What is the frequency of upstream oscillation in $\frac{rad}{hr}$?

b. What is the transfer function for the system in the form $\frac{K}{\tau s+1}$ relating the upstream input flow rate to the tank liquid level?

c. For this system, what is the expression for the Amplitude Ratio as a function of ω?

d. What is the area of the tank in m^2 that will limit level variation to $\pm 0.1\,m$?

4. Your boss at the Ideal Gas Company put you in charge of analyzing two tanks, each with cross sectional area of $2\,m^2$. The tanks are arranged in series. The flow from tank 1 to tank to is $F_1 = kh_1$ and the flow from tank 2 is $F_2 = kh_2$. The flow into the first tank is known to vary with a frequency of $0.5\,\frac{rad}{hr}$. You are told that $k = 2\frac{m^2}{hr}$.

a. What is the transfer function for the process relating the flow into tank 1 to the flow out of tank 2?

b. For this system, what is the expression for the Amplitude Ratio and Phase Angle as a function of ω?

c. What is amplitude of the variation in the flow out of tank 2 as a function of ω?

d. For a frequency of oscillation of $10\,\frac{rad}{hr}$ What is amplitude of the variation in the flow out of tank 2?

Chapter 7

Multivariable Systems

Multivariable System Modeling

Multivariable systems can be modeled as dynamic systems using transfer functions the same way that SISO systems can be modeled using transfer functions. In multivariable systems, a vector of inputs goes into the transfer function and a vector of outputs comes out:

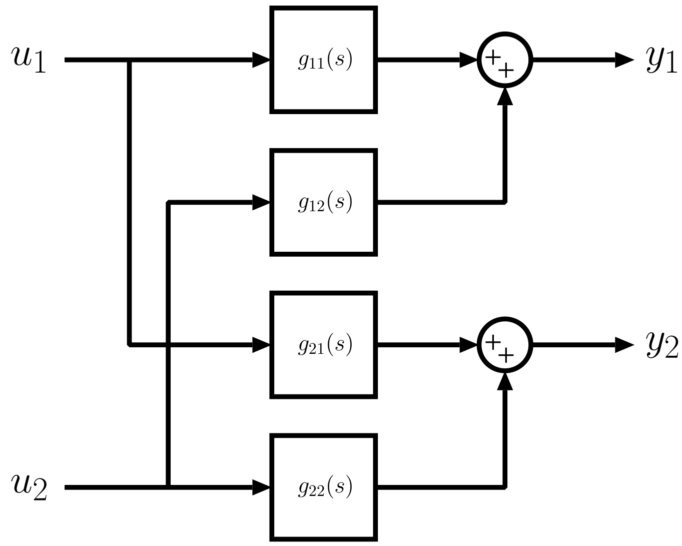
$$\underline{y}(s) = \underline{\underline{G}}(s)\ \underline{u}(s)$$

Just like in multivariable steady-state modeling:

$$\Delta \underline{y} = \underline{\underline{K}}\ \Delta \underline{u}$$

Where the multivariable system of equations represent steady-state relationships, in the dynamic case the multivariable transfer function represents dynamic relationships between the inputs and outputs. In the case of a 2×2 system, $\underline{\underline{G}}(s)$ will be a 2×2 matrix with four transfer functions, $g_{11}(s)$, $g_{12}(s)$, $g_{21}(s)$, and $g_{22}(s)$. The first row is for the first set of equations relating the first output to the rest of the inputs.

$$
\boxed{\begin{aligned} y_1(s) &= g_{11}(s)\, u_1(s) + g_{12}(s)\, u_2(s) \\ y_2(s) &= g_{21}(s)\, u_1(s) + g_{22}(s)\, u_2(s) \end{aligned}}
$$

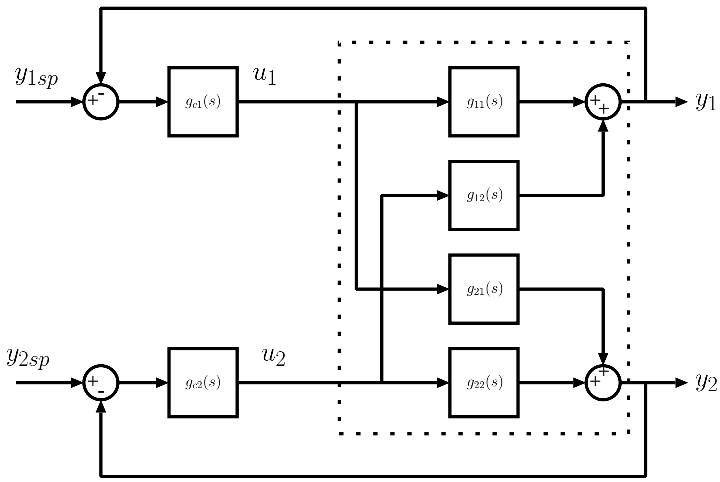


The multiple transfer functions can be developed in the usual manner. Open-loop step tests for each process input could be used to determine gain, time-constant and time delay for simplified FOTD models, or fundamental mass and energy balances could be used to develop dynamic equations that can then be linearized and transformed into the LaPlace domain.

7.1 Relative Gain Array

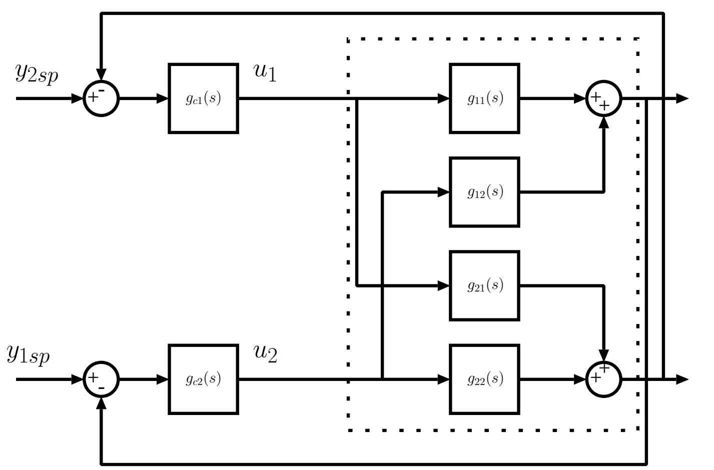
The Relative Gain Array (RGA) is a tool that can be used to help analyze multivariable systems. When considering control of multivariable control systems, one must consider interaction. In a 2×2 MIMO system, changing u_1 will usually affect both y_1 and y_2. Likewise, changing u_2 will usually affect both y_1 and y_2. Using our traditional SISO PID controllers, this can lead to problematic situations where two controllers “fight” each other significantly. The RGA can be used to help determine loop pairings for SISO controllers in a MIMO process.

For example, in the 2×2 system there are only two options: Option 1, pair $u_1 \leftrightarrow y_1$, $u_2 \leftrightarrow y_2$ OR Option 2, $u_1 \leftrightarrow y_2$, $u_2 \leftrightarrow y_1$.



Option 1

$y_1 \leftrightarrow u_1 \;\; y_2 \leftrightarrow u_2$



Option 2

$y_1 \leftrightarrow u_2 \;\; y_2 \leftrightarrow u_1$

In a 3×3 MIMO system, there would be six options for loop pairing, this grows as $n!$.

The RGA can be calculated for a 2×2 system as follows. First, calculate the stead-state gain matrix, $\underline{\underline{K}} = \underline{\underline{G}}(s = 0)$. Next, determine ζ where

$$\zeta = \frac{K_{12}K_{21}}{K_{11}K_{22}}$$

Then determine the RGA matrix, Λ

$$\Lambda = \begin{bmatrix} \frac{1}{1-\zeta} & \frac{-\zeta}{1-\zeta} \\ \frac{-\zeta}{1-\zeta} & \frac{1}{1-\zeta} \end{bmatrix}$$

For a general $n \times n$ system, the RGA is given as follows:

$$\Lambda = K \times (K^{-1})^T$$

The $\times$ operator represents element by element multiplication of the two $n \times n$ matrices. In Matlab, this can be done as follows:

```
R=K.*inv(K)'
```

Note that in the general $n \times n$ case you are taking the inverse of the steady-state gain matrix. If the square system has no inverse, you cannot calculate the inverse. This also means that your equations are linearly dependent, implying that a linear combination of your inputs can be equivalent. For example, increasing u_1 and u_2 have the same effect on the outputs. This type of system cannot be controlled in all output directions.

7.1.1 RGA Rules

These are approximate rules for loop pairing. The RGA is a steady-state analysis tool and may not hold true in all situations. These are guidelines for first considerations in multivariable systems.

1. If the λ_{ij} element is less than or equal to zero, avoid pairing output i with input j. This is the worst case for pairing and should be avoided.

2. If the λ_{ij} element is equal to one, pair output i with input j.

3. If possible avoid cases of $0 < \lambda_{ij} < 0.5$.

4. In all other cases, there will be interaction, but the quality of the closed-loop response depends on the controller tuning, the amount of nonlinearity, the magnitude of disturbances, and the process measurement noise.

7.1.2 Examples

Example 1

$$\underline{\underline{K}} = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\zeta = \frac{K_{12}K_{21}}{K_{11}K_{22}} = \frac{2\,(4)}{-1\,(3)} = -\frac{8}{3}$$

Then determine the RGA matrix, Λ

$$\Lambda = \begin{bmatrix} \frac{1}{1-\zeta} & \frac{-\zeta}{1-\zeta} \\ \frac{-\zeta}{1-\zeta} & \frac{1}{1-\zeta} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \frac{1}{1+\frac{8}{3}} & \frac{\frac{8}{3}}{1+\frac{8}{3}} \\ \frac{\frac{8}{3}}{1+\frac{8}{3}} & \frac{1}{1+\frac{8}{3}} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \frac{3}{11} & \frac{8}{11} \\ \frac{8}{11} & \frac{3}{11} \end{bmatrix}$$

Implying that you should pair $u_1 \leftrightarrow y_2$, $u_2 \leftrightarrow y_1$ since the (1,2) element (row 1, column 2) and (2,1) elements are $\frac{8}{11}$, close to 1.

Example 2

$$\underline{\underline{K}} = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}$$

$$\zeta = \frac{K_{12}K_{21}}{K_{11}K_{22}} = \frac{2\,(4)}{-1\,(-3)} = \frac{8}{3}$$

Then determine the RGA matrix, Λ

$$\Lambda = \begin{bmatrix} \frac{1}{1-\frac{8}{3}} & \frac{-\frac{8}{3}}{1-\frac{8}{3}} \\ \frac{-\frac{8}{3}}{1-\frac{8}{3}} & \frac{1}{1-\frac{8}{3}} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} -\frac{3}{5} & \frac{8}{5} \\ \frac{8}{5} & -\frac{3}{5} \end{bmatrix}$$

Here, the (1,1) and (2,2) elements are negative. Avoid the $u_1 \leftrightarrow y_1$, $u_2 \leftrightarrow y_2$ pairing in this case, so you should use the $u_1 \leftrightarrow y_2$, $u_2 \leftrightarrow y_1$ pairing.

Example 3

$$K = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 2 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 0.25 & -2.25 & 3 \\ 0 & 3.5 & -2.5 \\ 0.75 & -0.25 & 0.5 \end{bmatrix}$$

In row 2, the only good option appears to be pair y_2 with u_2. There will be interaction on this loop, as the value of 3.5 predicts. There are now two different ways to consider the problem. If you consider column 1 first, you would pair y_3 with u_1 as a value of 0.75 is better than 0.25 , then end up with y_1 paired with u_3 for a value of 3. The alternative that would also be valid is pair y_1 with u_1 for a value of 0.25 and y_3 with u_3 for a value of 0.5. Either option is valid.

Example 4

$$K = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 2 & 2 \\ -3 & -3 & 2 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} -1 & 2 & 0 \\ 3.2 & -2.8 & 0.6 \\ -1.2 & 1.8 & 0.4 \end{bmatrix}$$

First, consider column 1. Row elements (1,1) and (3,1) are both negative, implying that you should pair y_2 with u_1. Now, examine row 1. u_1 is already paired with y_2, so y_1 should be paired with u_2 since the (1,3) element is 0. This leave y_3 to be paired with u_3 for a value of 0.4. Every pairing will have interaction. This could be foreseen to some extent. Examine the "direction" of columns 1 and 2. Increasing either u_1 or u_2 will force the output measurements in almost the same direction.

Chapter 8

Phase Plane Analysis

8.1 Linear Phase Plane

8.2 Nonlinear Phase Plane

Chapter 9

Numerical Optimization

Objective: Introduce basic theory and formulation of numerical optimization problems.

Optimization methods attempt to find the best solution to a problem. In some cases, the solution may have limits on the possible values for the solution.

Optimization plays a vital role in many situations. Everyday tasks such as walking across campus can be seen as optimization problems: minimize the distance traveled while staying within the bounds of the sidewalks. For engineers working in industry, each company expects employees to help maximize the profit for the company, within legal and ethical constraints. For specific engineering tasks, numerical optimization methods become very useful for finding the best solution to a problem.

Numerical optimization methods typically assume that a scalar value that is maximized or minimized can be calculated for the problem. This is considered the *cost function* or the *objective function*. The cost function is a function of *decision variables*. Let us define the cost function as a function of the decision variables x as $\Phi(x)$. An optimization routine must search the allowable solution space of the decision variables to find the best value of the objective function that satisfies the problem constraints. The general mathematical form of the problem could be written:

$$\min \Phi(x)$$

$$\text{subject to constraints on } x$$

9.1 Scalar Nonlinear Function Optimization

In calculus, you have seen function minimization and maximization. To find the maximum or minimum of a scalar function, $y = f(x)$, the critical points of the function can be evaluated. The cost function in this example would be $\Phi(x)$, $x \in X \subset \mathbb{R}$, $\Phi : X \to \mathbb{R}$. The first derivative can be calculated and set to zero, $\frac{d\Phi}{dx}(x) = 0$. Assuming that in the solution of this equation, the value for x is value is not an inflection point the solution will be a maximum or minimum value for the function. Finding the solution to this equation can be done analytically in some cases (quadratic formula, etc.).

In cases where $\frac{d\Phi}{dx}(x) = 0$ is very difficult to solve analytically, the numerical solution can be found. Solving $\frac{d\Phi}{dx}(x) = 0$ can be seen as finding the zero of a function ($f(x) = 0$). Newton's method can be used to find the solution to this new problem.

Additionally, gradient "hill climbing" method can be used to find the maximum value of $\Phi(x)$. The derivative $\frac{d\Phi}{dx}(x)$ is the rate of increase for the function. An iterative procedure could be used where $x_{new} = x_{old} + K\,\frac{d\Phi}{dx}(x_{old})$. Here, K is size of the step in the direction of increasing objective function value. In some cases, even $\frac{d\Phi}{dx}$ may be difficult to evaluate analytically. A first-order Taylor series expansion can be used to find $\frac{d\Phi}{dx}(x) = \frac{\Phi(x+\Delta x)-\Phi(x)}{\Delta x}$.

Newton's method or gradient search methods do not guarantee that the result is the optimal value in a global sense. The resulting solution/s could be considered a local maxima or local minima. Additionally, in cases where the range of x is limited, the optimal value may actually be found at the constraints.

9.1.1 Example Problem 1 - Multivariable Function Optimization

Given a cost function (objective function) that is a function of many variables, attempt to find the minimal value of the function. For this example, $\Phi(x) = (x_1+x_2-10)^2+(x_1*x_2-5)^2$. First, create a Matlab function *f.m* that returns the objective function evaluated at a given value of x:

```
function objective=f(x)
x1=x(1);
x2=x(2);
objective=(x1+x2-10)^2 + (x1*x2-5)^2 ;
```

This function can be used to find the minimum value of the function using the unconstrained optimization function *fminunc* in Matlab's Optimization toolbox. To find the minimum value, an initial guess for x must be supplied. To use an initial guess $x = [1\,1]$, the command would be:

```
xnew=fminunc('f',[1 1])
```

9.2 Unconcstrained Nonlinear Function Optimization

General form for an unconstrained optimization problem is:

$$\max \Phi(x)$$

where $x \in X \subset \mathbb{R}$, $\Phi : X \to \mathbb{R}$. This means x is now a vector and $\Phi(x)$ is a scalar valued function. **Note that you can perform minimization of a function by maximization of $-\Phi(x)$.** The critical points for this function occur when the gradient of $\Phi(x)$ are equal to 0, $\frac{\partial\Phi}{\partial x}(x) = 0$.

The gradient of the objective function, $\frac{\partial\Phi}{\partial x}(x) = \nabla\Phi(x)$, is a vector function of x. The direction of this vector points in the direction of steepest increasing value of $\Phi(x)$ at the point x. From a starting point x_0, one could perform iterative search looking for $\nabla\Phi(x) = 0$ using the formula $x_{new} = x_{old} + K\,\frac{\partial\Phi}{\partial x}(x_{old})$. For some values of K this numerical method may be unstable. Remember that the result may only be a local optima. Additionally, care should be taken to avoid saddle points.

9.3 Convexity

At this point, issues involving convexity should be addressed.

A convex set X satisfies $\lambda x_1 + (1-\lambda)x_2 \in X$ for all $0 \leq \lambda \leq 1, \forall x_1, x_2 \in X$.

A convex set can be constructed from a convex function by evaluating the epigraph of a convex function. If $x \in X \subset \mathbb{R}^n$, $f : X \to \mathbb{R}$, $epi(f) \in \mathbb{R}^{n+1}$.

As in convexity results from calculus, convexity of a function requires analysis of a second-order condition. The Hessian matrix H can be calculated for $\Phi(x)$ as:

$$H(x) = \begin{bmatrix} \frac{\partial^2\Phi}{\partial x_1 \partial x_1} & \frac{\partial^2\Phi}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2\Phi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\Phi}{\partial x_2 \partial x_1} & \frac{\partial^2\Phi}{\partial x_2 \partial x_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2\Phi}{\partial x_n \partial x_1} & \cdots & \cdots & \frac{\partial^2\Phi}{\partial x_n \partial x_n} \end{bmatrix}$$

A given function $f(x)$ is convex if the Hessian of the function is positive semi-definite. This means that all the eigenvalues of $H(x)$ are ≥ 0. Note that the Hessian matrix may be a function of x, making calculation of the eigenvalues quite difficult. Some methods exist to bound the smallest eigenvalue for a general Hessian matrix using interval analysis methods.

Example: $f(x) = (x_1)^3 + x_2$, $\frac{df}{dx} = \begin{bmatrix} 2x_1^2 \\ 1 \end{bmatrix}$, $H(x) = \begin{bmatrix} 4x_1 & 0 \\ 0 & 0 \end{bmatrix}$ For the Hessian, the eigenvalues are known to be $4x_1$ and 0. Note that the minimum eigenvalue depends on the range of values for x_1. If the lower bound on x_1 is ≥ 0, the minimum eigenvalue is 0 making the Hessian positive semidefinite and the function convex over the range of x. If the lower bound on x_1 is < 0, the Hessian is not semidefinite and the function is nonconvex over the range of x.

Note that nonlinear equality constraints are nonconvex. A nonlinear functions $f(x) = 0$ can be written as two inequality constraints:

$$0 \leq f(x) \leq 0$$

This implies that if $f(x)$ is nonlinear and convex, one of the two preceding inequality constraints would be nonconvex.

$$\begin{aligned} 0 &\leq f(x) \\ f(x) &\leq 0 \end{aligned}$$

Is the same as

$$\begin{aligned} -f(x) &\leq 0 \\ f(x) &\leq 0 \end{aligned}$$

Therefore one constraint must be nonconvex.

9.4 Vector Nonlinear Function, Constrained

A general form for the constrained optimization problem is:

$$\max \Phi(x)$$

$$\text{subject to } g(x) \leq 0$$

with the constraint functions $g : X \to \mathbb{R}^m$ for m separate constraints. Consider the case where $g(x)$ is a single constraint, a scalar function of the vector x. The problem can be written using a Lagrangian relaxation to form an unconstrained problem. The new problem becomes:

$$\max \Phi(x) - ug(x)$$

where u is a positive value. Think of this as a penalization for violating the constraint $g(x) \leq 0$. When $g(x)$ is positive, the objective function increases, so it is desirable to have $g(x)$ be negative. The unconstrained optimization problem can be solved iteratively, changing the values for u until the minimum value for u is found to keel $g(x) \leq 0$.

Now, consider the case where $g(x)$ is a vector function of the vector x. The problem can be also written using Lagrangian relaxations to form an unconstrained problem. The new problem becomes:

$$\max \Phi(x) - u_1 g_1(x) - ... - u_m g_m(x)$$

where u_m are positive values. In the case where multiple constraints are written on the problem, the problem can become infeasible if no feasible solution can be found.

9.5 KKT Conditions

For a potential solution $\hat{x}$, the following conditions hold. The set I specifies the binding constraints at $\hat{x}$, $I = \{i : g_i(\hat{x}) = 0\}$. Additionally, $\nabla g_i(\hat{x})$ are linearly independent. If the following conditions hold at $\hat{x}$, then $\hat{x}$ is a KKT point and a local solution.

$$\nabla f(\hat{x}) + \sum_{i \in I} u_i \, \nabla g_i(\hat{x}) = 0$$

$$u_i \geq 0$$

Note that this does not specify how to find a KKT point. Also note that a KKT point is not necessary to minimize a convex problem.

9.6 Special Types of Optimization

Linear Programming (LP) In some cases, the general optimization form has linear objective functions and linear constraints. An optimization problem can be found in the form:

$$\text{min } Cx$$

$$\text{subject to } Ax \leq b,\ lb \leq x \leq ub$$

This is a special linear constrained case where the objective function is a linear function of the decision variable vector x and the constraints are also linear. This can readily be solved using the *alp* command in Matlab, even for large scale problems. Generally the simplex method is used, but interior point methods are gaining popularity.

Quadratic Programming (QP) For cases with a convex quadratic objective of the form:

$$\text{min } \frac{1}{2}x^T Hx + Cx$$

$$\text{subject to } Ax \leq b,\ lb \leq x \leq ub$$

The problem is termed a Quadratic Program (QP). The Matlab command *qp* can be used to solve this type of problem.

Mixed Integer Programming For problems where some variables can only take binary values, the problem is considered a Mixed Integer problem. A common mixed integer problem is the Mixed Integer Linear Programming (MILP) problem of the form

$$\text{min } Cx$$

$$\text{subject to } Ax \leq b,\ lb \leq x \leq ub,\ x_i \in \{0, 1\}$$

There exist specialized methods for solving problems where the decision variables are only allowed to take values of 0 or 1 rather than values between 0 and 1 (inclusive).

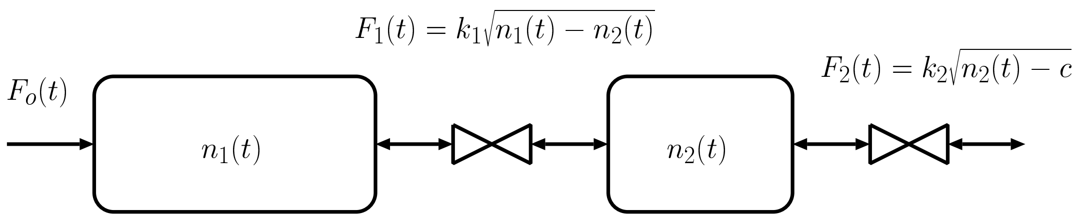
Chapter 10

Nonconvex Numerical Optimization

When an optimziation problem involves nonconvex algebraic functions in the objective function or the constraints, the problem is said to be nonconvex and may suffer from local minima. When a problem includes binary or integer variables, the decision space is also noconvex, but the idea of local optimality is not so clear.

Chapter 11

Tank Case Study



A two tank system is arranged in series as shown in the above figure. The molar gas flow rate into tank 1 can be changed by the operator. The tanks are constant volume and isothermal. The volumetric flow of gas across a valve is usually written $F = k\sqrt{\Delta P}$. In this case, using ideal gas law $PV = nRT$ you should realize that the molar amount of gas in each tank is proportional to the pressure in the tank, $n = \frac{V}{RT}\,P$ where V, R, and T are constant. As a result, the molar flow rate between the two tanks across a valve can be written as $F_1(t) = k_1\sqrt{n_1(t) - n_2(t)}$ and the flow across a valve to the atmosphere can be written $F_2(t) = k_2\sqrt{n_2(t) - c}$

1. Develop a dynamic mass balance for the two tank system.
2. Linearize any nonlinear terms.
3. Develop linear dynamic approximation for the system.
4. Take the Laplace transform of your linear ordinary differential equations.
5. Derive the transfer function relating the input flow to the number of moles in tank 2.
6. Determine the analytical response for the number of moles in tank 2 for a unit step change in $F_o(t)$ at time $t = 0$.
7. Sketch the bode plot for this system.
8. Assuming a feedback controller of the form

   $$g_c = K_c + \frac{K_c}{\tau_I\, s}$$

   derive the closed-loop transfer function.

Chapter 12

ECHE 550 Topics

General Control Configurations
Jargon: MV, CV, DV
Feedback on PFD
Feedforward on PFD
Cascade on PFD
Linear Algebra
Steady state modeling ($\underline{\Delta y} = \underline{\underline{K}}\,\underline{\Delta u}$)
Solving $\underline{\underline{A}}\,\underline{x} = \underline{b}$ by row reduction
Solving $\underline{\underline{A}}\,\underline{x} = \underline{b}$ by calculating $\underline{\underline{A}}^{-1}$
Matrix multiplication
Determinant / Eigenvalues of $\underline{\underline{A}}$
Dynamic Modeling (Open-loop)
Dynamic mass and energy balances
State Space Representation for ODEs
Laplace Transforms
step, delayed step, impulse
ramp, sinusoid, exponential
time delay and Heavyside function
derivative, integral of function
Solving Ordinary Differential Equations (ODEs)
Step response of First-Order system
Partial Fraction Expansion
Linearity applied to complex functions
$f(t) = f_1(t) + f_2(2) \Rightarrow f_1(s) + f_2(s) = f(s)$
Compound / Composite functions
Dynamic Modeling (Open-loop)
Dynamic mass and energy balances
CSTR, Mixing Tank, Tank Level
Transfer Function Representation
$y(s) = g(s)u(s) + g_d(s)d(s)$
Block diagrams
Poles and Zeros of transfer functions

Low Order Systems
- First Order
- Pure Gain
- Pure Capacity
- Lead Lag

High Order systems
- Two first order in series
- Interacting tanks
- General 2nd order
- Higher order

Inverse Response (RHP zero)

Time Delay

Stability
- poles and eigenvalues for stability
- BIBO stab. of oscillatory systems (pole at $s = 0$)

Poles and Zeros of state space representation

Frequency Response
- Amplitude Ratio and Phase Angle for $g(s)$
- Basic Bode Plots given $g(s)$
- Complex Bode Plots for $g_1(s)g_2(s)...g_n(s)$
- Developing models from frequency response

Linearization of nonlinear ODEs

Model Identification

Feedback Control
- Process Reaction Curve (K, τ, α)
- Basic PID Controller Tuning (K_c, τ_I, τ_D)
- PID Transfer Function for $g_c(s)$
- Internal Model Control
- Direct Synthesis

Feedforward Control

Cascade Control

Multivariable Open-loop Modeling
- Transfer function based
- State space
- Multivariable system poles and zeros

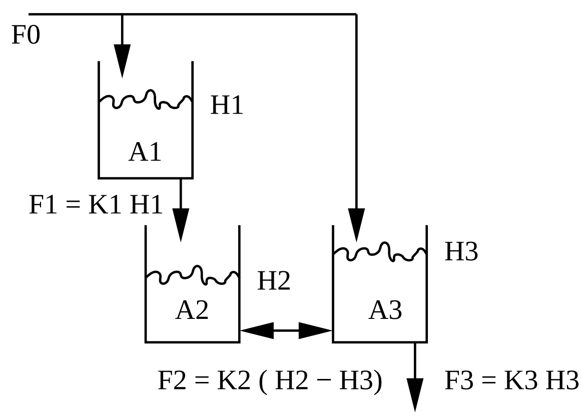
Multivariable Control Issues
- Relative Gain Array and loop pairing
- Decoupling control
- Actuator constraints
- Moving horizon control
- Optimization

Chapter 13

Various Problems

13.1 Tank Modeling Problem With Explanation

1. **(25 pts.)** A system consists of three tanks as shown below. The flow rate F_0 can be manipulated. A fraction of the flow rate F_0 into the system goes into tank 1 and the rest of the flow enters into tank 3 as shown. The fraction of flow F_0 into tank 1 is γ, with $0 \leq \gamma \leq 1$ and γ remaining constant. The flow rate from tank 1 to tank 2 is given as $F_1 = k_1 h_1$. The flow rate into tank 3 from tank 2 is $F_2 = k_2(h_2 - h_3)$. The flow rate out of tank 3 is $F_3 = k_3 h_3$. The constant cross sectional tank areas are A_1, A_2, and A_3, respectively.



a. Derive the differential equation model for the system.

b. Put your differential equation model into State Space form ($\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}u,\ y = \underline{c}^T x$) for the system, given that $u = F_0$, $y = h_3$, and $\underline{x}$ with $\underline{x}$:

$$\underline{x} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

SOLUTION AND EXPLANATION: First of all, you must realize that you need to perform a dynamic mass balance for this system. Dynamic balances include a nonzero accumulation term and can result in a differential equation model of your process.

To start the mass balance, remember that you should perform balances around individual systems. In this case, you will need three balances, one over each tank system.

The amount of "stuff" in the first tank is $\rho V_1(t)$ or more simply $\rho A_1 h_1(t)$. As the density and cross-sectional area are not functions of time, the accumulation term (or rate of change of "stuff" in the tank") can be written as:

$$\rho A_1 \frac{dh_1}{dt}(t)$$

Assuming the flow rates are all in volumetric terms, the mass balance on the first system can be written as:

$$\rho A_1 \frac{dh_1}{dt}(t) = \rho\gamma F_0(t) - \rho k_1 h_1(t)$$

Note that for this tank, you have a flow rate in term and a flow rate out term. Also note that only a portion of the flow into the system goes into tank 1. There is no reaction taking place in this system. Similarly, for the other two tanks you can write similar mass balances:

$$\begin{aligned}
\rho A_2 \frac{dh_2}{dt}(t) &= \rho k_1 h_1(t) - \rho k_2 \ (h_2(t) - h_3(t)) \\
\rho A_3 \frac{dh_3}{dt}(t) &= \rho(1-\gamma) F_0(t) + \rho k_2 \ (h_2(t) - h_3(t)) - \rho k_3 h_3(t)
\end{aligned}$$

Such that the overall model is in the form:

$$\begin{aligned}
\rho A_1 \frac{dh_1}{dt}(t) &= \rho\gamma F_0(t) - \rho k_1 h_1(t) \\
\rho A_2 \frac{dh_2}{dt}(t) &= \rho k_1 h_1(t) - \rho k_2 \ (h_2(t) - h_3(t)) \\
\rho A_3 \frac{dh_3}{dt}(t) &= \rho(1-\gamma) F_0(t) + \rho k_2 \ (h_2(t) - h_3(t)) - \rho k_3 h_3(t)
\end{aligned}$$

Note the sign difference in terms. If something is assumed to flow out of one tank and into another, the same term should appear in both mass balances, only with a different sign in each. Also note that the flow from tank 2 to tank 3 is assumed to be positive (so long as $h_2 > h_3$). The term appears with a negative sign in the second balance and with a positive sign in the third balance. In some cases, h_3 may exceed h_2. In such a case, the sign of the term would automatically change, taking care of the reverse flow in the model. The negative term for flow out: $-\rho k_2 \ (h_2(t) - h_3(t))$ for the tank 2 balance would become positive value if h_3 exceeds h_2 and the flow out term would actually become a flow in term. Nothing special must be done in these cases, except making sure the terms have different signs if they appear in different balances.

You now have a full differential mass balance. Now we would like to get our equations in state space form. You have three accumulation terms, so you should have three states:

$$\underline{x}(t) = \begin{bmatrix} h_1(t) \\ h_2(t) \\ h_3(t) \end{bmatrix}$$

Next, simplify the equations. First, divide out the density from all the terms.

$$
\begin{aligned}
A_1 \frac{dh_1}{dt}(t) &= \gamma F_0(t) - k_1 h_1(t) \\
A_2 \frac{dh_2}{dt}(t) &= k_1 h_1(t) - k_2 \ \left(h_2(t) - h_3(t)\right) \\
A_3 \frac{dh_3}{dt}(t) &= (1-\gamma) F_0(t) + k_2 \ \left(h_2(t) - h_3(t)\right) - k_3 h_3(t)
\end{aligned}
$$

Next, get the accumulation terms to all have 1 as the leading coefficient. This means divide each equation by the cross sectional area in this case:

$$
\begin{aligned}
\frac{dh_1}{dt}(t) &= \frac{\gamma}{A_1} F_0(t) - \frac{k_1}{A_1} h_1(t) \\
\frac{dh_2}{dt}(t) &= \frac{k_1}{A_2} h_1(t) - \frac{k_2}{A_2} \ \left(h_2(t) - h_3(t)\right) \\
\frac{dh_3}{dt}(t) &= \frac{(1-\gamma)}{A_3} F_0(t) + \frac{k_2}{A_3} \ \left(h_2(t) - h_3(t)\right) - \frac{k_3}{A_3} h_3(t)
\end{aligned}
$$

Now, write all the the equations in terms of all the states and the inputs, including the measurement equation, $y(t) = h_3(t)$

$$\begin{aligned}
\frac{dh_1}{dt}(t) &= -\frac{k_1}{A_1}h_1(t) + 0h_2(t) + 0h_3(t) + \frac{\gamma}{A_1}F_0(t) \\
\frac{dh_2}{dt}(t) &= \frac{k_1}{A_2}h_1(t) - \frac{k_2}{A_2}h_2(t) + \frac{k_2}{A_2}h_3(t) + 0F_0(t) \\
\frac{dh_3}{dt}(t) &= 0h_1(t) + \frac{k_2}{A_3}h_2(t) - \frac{k_2}{A_3}h_3(t) - \frac{k_3}{A_3}h_3(t) + \frac{(1-\gamma)}{A_3}F_0(t) \\
y(t) &= 0h_1(t) + 0h_2(t) + 1h_3(t) + 0F_0(t)
\end{aligned}$$

Now, it is easier to pick out your state space matrices, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, $\underline{\underline{D}}$.

$$\underline{\underline{A}} = \begin{bmatrix} -\frac{k_1}{A_1} & 0 & 0 \\ \frac{k_1}{A_2} & -\frac{k_2}{A_2} & \frac{k_2}{A_2} \\ 0 & \frac{k_2}{A_3} & -\frac{k_2}{A_3} - \frac{k_3}{A_3} \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} \frac{\gamma}{A_1} \\ 0 \\ \frac{(1-\gamma)}{A_3} \end{bmatrix}$$

$$\underline{\underline{C}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{D}} = [0]$$

And the matrices should fit together nicely in the form:

$$\begin{array}{ccc} A & || & B \\ == & || & = \\ C & || & D \end{array}$$

Which means A and C should have the same number of columns (= # states), while A and B should have the same number of rows (= # states). $B$and D should have the same number of columns (= the number of inputs) while C and D should have the same number of rows (= the number of output measurements).

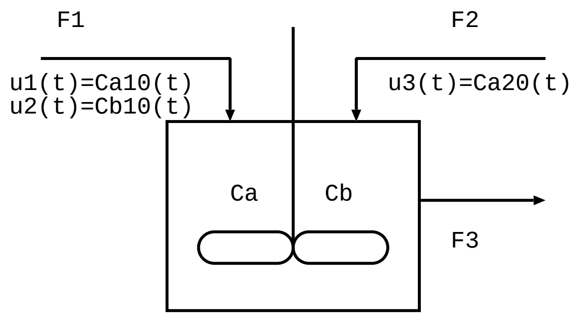
Now, after you finish the mass / energy balance but before you put your equations in state space, I could have asked you to take the Laplace transform and get a transfer function for the relationship between $F_0(t)$ and $h_3(t)$. When you take the Laplace transform of your differential equations and measurement equations, you will have a number of different equations in the s domain, like:

$$sh_1(s) = \frac{\gamma}{A_1}F_0(s) - \frac{k_1}{A_1}h_1(s)$$

And a couple of other equations. You would have to use the equations to eliminate the variables you don't want. In this case, you only want $F_0(s)$ and $h_3(s)$, so you would have to eliminate $h_1(s)$and $h_2(s)$. This would be a mess on this problem, but you could feasibly do it.

13.2 CSTR Modeling Problem With Explanation

1. At the Ideal Gas Company, you are in charge of operating a reactant mixing system. Your boss wants a dynamic model of the system to be used for process control and process optimization. The constant volume mixing tank has two feed streams with constant volumetric flowrates of F_1 and F_2. Feed stream 1 contains both species A and species B, while stream 2 only contains species A. You can modify the initial concentrations of the two species coming into the tank system, $u_1(t) = C_{A10}(t)$, $u_2(t) = C_{B10}(t)$, $u_3(t) = C_{A20}(t)$. At the exit stream, due to instrumentation limitations, you can only measure the total concentration of both components, $y(t) = C_A(t) + C_B(t)$.



a. **(4 points)** What is the dynamic mass balance describing the concentrations of species A and species B at the exit of the mixing tank?

b. **(4 points)** Put your model in state space form. Clearly identify $\underline{x}$, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$. Example state space form:

$$\begin{aligned} \dot{x} &= \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u} \\ y &= \underline{\underline{C}}\underline{x} + \underline{\underline{D}}\underline{u} \end{aligned}$$

SOLUTION AND EXPLANATION: Ok, one tank, two species, three inputs, one measurement. Two dynamic mass balances should work. F_1, F_2, F_3, and V are all constant. No reaction, this is a mixing tank. One balance will consider the amount of species A in the system, while the other will model the amount of species B. The total amount of A in the system is:

$$VC_A(t)$$

And the accumulation term for A will be:

$$V\frac{dC_A}{dt}(t)$$

So the mass balances become:

$$
\begin{aligned}
V\frac{dC_A(t)}{dt} &= F_1 C_{A10}(t) + F_2 C_{A20}(t) - F_3 C_A(t) \\
V\frac{dC_B}{dt}(t) &= F_1 C_{B0}(t) - F_3 C_B(t)
\end{aligned}
$$

The states (concentrations in the reactor) and inputs (inlet concentrations for inlet flows) in this problem are:

$$\underline{x}(t) = \begin{bmatrix} C_A(t) \\ C_B(t) \end{bmatrix} \ \underline{u}(t) = \begin{bmatrix} C_{A10}(t) \\ C_{B10}(t) \\ C_{A10}(t) \end{bmatrix}$$

The measurement equation is a little tricky... You can measure the total concentration of both components:

$$y(t) = C_A(t) + C_B(t)$$

For state space, divide both equations by V, write all equations in terms of all states x and all inputs u.

$$\begin{aligned} \frac{dC_A(t)}{dt} &= -F_3/VC_A(t) + 0C_B(t) + F_1/VC_{A10}(t) + 0C_{B10}(t) + F_2/VC_{A20}(t) \\ \frac{dC_B}{dt}(t) &= 0C_B(t) - F_3/VC_B(t) + 0C_{A10}(t) + F_1/VC_{B0}(t) + 0C_{A20}(t) \end{aligned}$$

And the matrices should fit together nicely in the form:

$$\begin{array}{ccc} A & || & B \\ == & || & = \\ C & || & D \end{array}$$

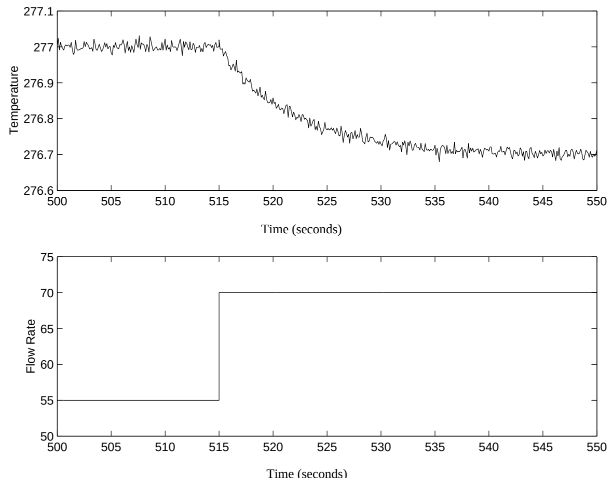
so that:

$$\begin{array}{cccccc} -\frac{F_3}{V} & 0 & || & \frac{F_1}{V} & 0 & \frac{F_2}{V} \\ 0 & -\frac{F_3}{V} & || & 0 & \frac{F_1}{V} & 0 \\ == & == & || & = & = & = \\ 1 & 1 & || & 0 & 0 & 0 \end{array}$$

Note the eigenvalues of the A matrix. These are the poles of your system. Note they are identical. $C_A(t)$ has no effect on $C_B(t)$.

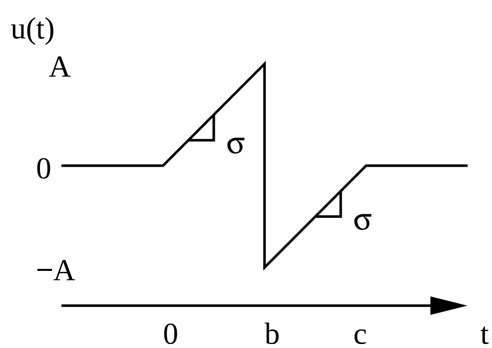
13.3 Fall 2001 Exam 1

1. **(15 pts.)** The Ideal Gas Company is attempting to develop a dynamic process model for a combustion chamber which burns a stream of aqueous liquid waste. The process output and the process input are shown for a input step change. What is the transfer function for this system, assuming it is a first order process?

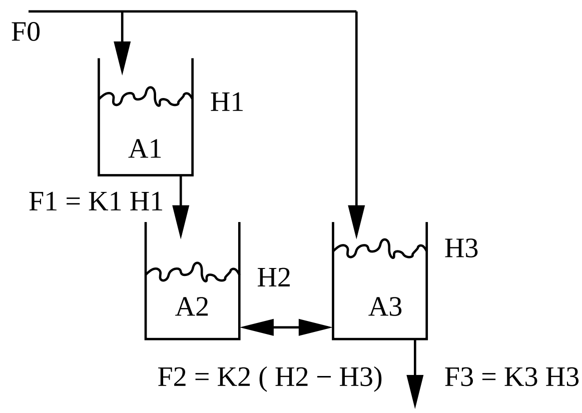


2. **(15 pts.)** What is the Laplace transform $u(s)$ of the following function?

$$u(t) = \begin{cases} 0 & t < 0 \\ \sigma t & 0 \le t < b \\ -2A + \sigma t & b \le t < c \\ 0 & c \le t \end{cases}$$



3. **(25 pts.)** A system consists of three tanks as shown below. The flow rate F_0 can be manipulated. A fraction of the flow rate F_0 into the system goes into tank 1 and the rest of the flow enters into tank 3 as shown. The fraction of flow F_0 into tank 1 is γ, with $0 \leq \gamma \leq 1$ and γ remaining constant. The flow rate from tank 1 to tank 2 is given as $F_1 = k_1 h_1$. The flow rate into tank 3 from tank 2 is $F_2 = k_2(h_2 - h_3)$. The flow rate out of tank 3 is $F_3 = k_3 h_3$. The constant cross sectional tank areas are A_1, A_2, and A_3, respectively.

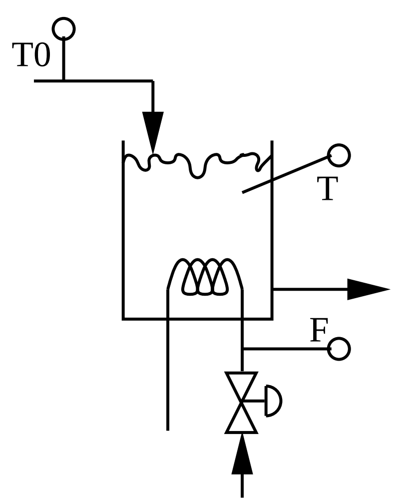


a. Derive the differential equation model for the system.

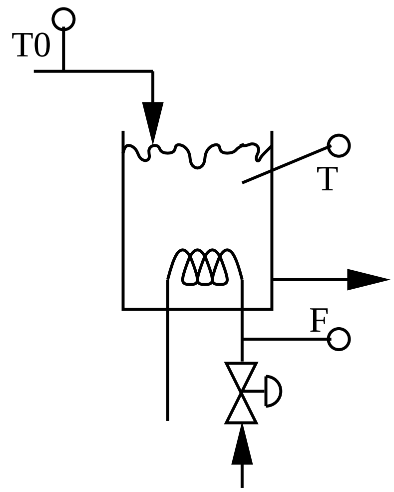
b. Put your differential equation model into State Space form ($\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}u$, $y = \underline{c}^T x$) for the system, given that $u = F_0$, $y = h_3$, and $\underline{x}$ with $\underline{x}$:

$$\underline{x} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

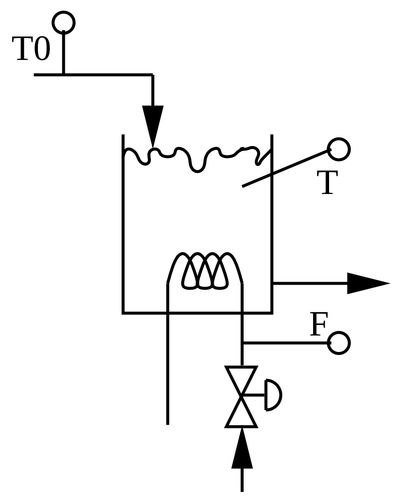
4. **(20 pts.)** For the following system, steam is used to heat the liquid in a constant volume tank. The available measurements include the temperature of the liquid in the tank, the temperature of the feed flowing into the tank, and the steam flow rate. The steam valve can be manipulated. It is desired to regulate the temperature of the exit flow from the tank at a constant value.



a. In the figure above, draw a feedback control loop for the system



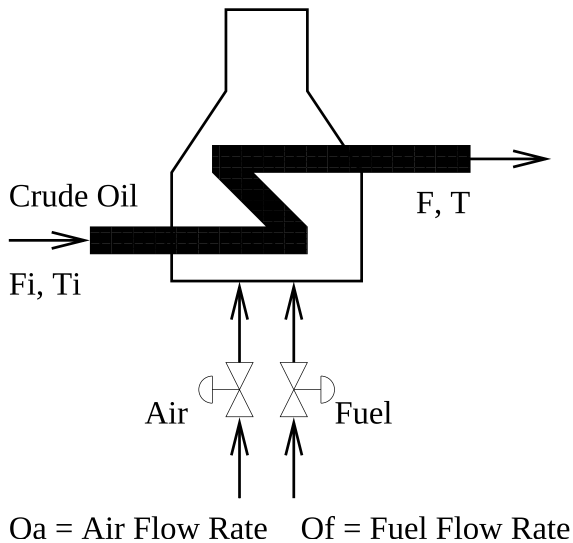
b. In the figure above, draw a feed forward control loop, assuming the feed temperature acts as the disturbance.



c. In the figure above, assuming the steam flow rate varies unpredictably, draw a cascade configuration using two feedback controllers.

13.4 Fall 2001 Quiz 1, Practice

1. **(4 pts.)** A preheater furnace is used increase the temperature of crude oil from T_i to T, the target value. The preheated hot crude oil is then sent down stream to a reactor. The crude oil enters the furnace at the flow rate F and leaves at the same rate. Fuel and air are mixed and burned in the furnace to heat the crude oil. See diagram below.



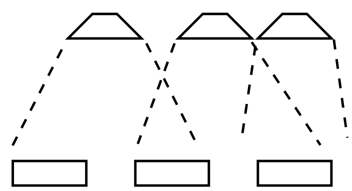
Construct two different feedback control configurations. Also, construct two different feedforward control configurations. Clearly label what is measured and what is manipulated.

3. **(2 pts.)** What are the eigenvalues of the following matrix?

$$\begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

13.5 Fall 2001 Quiz 1

1. **(4 pts.)** An agricultural process requires that trays of plants be maintained at specified temperatures. Three lamps are used to warm three plant trays as seen below.



A 3x3 steady state model is desired relating the change in voltages ΔV_i (for each lamp i) to the change in plant temperature ΔT_j (for each plant j). It is known that increasing the voltage for Lamp A by 1 volt increases the temperature of Plant A by 3.3 degrees and increases the temperature of Plant B by 2.1 degrees. Increasing Lamp B voltage by 1 volt increases both Plant B and Plant C by 2 degrees. Increasing Lamp C voltage by 1 volt increases the temperature of Plant C by 4 degrees.

Develop a model in the form $\underline{\underline{A}}\underline{x} = \underline{b}$ and identify $\underline{\underline{A}}$, $\underline{x}$, and $\underline{b}$.

You may want to check your model by assuming arbitrary values for the change in lamp voltages, then verifying the expected change in plant temperatures.

3. **(2 pts.)** a. What is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -6 & 7 \\ -1 & -2 & 3 \end{bmatrix}$$

13.6 Fall 2002 Quiz 1

You must develop a model of paper machine sheet forming process. A simple schematic is shown below. A feed stream of pulp (wood fibers and water) is sprayed onto a moving screen (conveyor belt). As the screen moves, water drains out of the pulp, through the screen. At the product end of the paper machine, the pulp is effectively just wet paper.

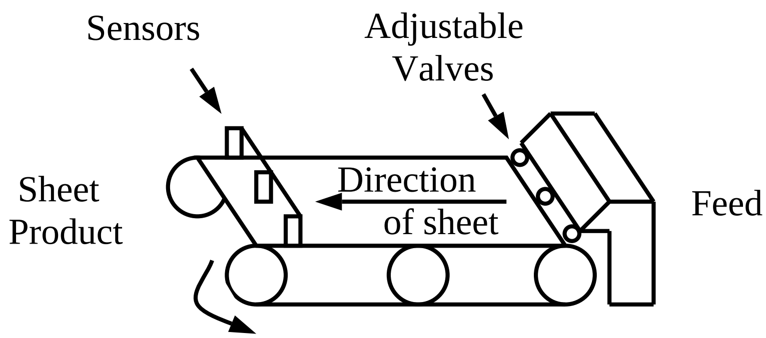


Diagram of a sheet forming process.

Three valves are available to adjust the flowrate of pulp when the pulp concentrations change. Three sensors measure the thickness of the wet paper. Increasing the valves on the edge by 1% (v_1 and v_3) increases the thickness in the corresponding paper location by 2mm. A 1% increase in v_1 and v_3 will also decrease the thickness in the center position by 1 mm. A 1% increase in v_2 will increase the thickness in the center by 3 mm and reduce the edge thickness by 0.5mm.

1. **(1pt)** What are the controlled variables, manipulated variables, and disturbances for this paper making process?

2. **(3pts)** Develop a model of this process relating $\underline{\Delta s}$ and $\underline{\Delta v}$.

3. **(2pts)** Put your model in the form $\underline{\underline{A}}\underline{x} = \underline{b}$ and clearly identify $\underline{\underline{A}}$, $\underline{x}$, and $\underline{b}$.

4. **(2pts)** What is the determinant of the following matrix??

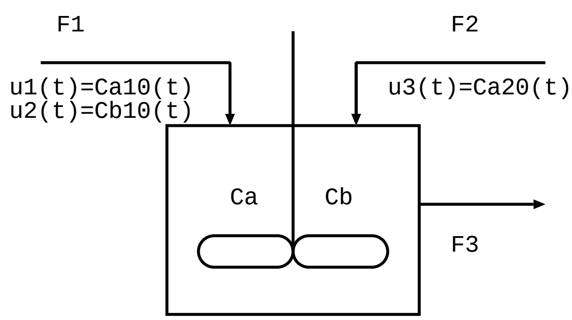
$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 4 \\ 1 & 3 & 1 \end{bmatrix}$$

5. **(2pts)** What are the eigenvalues of the following matrix?

$$\begin{bmatrix} -5 & -2 \\ 3 & -10 \end{bmatrix}$$

13.7 Fall 2002 Quiz2

1. At the Ideal Gas Company, you are in charge of operating a reactant mixing system. Your boss wants a dynamic model of the system to be used for process control and process optimization. The constant volume mixing tank has two feed streams with constant volumetric flowrates of F_1 and F_2. Feed stream 1 contains both species A and species B, while stream 2 only contains species A. You can modify the initial concentrations of the two species coming into the tank system, $u_1(t) = C_{A10}(t)$, $u_2(t) = C_{B10}(t)$, $u_3(t) = C_{A20}(t)$. At the exit stream, due to instrumentation limitations, you can only measure the total concentration of both components, $y(t) = C_A(t) + C_B(t)$.

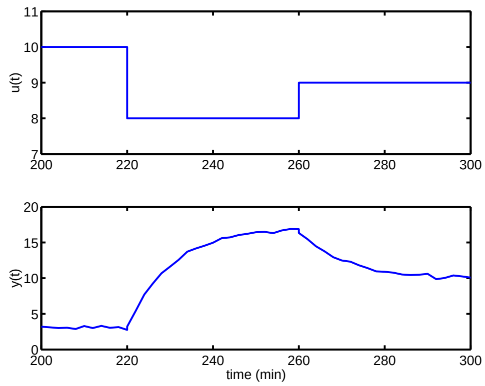


a. **(4 points)** What is the dynamic mass balance describing the concentrations of species A and species B at the exit of the mixing tank?

b. **(4 points)** Put your model in state space form. Clearly identify $\underline{x}$, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$. Example state space form:

$$\begin{aligned} \dot{x} &= \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u} \\ y &= \underline{\underline{C}}\underline{x} + \underline{\underline{D}}\underline{u} \end{aligned}$$

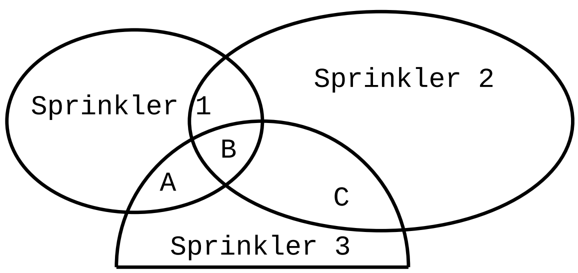
2. **(2 points)** After running some step tests for your system varying $u_1(t)$ and measuring the output $y(t)$ you have the following process data. Identify the approximate process gain for this Single-Input-Single-Output system.



3. Bonus - Dr. Gatzke has a flower bed with three sprinkler heads. In one minute, sprinkler 1 delivers $2\,mm$ of water to its coverage area, sprinkler 2 delivers $0.5\,mm$ of water to its coverage area, and sprinkler 3 delivers $4\,mm$ of water to its coverage area. Plant A is covered by sprinkler 1 and 3, plant B is covered by all sprinkler, and plant C is covered by sprinkler 2 and 3. The system is currently set to operate at normal operating program times. Develop a steady state model relating possible changes in sprinkler operating times to changes in amount of water delivered to each plant. Put you model in the form:

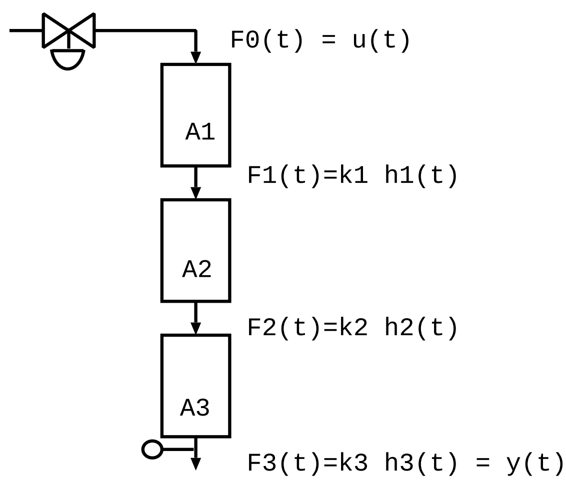
$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

3b. Assume that plant A needs an additional $2\,mm$ of water, plant B needs $1\,mm$ less, and Plant C is fine the way it is. How does the problem change? What changes to the sprinkler operating times would make this change? Solve using Row Reduction Methods.



13.8 Fall 2002 Exam 1 Practice Problems

1. A series of tanks are shown below. You can manipulate $F_0(t)$ and you can measure the flow rate out of tank 3, $F_3(t)$.



a. Assuming constant density, develop a mass balance for the system.

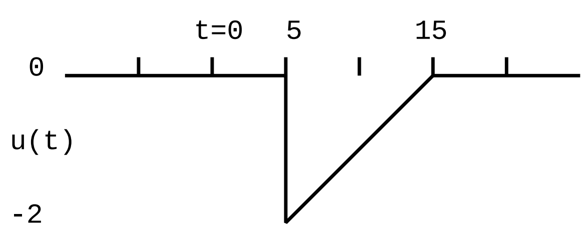
b. Put your model in state space form. Clearly identify $\underline{x}$, $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$. Example state space form:

$$\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u} \\ \underline{y} &= \underline{\underline{C}}\underline{x} + \underline{\underline{D}}\underline{u} \end{aligned}$$

c. What are the eigenvalues of your $\underline{\underline{A}}$ matrix from your system?

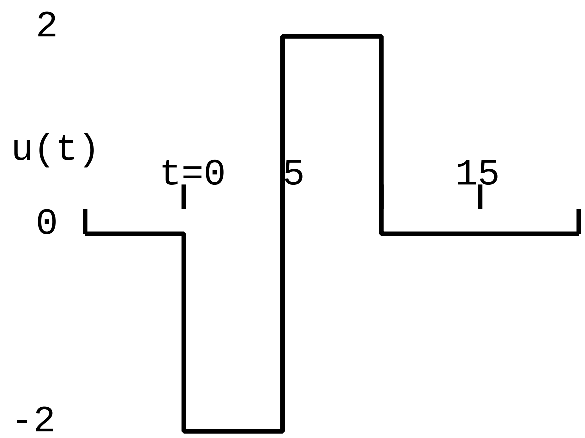
d. From part (a.) take the Laplace transform of your dynamic model assuming the tanks are empty initially. Sove the three equations for the relationship between $y(s)$ and $u(s)$.

2a.Express the following function as a simple function of time (You may need to use the heaviside function multiplied by other functions.)



b. Establish the Laplace transform of the function, $u(s)$.

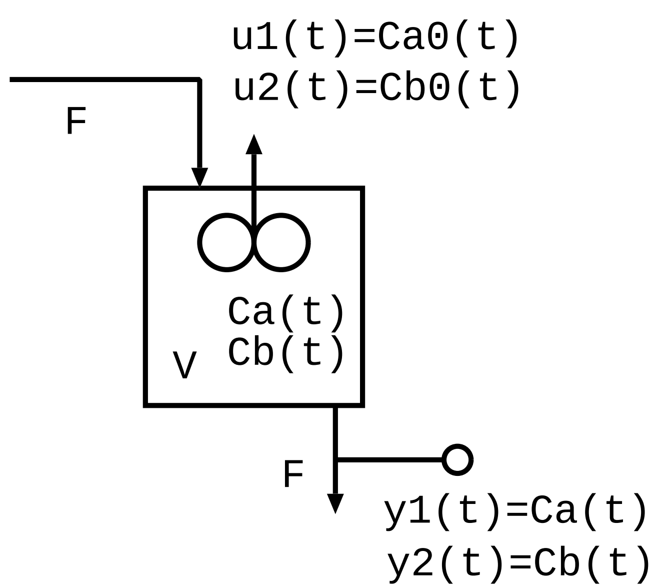
3a.Express the following function as a simple function of time.



b. Establish the Laplace transform of the function, $u(s)$.

c. Assuming this function $u(t)$ is the input to a first-order system, $g(s) = \frac{5}{10s+1}$, $y(s) = g(s)u(s)$. Establish $y(s)$ and $y(t)$.

4. Assuming a constant volume mixing tank for two species, A and B. Assuming you can change the inlet concentrations of A and B and measure the outlet concentrations of A and B, develop a dynamic mass balance and put your equations in state space form.



13.9 Fall 2002 Exam 1

Chemical Process Dynamics and Control

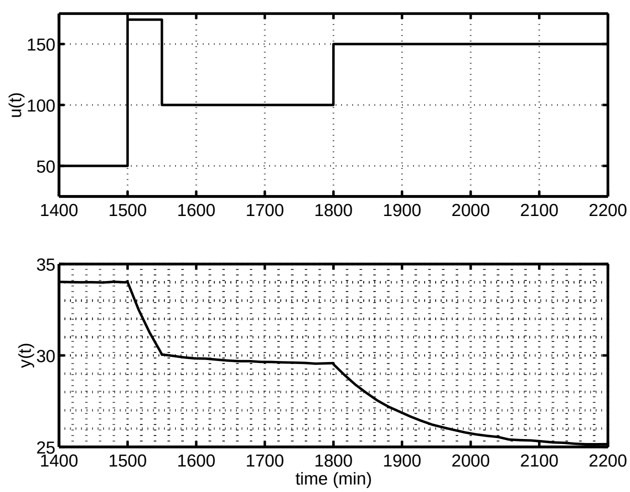
Exam #1

September 25, 2002

1. **(15 pts.)** The Ideal Gas Company is attempting to develop a dynamic process model for a chemical reactor. The process output and the process input are shown below for a input step change.

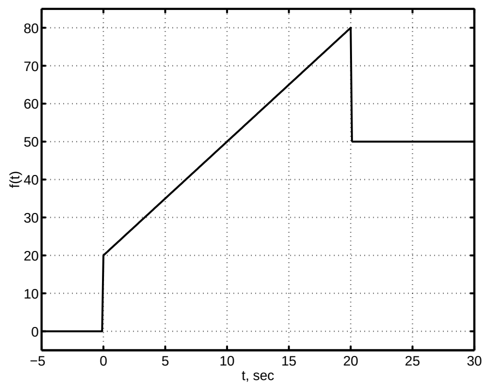
a. Determine the process gain (K), the process time constant (τ) and the process dead time (α) for the system.

b. What is the transfer function for this system, $g(s)$, assuming it is a first order process?



2. **(15 pts.)** What is the the time domain expression for the following function expressed using the Heaviside function? What is the Laplace transform $u(s)$ of the following function?

$$u(t) = \begin{cases} 0 & t < 0 \\ 20 + 3t & 0 \leq t < 20 \\ 50 & 20 < t \leq \infty \end{cases}$$



f(t)
t, sec
80
70
60
50
40
30
20
10
0
−5
0
5
10
15
20
25
30

3. **(25 pts. total)** A system consists of two mixing tanks in series, as pictured below. Two manipulated inputs are available: the initial concentration of species A entering tank 1 and the initial temperature of the liquid entering tank 1. You can measure the temperature at the exit of tank 1 and the concentration of A at the exit of tank 2. You may assume well-mixed tanks, constant volumetric flow rates, constant volume tanks, constant density, and constant heat capacity. You may use the reference temperature T^*.

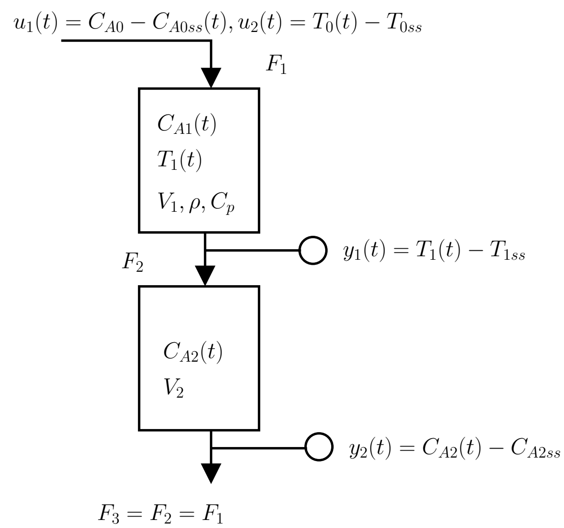


Figure 13.1: Two mixing tanks in series.

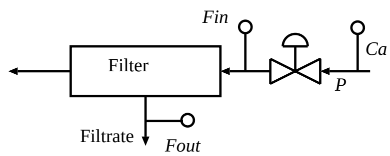
a. **(12 pts.)** Derive the differential equation model for the system.

b. **(6 pts.)** Put your differential equation model into deviation variables by subtracting the steady state equations. Use the following variables:
$u_1(t) = C_{A0}(t) - C_{A0ss}$, $u_2(t) = T_o(t) - T_{oss}$, $x_1(t) = C_{A1}(t) - C_{A1ss}$ $x_2(t) = T_1(t) - T_{1ss}$, and $x_3(t) = C_{A2}(t) - C_{A2ss}$.

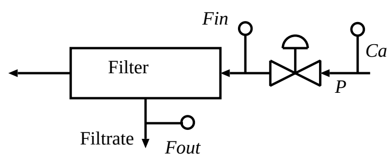
c. **(7 pts.)** Put your deviation differential equation model into State Space form ($\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$, $\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$,) for the system, given $y_1(t) = T_1(t) - T_{1ss}$, and $y_2(t) = C_{A2}(t) - C_{A2ss}$. $\underline{x}$:

$$\underline{x} = \begin{bmatrix} C_{A1}(t) - C_{A1ss} \\ T_1(t) - T_{1ss} \\ C_{A2}(t) - C_{A2ss} \end{bmatrix}$$

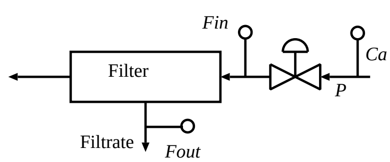
4. **(15 pts.)** For the following system, a stream containing radioactive solids is passed through a crossflow filter. The clean filtrate is separated from the radioactive slurry. The available online measurements include the concentration of the slurry entering the filter, the flow rate of the slurry entering the filter, and the flow rate of the filtrate exiting the filter. The inlet valve can be manipulated. It is desired to regulate the filtrate flow, keeping it at a constant value.



a. In the figure above, draw a feedback control loop for the system



b. In the figure above, draw a feed forward control loop, assuming the feed concentration changes unpredictably.



c. In the figure above, assuming the feed stream pressure changes and will affect the flow through the valve, draw a cascade configuration using two feedback controllers.

5. **(30 pts. total)** A constant volume salt mixing tank system can be modeled by a first order transfer function relating the measurement $C_A(t)$ to the manipulated input $C_{Ao}(t)$:

$$\frac{V}{F}\frac{dC_A}{dt}(t) + C_A(t) - C_{Ao}(t) = 0$$

a. **(4 pts.)** From this differential equation, what are the values for the process time constant, τ, and process gain, K in terms of tank volume V and flow rate F ($F_{in} = F_{out} = F$) ?

b. **(5 pts.)** Justify using physical arguments the value of the steady state process gain.

c. **(5 pts.)** Assuming $C_A(t = 0) = 0$, $V = 2m^3$ and $F = 0.05\frac{m^3}{sec}$, express $C_A(s)$ in terms of s and $C_{Ao}(s)$ by taking the Laplace transform of the differential equation.

d. **(6 pts.)** When salt is not being added to the system, the inlet flow to the tank gets clogged with dried salt. When the salt is first added to the system, the salt plug flows into the tank and the inlet concentration spikes to a value of 110 for 1 second, then returns to the desired value of 10. Assume that the short time, high level rectangular pulse in $C_{Ao}(t)$ can be expressed as an impulse. For this start up procedure, sketch $C_{Ao}(t)$, determine $C_{Ao}(t)$ in terms of $H(t)$ and $\delta(t)$, and determine the Laplace transform of this function, $C_{Ao}(s)$. Assume the impulse occurs at time $t = 0$.

e. **(4 pts.)** What is the exit concentration response, $y(s)$, realizing $y(s) = g(s)u(s)$?

f. **(6 pts.)** This response can be broken into two portions, $y_1(t)$ and $y_2(t)$. Determine $y_1(t)$ and $y_2(t)$ then sketch $y_1(t)$, $y_2(t)$, and the overall response $y(t)$.

13.10 Fall 2003 Exam 1

Chemical Process Dynamics and Control

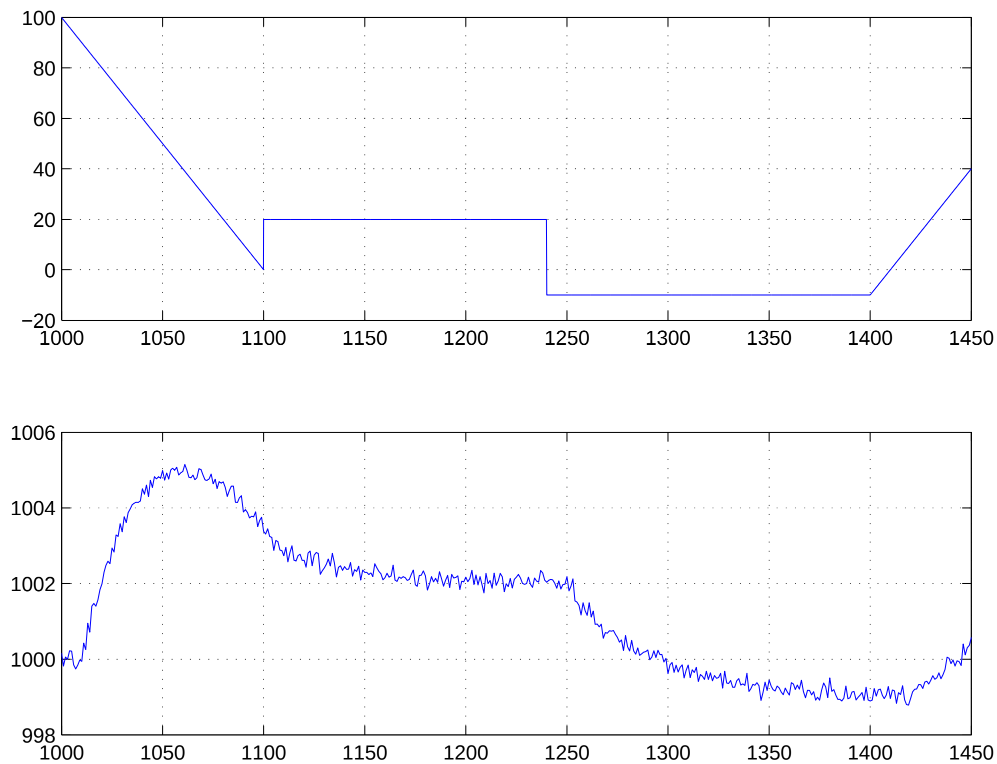
Exam #1

September 28, 2002

1. **(15 pts. total)** The Ideal Gas Company is attempting to develop a dynamic process model for a chemical reactor. The process output and the process input are shown below for a input step change.

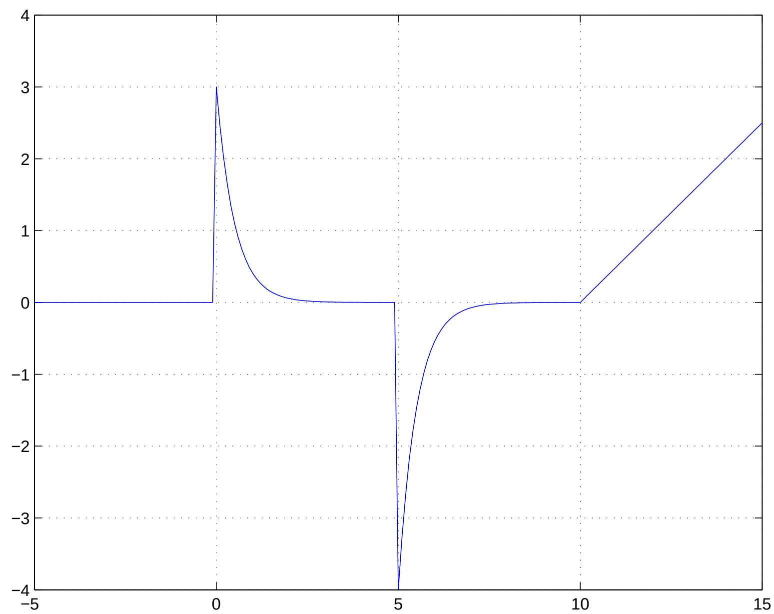
a. **(12 pts.)** Determine the process gain (K), the process time constant (τ) and the process dead time (α) for the system. Time units on the graph are in minutes.

b. **(3 pts.)** What is the transfer function for this system, $g(s)$, assuming it is a first order process with time delay?



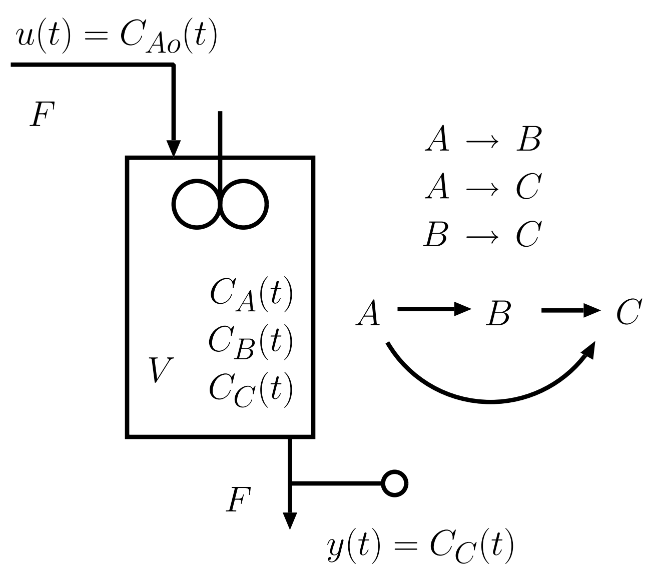
2. **(15 pts.)** What is the Laplace transform $u(s)$ of the following function, given that $u(t) = 0$ for $t < 0$?

$$u(t) = \begin{cases} 3e^{-2t} & 0 \leq t < 5 \\ 3e^{-2t} - 4e^{-2(t-5)} & 5 \leq t < 10 \\ 3e^{-2t} - 4e^{-2(t-5)} + \frac{1}{2}(t-10) & 10 \leq t < \infty \end{cases}$$



3. **(30 pts. total)** A reactor is to be used to produce a new product. The reactor is a constant volume system, with constant volume V and constant flow rate in / out F. Three species are present. Species A can react to form species B at a rate of $r_{AB} = k_1 C_A$. Species A can ALSO react to form species C at a rate of $r_{AC} = k_2 C_A$. Species B will react to form species C at a rate of $r_{BC} = k_3 C_B$. Reaction rates are volumetric, $(\frac{mol}{L\ min})$. Only species A is entering the system. You can adjust the concentration of species A entering the system, $C_{Ao}(t)$. You can measure the concentration of C leaving the system. You do not need to carry unit throughout the problem, just make sure you have the correct terms in each balance.

- $A \rightarrow B$ with reaction rate $r_{AB} = k_1 C_A$
- $A \rightarrow C$ with reaction rate $r_{AC} = k_2 C_A$
- $B \rightarrow C$ with reaction rate $r_{BC} = k_3 C_B$
- Constant volume V and flows F
- Single input, $u(t) = C_{Ao}(t)$
- Single measurement, $y(t) = C_C(t)$



a. **(3 pts.)** What are some additional assumptions you will use to model this system?

b. **(15 pts.)** Derive a differential equation model for the system.

c. **(12 pts.)** Put your deviation differential equation model into State Space form ($\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$, $\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$,) for the system, given that concentrations are all = 0 initially (no deviation variables needed in this case).

4. **(40 pts. total)** At the Ideal Gas Company, a model of a simple chemical reactor system was developed by a previous employee. You are expected to verify the model and determine the time domain response of the model for changes in the input value.

a. **(10 pts.)** For the following differential equation:

$$\frac{d^2y}{dy^2}(t) + 7\frac{dy}{dy}(t) + 12y(t) = \frac{du}{dt}(t) + u(t)$$

show that for the initial conditions $y(t = 0) = 0$, $\frac{dy}{dt}(t = 0) = 0$ $u(t = 0) = 0$, and $\frac{du}{dt}(t = 0) = 0$, the following transfer function relationship holds:

$$y(s) = \frac{s+1}{s^2 + 7s + 12}\, u(s)$$

b. **(5 pts.)** Given that the input to the system model is a unit impulse at time $t = 1$ (NOT at t=0) show that $y(t = 0) = 0$ using the Initial Value Theorem.

c. **(5 pts.)** Given that the input to the system model is a unit impulse at time $t = 1$ (NOT at t=0) show that $y(t = \infty) = 0$ using the Final Value Theorem.

d. **(20 pts.)** Given that the input to the system is a unit impulse at time $t = 1$ (NOT at t=0) find the analytical response $y(t)$ of the system to the unit impulse as an explicit function of time.

BONUS, sketch $u(t)$ and $y(t)$.

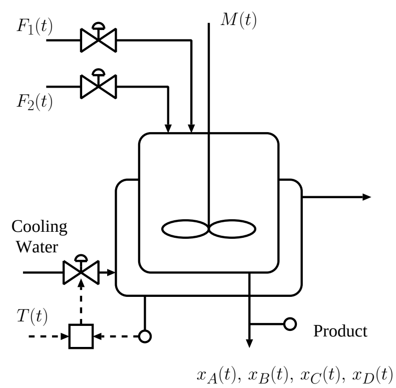
13.11 Fall 2003 Quiz 1

Chemical Process Dynamics and Control

Quiz #1

September 5, 2001

1. **(4 pts.)** A continuous polymerization reactor has two feed streams. Four species are measured at the exit of the reactor. The temperature of the reactor can be modified using a cooling jacket. Additionally, the mixing speed can be modified.



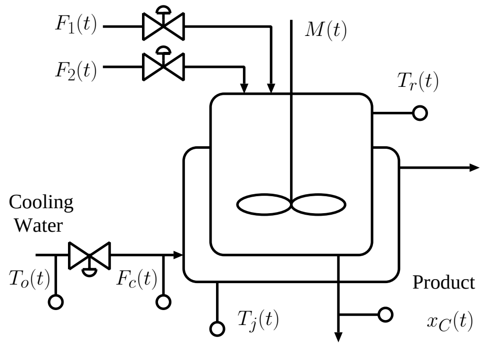
A 4x4 steady state model is desired relating the manipulated variables changes to the change in the output product concentrations, Δx_A, Δx_B, Δx_C, Δx_D. Input flows (ΔF_1 and ΔF_2), the change in the reactor jacket temperature (ΔT), and the change in mixing speed (ΔM) affect product quality in the following manner:

- A +10 GPH change in F_1 increases x_C by 2 %
- A +10 GPH change in F_1 increases x_D by 4 %
- A +10 GPH change in F_1 **decreases** x_B by 3 %
- A +10 GPH change in F_2 increases x_A by 1 %
- A +10 GPH change in F_2 increases x_C by 1 %
- A +1 change in T increases all concentrations by 0.2%
- A +2 RPM increase in M increases x_B by 5%

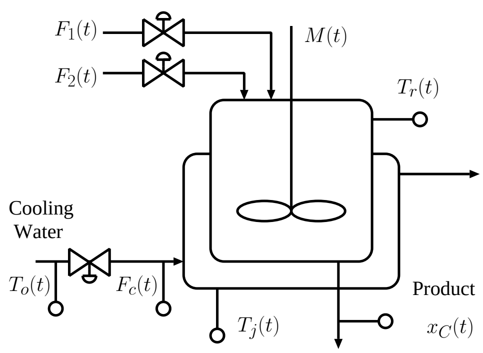
Develop a model in the form $\underline{\underline{K}}\ \underline{\Delta u} = \underline{\Delta y}$ and identify $\underline{\underline{K}}$, $\underline{\Delta u}$, and $\underline{\Delta y}$.

You may want to check your model by assuming arbitrary values for the change in inputs, then verify the expected change in concentrations.

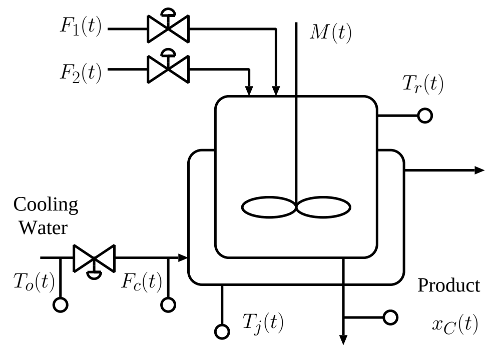
2. **(2 pts.)** Given the following system, draw a simple feedback control scheme to control the product quality x_C by manipulating the cooling water flow.



3. **(2 pts.)** Given the following system, draw a simple feedforward control scheme to control the product quality x_C by manipulating the cooling water flow given variations in the inlet cooling water temperature.



4. **(2 pts.)** Given the following system, draw a cascade control scheme to control the product quality x_C.



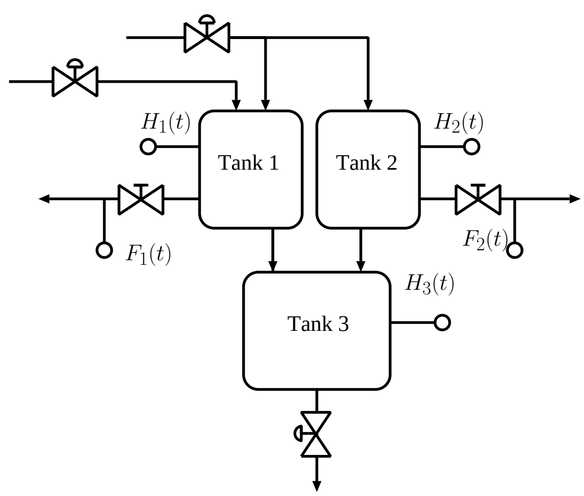
13.12 Fall 2004 Quiz1

Chemical Process Dynamics and Control

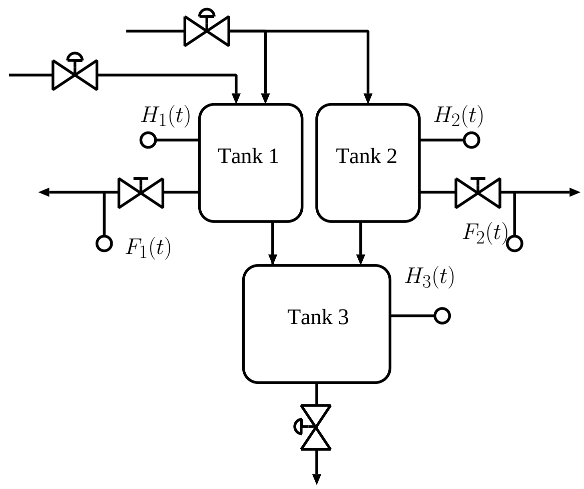
Quiz #1

September 3, 2004

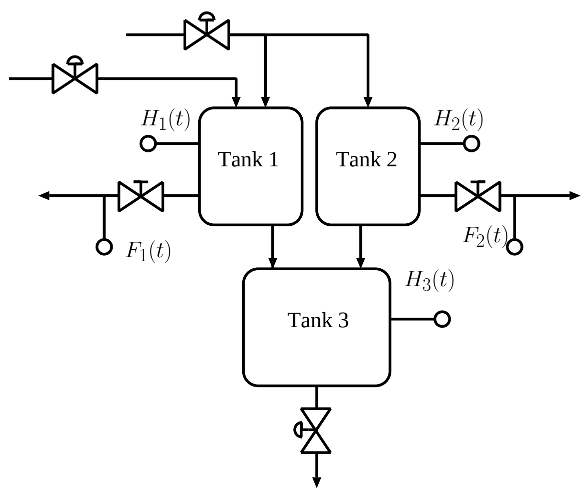
1. **(3 pts.)** Given the following process system, draw two separate simple feedback control loops to control tank levels in tanks 1 and 2. Be sure to use control valves in your loops.



2. **(2 pts.)** Given the following system, draw a simple feedforward control schemes to help minimize variation in tank levels given changes in $F_2(t)$. Be sure to use control valves in your loops.



3. **(2 pts.)** Given the following system, draw a cascade control scheme to control the level in tank 3, noting that the level in tanks 1 or 2 would have some effect on the level of tank 3.



4. **(2 pts.)** What are the eigenvalues of the following matrix? Please show your work.

$$\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$$

5. **(1 pts.)** What is the determinant of the following matrix? Please show your work.

$$\begin{bmatrix} 0 & -2 & -1 \\ 0 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

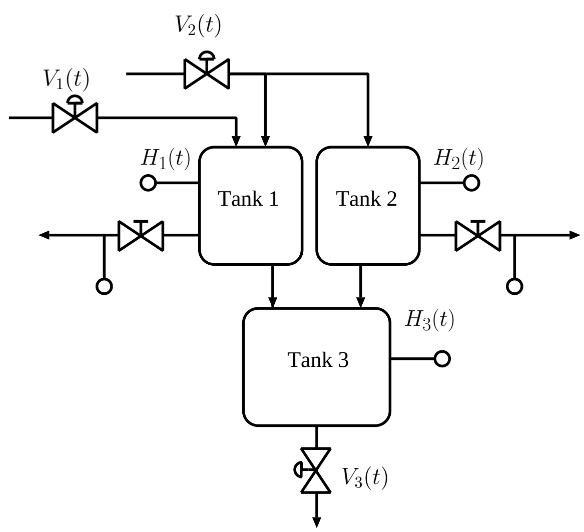
13.13 Fall 2004 Quiz 2

Chemical Process Dynamics and Control

Quiz #2

September 14, 2004

1. At the Ideal Gas Company, you are expected to develop a steady state model of the following process:

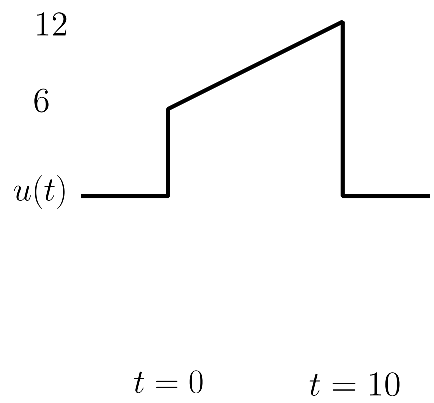


Given the following information:

- A 1% change in V_1 increases the level in Tank 1 by 3 inches
- A 1% change in V_1 increases the level in Tank 3 by 1 inch
- A 3% change in V_2 increases the level in Tank 1 by 4 inches
- A 3% change in V_2 increases the level in Tank 2 by 5 inches
- A 3% change in V_2 increases the level in Tank 3 by 6 inches
- A 1% change in V_3 decreases the level in Tank 3 by 2 inches

a. **(1 point)** What are $\underline{\Delta u}$ and $\underline{\Delta y}$?
b. **(4 points)** Develop a steady state multivariable model relating the inputs to the outputs.
c. **(1 point)** Put your model in the form $\underline{\Delta y} = \underline{\underline{K}}\ \underline{\Delta u}$ and clearly identify the $\underline{\underline{K}}$ matrix.

2. **(4 points)**What is the Laplace transform of the following input sequence, $u(t)$.



$$u(t) = \begin{cases} 0 & t < 0 \\ 6 + \frac{6}{10}t & 0 \leq t < 10 \\ 0 & 10 \leq t \end{cases}$$

3. **BONUS** In ***five words or less***, why can we analyze dynamic systems with complex composite forcing functions by treating each part separately?

13.14 Fall 2004 Exam 1

ECHE 550, Fall 2004

Chemical Process Dynamics and Control

Exam #1

September 27, 2004

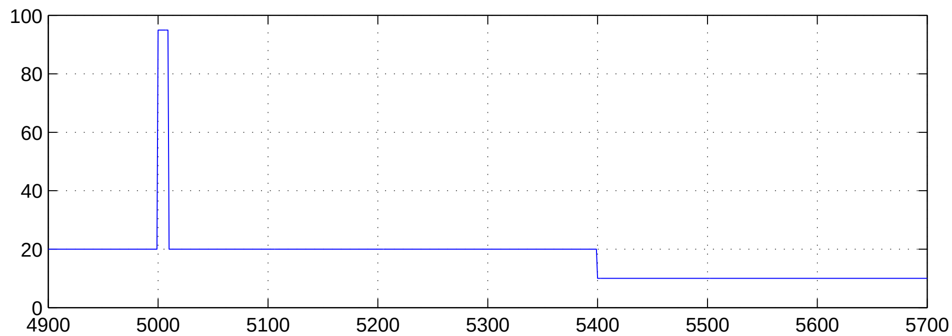
1. **(20 pts. total)** The Ideal Gas Company is attempting to develop a dynamic process model for a continuous processing nylon production system. Data from the process output and the process input are shown below for a step change and a simulated impulse.

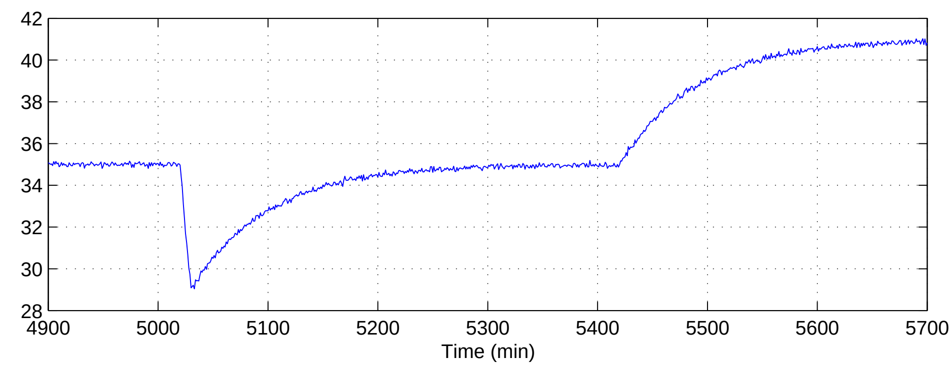
a. **(15 pts.)** Determine the

- process gain (K)
- process time constant (τ)
- process dead time (time delay) (α)

for the system. Time units on the graph are in minutes.

b. **(5 pts.)** Given a unit step change increase in the process input at time $t = 0$, what is the expected response of your model as a function of time?





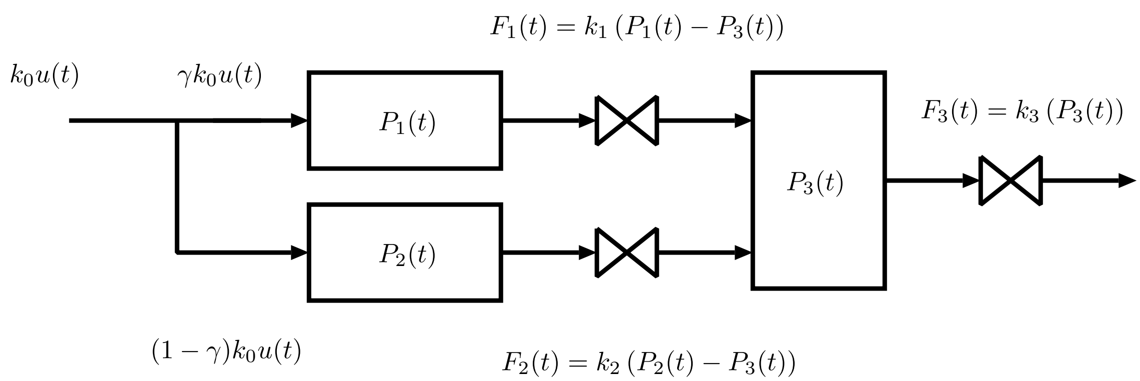
2. **(20 pts. total)** Determine the eigenvalues of the following matrix. Note, you probably should **not** use row reduction methods in the solution of this problem.

a. **(10 pts.)** Set up the problem to be solved.

b. **(10 pts.)** Find simplified numerical values for the eigenvalues.

$$\begin{bmatrix} 0 & -4 & 1 \\ 0 & -3 & 5 \\ 0 & -4 & 1 \end{bmatrix}$$

3. **(25 pts. total)** You must develop a dynamic model based on fundamental principles for a pressure tank system as pictured below. You may assume that the total number of moles of gas in tank i, $n_i(t)$, may also be expressed as $\frac{V_i}{RT}P_i(t)$ using the ideal gas law. All flow rates are molar flow rates. You can change the valve position on the inlet stream, $u(t)$. You do not need to carry units throughout the problem, just try to make sure you have the correct terms in each balance.



- The total molar flow rate into the system is $k_0u(t)$
- The molar flow rate into tank 1 is $\gamma k_0u(t)$, $0 \leq \gamma \leq 1$
- The molar flow rate into tank 2 is $(1-\gamma)k_0u(t)$
- The molar flow from tank 1 into tank 3 is $k_1(P_1(t) - P_3(t))$
- The molar flow from tank 2 into tank 3 is $k_2(P_2(t) - P_3(t))$
- The molar flow from tank 3 into the atmosphere is $k_3P_3(t)$
- The tanks are constant volume, V_1, V_2, V_3.
- The gas in the tanks is at a constant temperature.
- You can measure the pressure in tanks 2 and 3.

a. **(15 pts.)** Derive a differential equation model for the system.

b. **(10 pts.)** Put your deviation differential equation model into State Space form ($\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$, $\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$,) for the system, given that all pressures are = 0 initially (no deviation variables needed in this case).

4. **(35 pts. total)** At the Ideal Gas Company, a model of a simple chemical reactor system was developed by a previous employee. You are expected to verify the model and determine the time domain response of the model for changes in the input value.

a. **(7 pts.)** For the following differential equation:

$$6\frac{dy}{dt}(t) + 2y(t) = 4\frac{du}{dt}(t) + u(t)$$

show that for the initial conditions $y(t = 0) = 0$ and $u(t = 0) = 0$, the following transfer function relationship holds:

$$y(s) = \frac{4s + 1}{6s + 2}\, u(s)$$

b. **(6 pts.)** What are the poles of your transfer function? What are the zeros?

c. **(4 pts.)** What is the gain of this model?

d. **(5 pts.)** Given that you implement a negative step change of magnitude 3 at time zero in the input $u(t)$, what is the ultimate response? Use the Final Value Theorem to find $y(t = \infty)$.

d. **(13 pts.)** Given that you implement a negative step change of magnitude 3 at time zero in the input $u(t)$, find the analytical response $y(t)$ of the system.

BONUS, sketch $u(t)$ and $y(t)$.

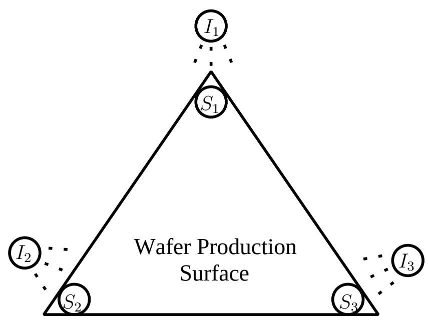
13.15 Fall 2005 Quiz 1

Chemical Process Dynamics and Control

Quiz #1

September 2, 2005

1. **(4 pts.)** Your first assignment for GameCockCo is in the silicon wafer production facility. Each wafer must be maintained at a high temperature during the etching process. Since very high temperatures are required, radiative heat transfer using high temperature lamps will be used to heat the chamber. The triangular chemical vapor decomposition chamber has three variable intensity lamps, one in each corner of the chamber, with intensities I_1, I_2, and I_3. The chamber also has three temperature sensors, denoted by S_1, S_2, and S_3. These sensors are also located in the corners of the chamber. You are told by a senior engineer in your department that a 5% increase in the intensity of any one of the three lamps results in a 8 degree increase in the corresponding temperature sensor location and a 3 degree increase in the temperature in the sensors in both of the opposite corners of the chamber.



Develop a model in the form $\underline{\underline{K}}\ \underline{\Delta u} = \underline{\Delta y}$ and clearly identify $\underline{\underline{K}}$, $\underline{\Delta u}$, and $\underline{\Delta y}$.

2. **(2 pts.)** During a production run, temperature sensor 1 is 6 degrees below optimal, sensor 2 is 1 degree above optimal, and sensor 3 is 9 degrees below optimal. What problem would you solve to get the chamber back to the nominal operating temperature (what is your $\underline{\Delta y}$ value?). How would you solve this problem? (What formula or method would you use?)

3. **(2 pts.)** What are the eigenvalues of the following matrix? Please show your work.

$$\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

4. **(2 pts.)** What is the determinant of the following matrix? Please show your work.

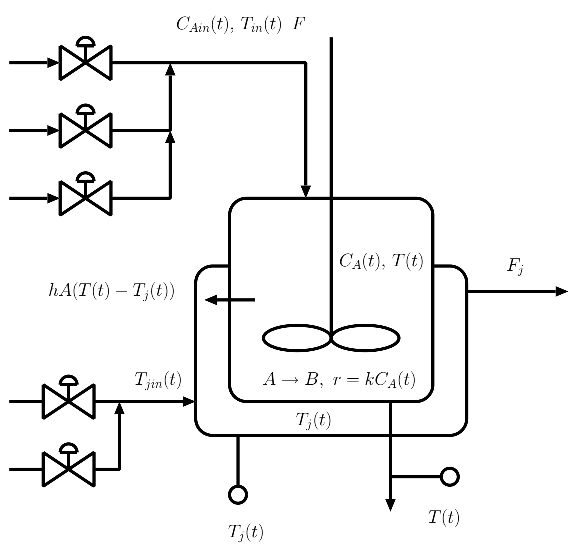
$$\begin{bmatrix} 1 & -4 & 0 \\ 0 & 3 & 0 \\ -1 & 3 & 2 \end{bmatrix}$$

BONUS, why are eigenvalues and determinants useful?
BONUS, solve problem 2 above for a real answer.

13.16 Fall 2005 Quiz 3

ECHE 550, Fall 2005
Chemical Process Dynamics and Control
Quiz #3, October 7, 2005

1. **(4 pts.)** A continuous bio-reactor for yeast fermentation has a single glucose feed stream. The growth reaction takes place in a jacketed CSTR. It is assumed that low-level control systems are in place such that the inlet glucose concentration and temperature can be specified (the mixing of pure cold water, pure hot water, and high concentration glucose ***is not*** to be considered here). Additionally, the temperature of water entering the jacket can be specified. Given these three manipulated input values, you are expected to develop a state-space model of the system.



The following is known about the system:

- The reactor is well-mixed with volume V
- The liquid in the jacket is well-mixed with volume V_J
- The reaction is a first-order reaction with volumetric reaction rate $kC_A(t)$
- The heat of reaction is $-\Delta H$
- The reactor volumetric flow rate in (and out) is F
- The jacket volumetric flow rate in (and out) is F_j
- The heat transfer from the reactor to the jacket is $hA(T(t) - T_j(t))$
- The liquids all have constant physical properties, C_p, ρ, μ

- The steady state input values for u_1, u_2, and u_3 are C_{AinSS}, T_{inSS}, and T_{jinSS} respectively
- The steady state state values are C_{ASS}, T_{SS}, and T_{jSS}
- The jacket temperature deviation value $T_j(t) - T_{jSS}$ and the reactor temperature deviation value $T(t) - T_{SS}$ are measured

a. Develop a dynamic mass and energy balance for the system.

b. Put your system in deviation variable form using the steady state values.

c. Identify your states and put your system in state-space form and identify $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$.

13.17 Fall 2005 Exam 1

ECHE 550, Fall 2005

Chemical Process Dynamics and Control

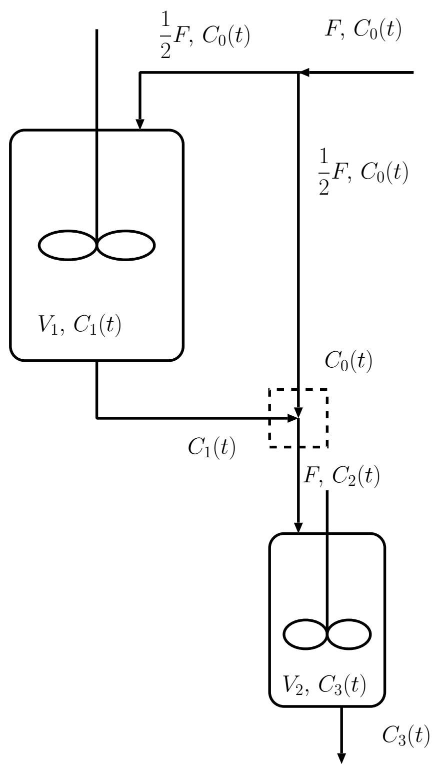
Exam #1

September 28, 2005

1. **(30 pts. total)** The Ideal Gas Company has a simple mixing system for preparation of a reactor feed. You must develop a dynamic model of this system.

- No reaction is taking place in either tank, both well-mixed
- The mixing tanks are constant volume, V_1 and V_2
- The volumetric flow rate F is fixed
- The inlet flow is equally split between the first tank and the side stream
- Initially all concentrations are 0

a. **(15 pts.)** Develop a dynamic model of the system. Note that instantaneous mixing occurs at the mixing point shown in the dotted box (no accumulation at the mixing point). State any additional assumptions you make for your system.



b. **(15 pts.)** Assuming that $F = 2\frac{m^3}{min}$, $V_1 = 20m^3$, $V_2 = 10m^3$, $C_0(t) = u(t)$ and $C_3(t) = y(t)$ take the Laplace transform of your model equations and show that the following transfer function holds:

$$\frac{y(s)}{u(s)} = g(s) = \frac{(10s+1)}{(5s+1)(20s+1)}$$

2. **(30 pts. total)** Assume that the inlet concentration momentarily changes, allowing some of the reactant to flow into the system. You can assume that $u(t) = \delta(t)$, with the system modeled as:

$$y(s) = \frac{(10s+1)}{(5s+1)(20s+1)}u(s)$$

a. **(2 pts.)** What is $u(s)$?

b. **(6 pts.)** Given this input, what is the initial value for y, $y(t=0)$?

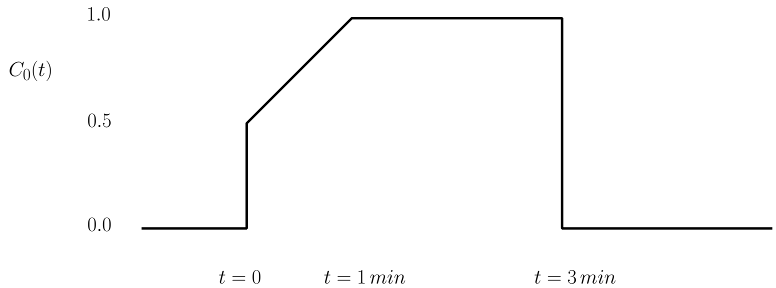
c. **(6 pts.)** Given this input, what is the final value for y, $y(t=\infty)$?

d. **(10 pts.)** What is the actual response of the outlet concentration as an analytical expression, $y(t)$? Note that:

$$L^{-1}\left\{\frac{1}{\tau s+1}\right\} = \frac{1}{\tau}e^{-\frac{t}{\tau}}$$

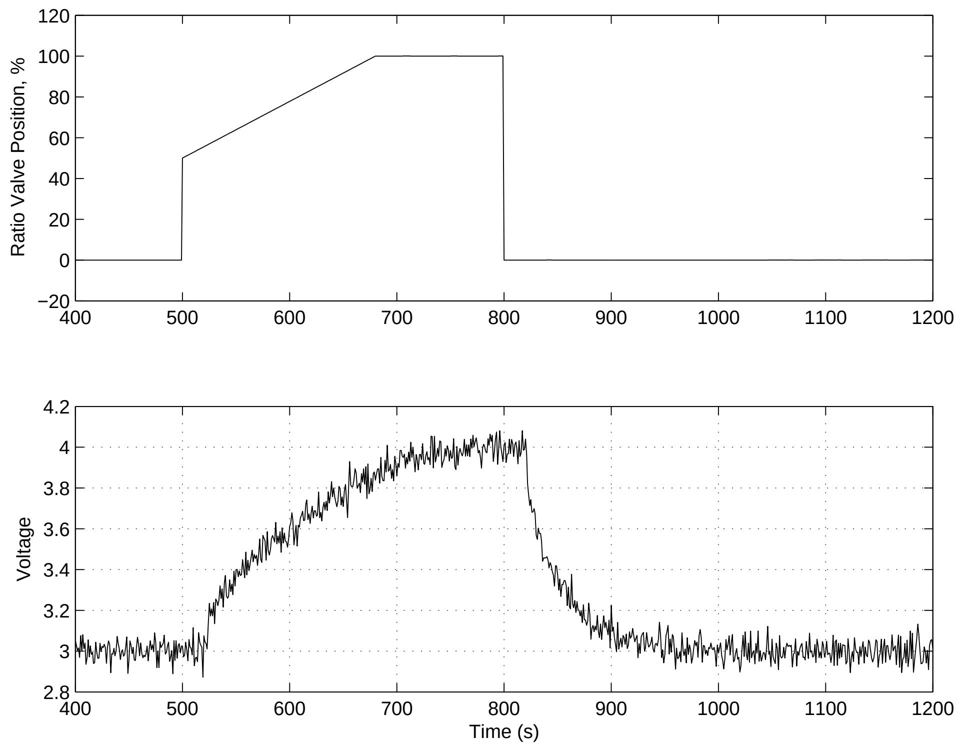
e. **(6 pts.)** Sketch $y(t)$.

3. **(20 pts. total)** The inlet concentration for your system can be manipulated, but there are limits to the response of the inlet concentration value. What is the Laplace transform of the following function of time, $C_0(s)$?



$$C_0(t) = \begin{cases} 0 & t \leq 0 \\ 0.5 + 0.5t & 0 \leq t \leq 1 \\ 1.0 & 1 \leq t \leq 3 \\ 0 & 3 \leq t \end{cases}$$

4. **(20 pts. total)** From the following dynamic response data, determine the values for the gain, time constant and time delay for the real experimental system (K, τ, α). The inlet concentration is manipulated using a ratio valve and the exit concentration measurement is reported as a signal voltage. The valve is limited in the ability to open, resulting in the abnormal $u(t)$ value.
b. **(5 points)** What is the transfer function, $g(s)$, for this system?



BONUS, Derive a state space model for your dynamic system from problem 2.

13.18 Fall 2006 Quiz 1

ECHE 550, Fall 2006

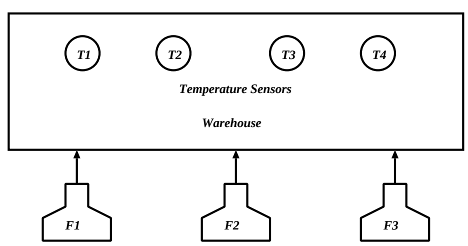
Chemical Process Dynamics and Control

Quiz #1

September 8, 2005

1. **(5 pts.)** As an intern at GameCockCo, you get stuck in the warehouse. The warehouse has had problems with product loss due to poor heating in the winter. The complex HVAC system is not maintaining a uniform temperature in the warehouse due to poor mixing (channeling in the warehouse ventilation flow). Rather than buy fans to force improved convection in the warehouse, you suggest an improved control system, since the current system runs all furnaces at the same rate. Four temperature sensors are available, T_1, T_2, T_3, and T_4. Three furnaces are available, F_1, F_2, and F_3. The furnaces run on a 0-100 scale and are controlled by a centralized computer system.

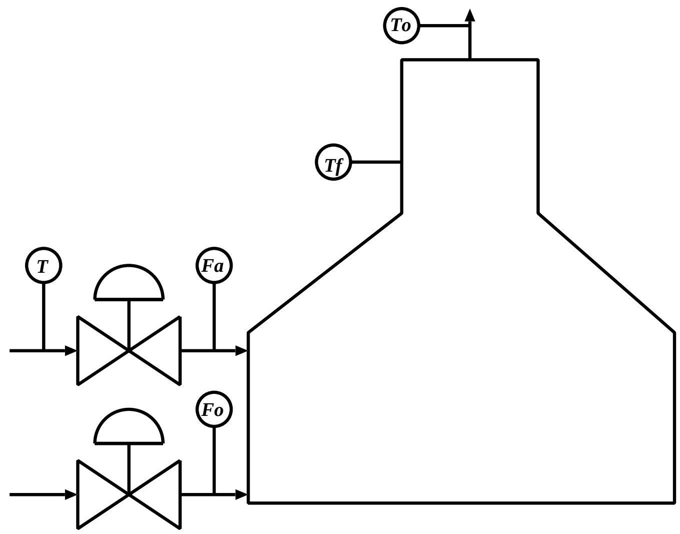
- Increasing Furnace 1 by 10 units increases T_1 by 3 degrees, $T_2$by 2 degrees, and T_3 by 1 degree
- Increasing Furnace 2 by 10 units increases T_1 by 1.5 degrees and T_2 by 5 degrees
- Increasing Furnace 2 by 10 units increases T_3 by 4 degrees, and T_4 by 2.5 degrees
- Increasing Furnace 3 by 10 units increases T_2 by 1.5 degrees, T_3 by 2.1 degrees, and T_4 by 3.2 degrees



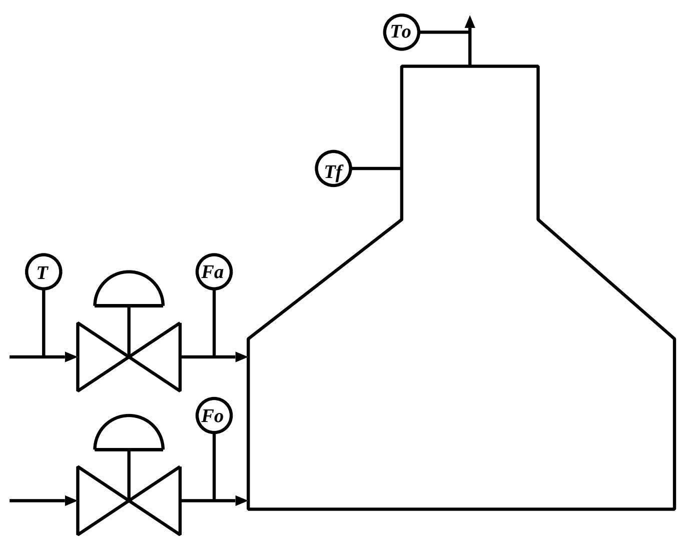
Develop a model in the form $\underline{\underline{K}}\ \underline{\Delta u} = \underline{\Delta y}$ and clearly identify $\underline{\underline{K}}$, $\underline{\Delta u}$, and $\underline{\Delta y}$.

2. **(1 pt)** During a production run, temperature sensor 1 is 2 degrees below optimal, sensor 2 is 1 degree above optimal, sensor 3 is 5 degrees below optimal, and sensor 4 is 2 degrees above optimal. What problem would you solve to get the chamber back to the nominal operating temperature (what is your $\underline{\Delta y}$ value?).

3. **(2 pts.)** For the furnace system, it is desired to regulate the outlet temperature T_o. Both air and oil are fed to the furnace, and both flow rates strongly influence the outlet temperature. Draw a simple feedback control system for the furnace below.



4. **(2 pts.)** For the furnace system, the inlet air temperature has some influence on the outlet temperature. Draw a feedforward control system to mitigate the effects of the incoming air temperature on the furnace outlet temperature.



13.19 Fall 2006 Quiz 2

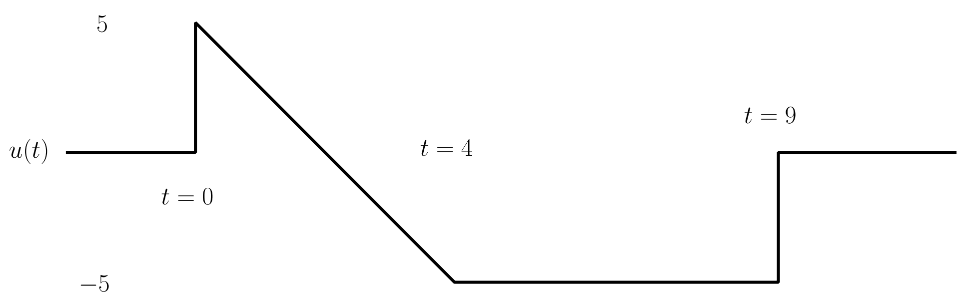
ECHE 550, Fall 2006

Chemical Process Dynamics and Control

Quiz #2

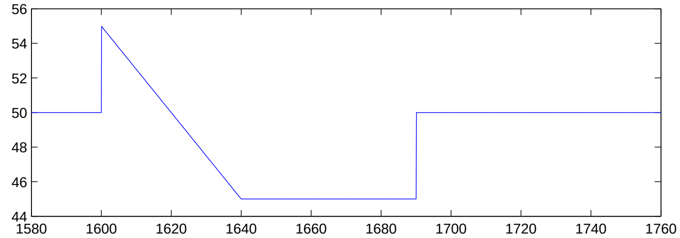
September 20, 2006

1. **(5 pts.)** For the following funciton of time:



a). Express $u(t)$ as a sum of simple functions of time.
b). Find $u(s)$, the Laplace transform of $u(t)$.

2. **(4 pts)** For the following process data, determine the gain, time constant, and time delay.
b).**(1 pt)** What is the first-order-plus-time-delay transfer function for this sytem?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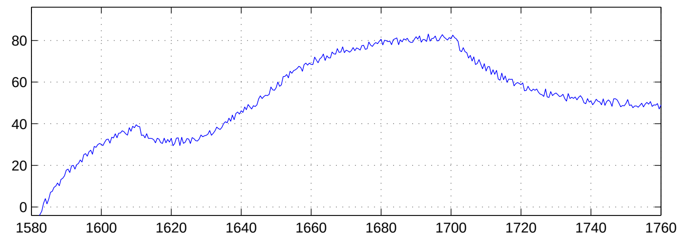
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13.20 Fall 2006 Exam 1

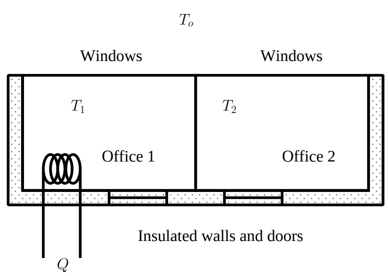
ECHE 550, Fall 2006

ECHE 550, Fall 2006

Chemical Process Dynamics and Control

Exam #1 - October 4, 20056

1. **(15 pts. total)** Your nice window office at GameCock Co. is on the west side of the building. Due to mistakes when installing the HVAC system, the temperature in your office is poorly regulated, since the cooling takes place only in the office next door and the windows are not insulated. Given the following steady state data:

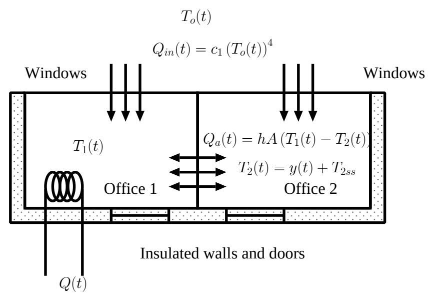


- A 3 degree increase in the external temperature T_o results in a 2 degree increase in T_1
- A 3 degree increase in the external temperature T_o results in a 1 degree increase in T_2
- A increase of 5 units in the chiller Q results in a 4 degree decrease in T_1
- A increase of 5 units in the chiller Q results in a 2.5 degree decrease in T_2

a. **(8 pts.)** Develop a model in the form $\underline{\underline{K}}\ \underline{\Delta u} = \underline{\Delta y}$ and clearly identify $\underline{\underline{K}}$, $\underline{\Delta u}$, and $\underline{\Delta y}$.

b. **(7 pts.)** Given that you want T_1 to remain constant and you want T_2 to decrease by 1 degree, what would have to happen to Q and T_o?

2. **(30 pts. total)** Develop a dynamic model of your office. Assume the following:



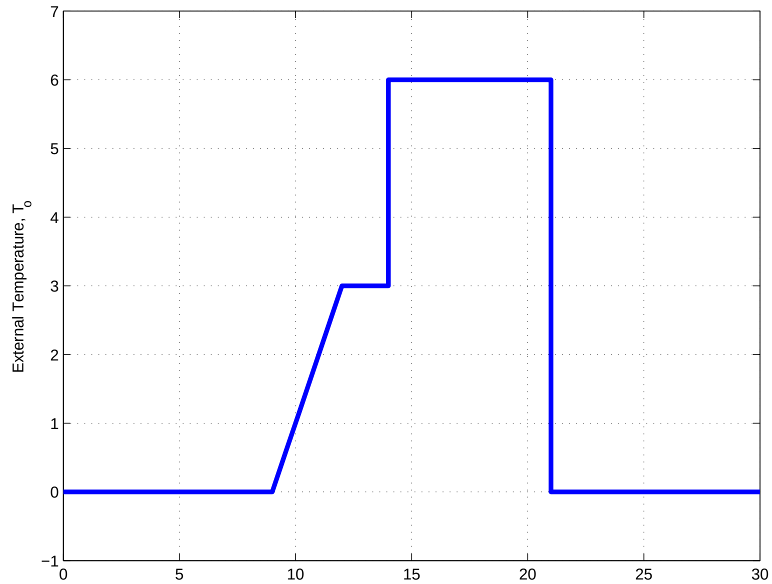
- The offices have no air moving in or out, but the air in the offices is well-mixed (fans)
- The volume of air in each office is V_1 and V_2 respectively
- The heat capacity and density of the air in each office is C_p and ρ
- The rate of energy entering the each office from the outside is equal: $Q_{in}(t) = c_1(T_o(t))^4$
- The rate of energy transferred across the thin wall is: $Q_a(t) = hA(T_1(t) - T_2(t))$
- Physical properties and parameters do not change with time
- Deviation values are:
  $x_1(t) = T_1(t) - T_{1ss}$
  $x_2(t) = T_2(t) - T_{2ss} = y(t)$
  $u_1(t) = T_o(t) - T_{oss}$
  $u_2(t) = Q(t) - Q_{ss}$

a. **(10pts.)** Develop a dynamic model for this system.

b. **(10pts.)** Develop a linear dynamic model for this system in deviation form.

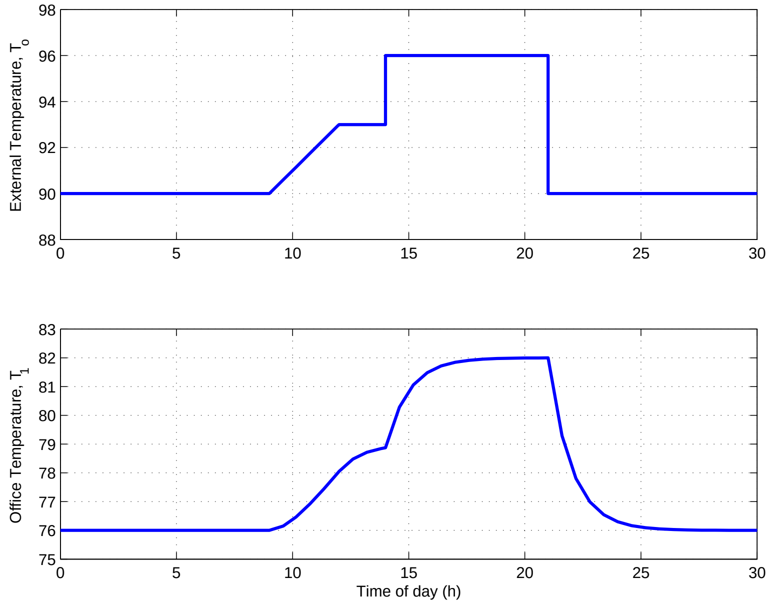
c. **(10pts.)** Put your linear model in state space form and clearly identify $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$.

3. **(15 pts. total)** The external temperature follows the following trajectory during the day. What is the Laplace transform of the following function of time, $u_1(s)$?



$$u_1(t) = \begin{cases} 0 & 0 \le t \le 9 \\ (t-9) & 9 \le t \le 12 \\ 3 & 12 \le t \le 14 \\ 6 & 14 \le t \le 21 \\ 0 & 21 \le t \end{cases}$$

4. **(15 pts. total)** From the following dynamic response data, determine an empirical transfer function for the system.



5. **(25 pts. total)** The previous employee that sat inf your office developed the following model for office temperature as a function of external temperature.

a. **(5 pts.)** For the following differential equation:

$$2\frac{d^2y}{dt^2}(t) + 3\frac{dy}{dt}(t) + 5y(t) = 5\frac{du}{dt}(t) + 10u(t)$$

show that for the initial conditions $y(t = 0) = \frac{dy}{dt}(t = 0) = 0$ and $u(t = 0) = 0$, the following transfer function relationship holds:

$$y(s) = \frac{5s + 10}{2s^2 + 3s + 5}\, u(s)$$

b. **(5 pts.)** What are the poles of your transfer function? What are the zeros?

c. **(5 pts.)** What is the gain of this model?

d. **(5 pts.)** Given that you implement a negative step change of magnitude 5 at time zero in the input $u(t)$, what is the initial value for y? Use the Initial Value Theorem to find $y(t = 0)$.

e. **(5 pts.)** Given that you implement a negative step change of magnitude 5 at time zero in the input $u(t)$, what is the ultimate response? Use the Final Value Theorem to find $y(t = \infty)$. Bonus: Does this model make sense for the system?

Chapter 14

Procedureal Programming Tutorial

14.1 INTRODUCTION

Engineers often use computers to solve problems. Sometimes engineers use a high level programs (like Aspen, pSPICE, or AutoCAD) but sometimes these tools don't do exactly what needs to be done. Various languages and environments allow you to use very general concepts to come up with solutions to problems. These concepts are quite portable, in that the same concepts for procedural problem solving exist in almost every language or environment. The following is a list of the basic concepts.

- Variables
- Input and Output
- Assignment Statements
- Data Structures
- IF statements
- FOR statements
- WHILE statements
- Scripts
- Functions
- Debugging
- Pseudo Code
- Compiling vs. Interpreting

14.2 Variables

Computers store information in memory. At a very low level, the operating system (Windows, Mac OS, Linux, Unix) keeps track of memory and what application is using memory. When an application needs more memory, it asks for a certain amount of memory. If the memory is available, a memory address is provided to the program to use. Luckily, we don't usually need to keep track of memory addresses. Generally, we use **variables** to reference values in the memory.

Variables usually have names that are mostly characters. Some environments have limitations for variables names. Some environments are case sensitive (counter and COUNTER represent different values). Some environment limit the length of variable names or limit the characters in a variable name.

It really is a personal decision for you to use whatever variable name you want to that is legal for the environment you are working in. The name should be descriptive enough so you have some idea what it is, but if it is too long you will have trouble typing it over and over and might make more mistakes.

The following are valid variable names in MATLAB

- counter
- Counter
- new_counter
- NewCounter
- New_Counter
- Counter1
- counter_1

Most environments require that variable names start with a character, so "1counter" would not be valid. Also, spaces in variable names are not usually allowed, like "counter 1".

Matlab does not require you to specify the type or size of variables before you use them. Some languages force you to specify exactly what type and size each variable is.

14.3 Assignment Statements

You can assign a variable a new value using an assignment statement. The value on the right hand side of the equality is evaluated and assigned to the variable named on the left hand side. Sometimes this is simple, like

```
a=1
b=2
```

Sometimes this is more complicated, like

```
c=a*b+3
```

Sometimes you use simple math functions provided by the environment

```
a=sin(2)
b=log(3)
```

Note that you don't have to use just numbers in the function call, you can use variables in the functions:

```
c=sin(a)+log(b)
```

You can call a function using a variable in the function call and reassign the variable value:

```
a=log(a)
```

Sometimes you write your own function to do something special:

```
c=MyFunction(3)
```

Sometimes a function takes multiple input arguments:

```
c=conv(a,b)
a=rand(3,2)
```

In Matlab, you can have multiple output arguments in a function, so two variables are assigned values after the function is evaluated:

```
[a,b]=find(c)
```

14.4 Input and Output

There are various ways to get information into and out of the computer. For input, sometimes you type in values at the keyboard when prompted, sometimes you load a data file. There are more interesting ways to get input values as well. Audio input can be read using a mic. Video input can be read using a camera. Various environmental sensors (for temperature, pressure, or others) can be read using specialized data acquisition devices (DAQ).

The following Matlab command would prompt the user for information and assign the value to the specified variable:

```
age=input('What is your age in years?')
```

The variable age would contain the user input value. Note that the user could type in a number or something not a number. For advanced programming, you may want to check to see that the user input actually is what you expect (a positive number between 0 and 120 in this case).

In many cases, you may have a data file that must be read. For simple numeric files with *m* columns and *n* rows of numeric data, Matlab can use the load command.

```
data=load('Filename')
```

This would read the specified file and put the $n \times m$ data values into the variable data. For more complex file reading, you can use fopen, fread, fwrite. For dealing with character strings, type “help strfun” to see what functions are available in Matlab.

As for output, there are various ways to present results. One easy way is to write text to the computer screen:

```
disp('Something is wrong!')
```

This would just print the message on the Matlab screen. Or for numerical warnings:

```
disp(['The reactor temp is ' num2str(temp) ])
```

This would acutally use the current number in the variable temp to make the output warning message.

You can save data to a file:

```
save filename a b c
```

This would save the values of the variables a b and c into the specified file. You could load those variables again using:

```
load filename
```

and you would have a, b, and c back in the memory of Matlab. Note that this file format is not human readable. To make a nice looking text file that you could open and look at in a text editor, use:

```
save filename a b c --ascii
```

Another type of output is visual. The plot command is very powerful in Matlab.

```
plot([1 2 3],[4 5 6],'x')
```

There are even ways to play audio data, producing output in the speakers

```
sound(rand(1000,1))
```

Using a DAQ system, you can send output to an actuator, like a valve or a motor.

14.5 Data Structures

There are some basic data types available to use for solving problems. The basic types are integers, real numbers, characters, and boolean values.

Integers are integral values, like 0, 1, -2, etc. Languages usually limit the maximum and minimum integer values. Matlab on some platforms is limited to approximately $\pm 1e300$.

Real numbers are numbers that are not integral, like 5.5 and -3.333. These are usually referred to as doubles, short for double precision. On 32 bit machines, a double precision value uses 64 0/1 bits to represent the value. One bit is for the sign, some of the bits are for the exponent value, and the rest represent the binary value of the mantissa.

Characters are basically anything on the keyboard.

Boolean values are either TRUE or FALSE. In Matlab, a positive numeric value means TRUE and 0 or anything negative is false. Logical operators return 0 or 1, but 5 or 0.5 would be seen as true, just like -2 would be seen as false.

Data structures are really more complicated data types. The main ones engineers use are arrays. Arrays are just indexed data values. A one dimensional array is a vector, while a two dimensional array is a matrix. Vectors are all matrices in Matlab, so they could be nx1 or 1xn (column or row vectors).

```
a=[5 6 7]
b=[5 ; 6 ; 7]
```

The previous would create a row vector and a column vector in matlab. You can access individual elements of an array, so a(2) would ask for the second element of a.

```
c=a(2)
```

Matlab also allows you to access multiple elements of an array,

```
c=a(2:3)
```

This would assign c as a vector with only the second and third elements of a.

Matrices are two dimensional arrays. Each element in a matrix has a row and a column index.

```
a=[ 4 5 6 ; 7 8 9]
```

This makes a matrix with two rows and three columns. You can access the row and column elements as a(row,col) so

```
a(2,3)
```

would access the second row, third column element. In this case, a value of 9.

You can use variables as the index when accessing elements:

```
a=[ 4 5 6 ; 7 8 9]
b=2
c=a(b,b)
```

Which would return the 2,2 element of a, in this case 8.

Note that Matlab will complain if you try to access an element of an array that has not been assigned yet, like a(5,5) in this example.

Characters can be used in an array. An array of characters is usually called a string:

```
a='This is a string.'
b=a(1:4)
```

This would put the first four elements of a into the variable b, in this case 'This'

Complex Data Structures

Matlab also allows you to have more complex data structures, so that multiple pieces of data are associated with a single variable.

```
person.age=30
```

```
person.name='Tom'
person.phone=5551234
```

This means you could pass the data structure to a function with a single variable name.

14.6 IF statements

IF statements allow you to check a logical condition. If the logical condition is met, you do something. If not, you may do something else.

```
IF (1<0)
     disp('Amazingly, 1 is less than 0')
     c=1
  END
```

Notice the indentation for the IF statement. Indentation helps read your code. As you see, you can put one statement to execute, or as many as you want.

```
IF (a<b)
     disp('Apparently, a is LESS than b')
  ELSE
     disp('Apparently, a is NOT LESS than b')
  END
```

You can put extra logical conditions in an IF statement as well.

```
IF (a<b)
     disp('Apparently, a is LESS than b')
  ELSEIF (a>b)
     disp('Apparently, a is GREATER than b')
  ELSE
     disp('Apparently, a must be EQUAL to b')
  END
```

If you have multiple IF and ELSEIF statements. If one condition is met, the conditions after and the ELSE code will never execute.

```
  IF (a<5)
     disp('Apparently, a is LESS than 5')
  ELSEIF (a>2)
     disp('Apparently, a is GREATER than 2')
  ELSEIF (a<7)
     disp('This will never execute, since one of the first two
conditions must be true')
  ELSE
     disp('Apparently, a must be EQUAL to b')
     disp('This will never execute, since one of the first two
conditions must be true')
```

```
END
```

You can have an IF statement inside an IF statement.

```
IF (a<b)
   IF (a>0)
      disp('Apparently, a is LESS than b and positive')
   ELSE
      disp('Apparently, a is LESS than b and not positive')
   END
ELSE
   disp('Apparently, a is NOT LESS than b')
END
```

You can also have more complicated logical statements using AND and OR operators

```
IF ((a<b) & (a>0))
   disp('Apparently, a is LESS than b and positive')
END
```

14.7 FOR statements

If you want to do something a few times, and you know how many times you want to do it, use a FOR statement.

```
FOR i=1:3
   a(i)=i*i;
END
```

This will make a into a vector [1 4 9].
You can use variables for the index as well.

```
l=length(a)
FOR i=1:length(a)
   b(i)=a(i)^2;
END
```

So, no matter how long the vector a is, this will put the elements of a, squared into b, and b will be the same length as the vector a.
You can also have nested FOR loops as well.

```
[rows,cols]=size(a)
FOR rowcounter=1:rows
   FOR colcounter=1:cols
      b(rowcounter,colcounter)=a(rowcounter,colcounter)+5
   END
END
```

14.8 WHILE statements

WHILE statements execute until some condition is met. This means they could execute forever, if the condition is not ever met. These statements are useful if you don't know how many times you want something to execute.

```
sum=0
data=1
WHILE (data>0)
   data=input('Enter a positive number, or 0 to quit')
   sum=sum+data
END
disp(['The resulting sum is ' num2str(data)])
```

This would keep prompting the user for another number, and add that number to sum. When the user enters 0 (or any negative number) the loop breaks out and continues on, displaying the result.

14.9 Scripts

In Matlab, you can start up the Matlab environment and start entering commands at the prompt, >>. This is great for simple things, but for anything slightly complex you may want to save your commands. Type "edit" at the prompt to open the text editor. You can type a bunch of commands in that text editor and save it on the computer. Usually the file name ends with a .m extension. To run your commands from the text file, you have multiple options.

Assuming the file name has no spaces and starts with a character, you can just type the file name at the prompt and all the commands in the file would execute (until it hits some sort of error or finishes). This also assumes Matlab is currently specified the directory where you saved your file. The "current directory" at the top right tells you where Matlab thinks it is right now. Hit the '...' button to change the directory.

For a few lines of text, you can highlight the selected commands from the text editor and copy-paste the text into the Matlab window. CTRL-C is copy and CTRL-V is paste. You can also highlight the selection, then right click on the text and select 'evaluate selection'.

14.10 Functions

Script files end with a .m extension and do something specified by the text in the file. In some cases, you may want to generalize a procedure to do something, like compute the mean of a vector. You can specify a function in a file with a .m extension similar to a script. The function must be saved in a file named procedurename.m. The first line of the file has a specific format. For example, the following would need to be saved in a file myfunction.m in the current directory of Matlab. (Typing `edit myfunction` at the command prompt will open the Matlab editor and create a new file myfunction.m)

```
function mean = myfunction(x)
```

```
n=length(x)
mean=0
for i=1:n
   mean=mean+x(i)
end
mean=mean/n
```

This function takes what ever input x, figures out the length of x, figures out the sum of the elements in the input, then calculates the mean of the vector. This assumes x is a vector (or a scalar value). You can call this function from another function or call it from a script or call it from the Matlab prompt.

The variable x is whatever you call the function with, so the following would work:

```
myfunction( [ 1 2 3 4 ] )
a = [ 3 4 5 6]
myfunction(a)
```

Note that the variables n and i are used inside the function. These are called local variables. If you had a variable named n or i outside of the function, calling the function would not change the value of the variables outside the function.

```
n=5
myfunction( [ 1 2 3 ] )
```

Here, n would still be 5, although inside `myfunction` n will have a value of 3.

You can specify multiple outputs for your functions as well.

```
function [minval, maxval]=minmaxfunction(x)
minval=x(1)
maxval=x(1)
n=length(x)
for i=2:n
   if (x(i)<minval)
      minval=x(i)
   end
   if (x(i)>maxval)
      maxval=x(i)
   end
end
```

Think about what happens in this function. If x is length 1, the resulting minval and maxval are just x. Otherwise, it goes through the indices of x from 2 to the end looking for a bigger and bigger or smaller and smaller values.

14.11 Debugging

Debugging just means fixing your code. For example, you may write code that goes past the end of a vector or does not produce the desired output. You must think about the variable values at each step in your code and think about what is happening at each step. This is called a variable trace. Sometimes it helps to print out variable values at many points in your code and see where things go wrong. Matlab will print out variable values if you type a variable name by itself.

```
>> a=2
a =
    2
>> a
a =
    2
```

You can suppress this normal output by using a semicolon at the end of a line.

```
function out=myfunction(x)
count=0
out=0
while (count<length(x))
   out=x(count)
   count=count+1
end
```

This function will not work. The count variable starts at 0, so Matlab will complain when you try to access the 0th element of the value *x*.

Pseudo Code

Pseudo Code is just a way of sketching out a solution methodology. Using pseudo code, you don't have to use accurate code syntax. You can summarize steps into a single idea, like "find the minimum and maximum values of the data" or "save the data in the specified format."

Compiling vs. Interpreting

On a PC, you have executable programs. These are special files with the commands needed to run something on the computer. Executable files on a PC end in .exe. These are created by compilers. You take source code written in C or C++ or Fortran or some other language and run it through a compiler to make an executable file. For example, MS Word is a .exe file and so is Matlab.

Matlab, MathCAD, and Java are all interpreted environments. Interpreted files rely on some executable to be running. Matlab figures out what to do for a given .m file. Interpreted environments are usually slower than compiled code.

14.12 Exercises

Start Matlab and try out the following examples. Type the bold lines that do not start with a % symbol in the Matlab prompt or in the editor, then run them. In some cases, make the specified function or script file and try running it in the Matlab prompt. Type 'edit' at the prompt to get the Matlab editor. You can type complex commands in the editor, then run them as a single .m file at the command prompt.

```
% Comments vary from language to language.
% Use comments to explain what your code is doing.
% In Matlab, anything to the right of a % is treated as a comment!


% DATA STRUCTURES

% We use variables to represent data of different types

% Traditional data structures include:

% integers

i=1
j=5

% real numbers (double precision)

pi=3.141
epsilon=0.001

% strings

name='bubba'
city='columbia'

% and boolean TRUE FALSE expressions.
% Note that in MATLAB, boolean is expressed as 0=false, 1= true.

flag1 = ( 1 < 0 )
flag2 = ( 1 > 0 )

% Also note that there are various Boolean operators you can
use.
%  These include <.  >, <=, >=, ==, ~=.
%  In Matlab & means logical AND, | means logical OR,
```

```
%   ~ means complement, and xor is XOR
%  See: help relop


% Special data structures

% Arrays - Arrays contain multiple pieces of data indexed
% along one or more dimensions for example, a vector can be
% seen as a 1D array of real numbers and a matrix can be
% described as a 2D array of real numbers.

b=[1 2 3 4]
b(1)
b(1:2)

A=[1 2 3;4 5 6]
A(1,1)
A(:,1)
A(:,1:2)

% Note that you can use N dimensional arrays,
%  but matrix multiplication won’t work:

C(2,2,2,2)=5

% Note that strings are really just a 1D array of single characters

name=’bubba’
name
name(2:4)


% You can use arrays of strings

names={’Bob’,’Sue’,’Tom’}
names(2)

% Structures - Structures provide convieninet representation

% for storage of data associated with a single name.

ssc.a=[1 0 ; 0 1]
ssc.b=[1 ; 0]
ssc.c=[1 0]
ssc.d=[0]
```

```
ssc.name='Test Model 1'
ssc.date='2/25/03'

% Most languages handle strings and structures differently,
% so watch out.

% MATLAB includes many nice functions for matrix manipulation

% and matrix operation that are not available in other languages.

% Matlab also includes data structures for complex numbers
% (also not available in most other languages.


% Assignment statements

% Most of you code is assignment statements.
% When you write a line of code, the name left of the = takes

% on the values of whatever is right of the =.

c=2+3
c=c+2+c*c

% Some functions are built in.

d=sqrt(5)
e=sin(2)
f=exp(3)


% Multiple expressions can be evaluated at once, be careful
% of brackets.

g=sin(exp(sqrt(6)))


% Order of operations - when making an assignment for a
% complex expression, you follow the standard order of
% operations:

% Please Excuse My Dear Aunt Sally

% Parens, inner first
% Exponents, Powers or root
```

```
% Multiply or
% Divide (left to right)
% Add or
% Subtract (left to right)

c= 100 - 10*(2 + 3) + 4
d= 36  /  4*(5 - 2) + 6

% If you have doubts, use more parens to specify the desired
order.


% FLOW CONTROL



% If statements
% If statements allow for sections of code to be executed only

% if a condition is met.  The condition must evaluate to a TRUE

% or FALSE value.

x=3

if (x>0)
   x=x^2
end



x=-3
y=-2

if ( (x<0)&(y<0) )
   x=x^2;
   y=y^2;
end

x,y

% Note the indentation of code inside the IF statement.
% This really helps code be legible.
```

```
% If / Else
% Usually, IF syntax includes an else condition.  Whenever the

% condition is not met, the second section of code executes.

x=0

if (x>0)
    disp('x is strictly positive')
else
    disp('x is 0 or negative')
end


% Usually, IF syntax includes elseif conditions.  The boolean

% values are checked in order.  Whenever a condition is met,

% the corressponding section of code is executed.  Note that

% even though a second conditional statement may evaluate to

% true, it never gets a chance to execute.

x=1

if (x==0)
    disp('x is 0')
elseif (x==1)
    disp('x is 1')
elseif (x==2)
    disp('x is 2')
elseif (x==3)
    disp('x is 3')
elseif (x==1)
    disp('x is 1, second time, will not run')
else
    displ('x is not 0, 1, 2, or 3')
end


% LOOPS
% Loops let you do repetitive stuff easily.

% FOR
```

```
% FOR loops are useful when you know how many times you may
% want to run the loop before you enter the loop.  This is
% especially good for manipulating data in an array.

x=[2 4 3 6 5 7 2 1]
s=length(x)

for i=1:1:s
    i
    x(i)=x(i)^2;
end
x


% In MATLAB, you can start the loop at any number and
% increment by any value.

x=[]
for i=2:5:30
    x=[x i]
end

% Note that array indices in MATLAB start at 1.   For an array

% (vector) of length s you will get an error if you try to
% access elements 0 or s+1.

x
s=length(x)
x(0)
x(s+1)



% WHILE
% WHILE loops continue to evaluate until the a boolean value

% is not longer positive These are usually used when the
% number of iterations in the loop are not know before
% entering the loop.

value=input('Input a number or Q to quit:   ','s');
value=str2num(value);
while (value)
   newvalue=value*value;
```

```
   disp(['New value is ' num2str(newvalue) ]);
   disp(' ');
   value=input('Input a number or Q to quit:   ','s');
   value=str2num(value);
end


% NESTED STATEMENTS

% You can have an if statement inside an if statement:

x=1
y=3
if (x<0)
    if (y<0)
        disp(' x and y are negative');
    else
        disp(' x negative, y positive or 0');
    end
else
    if (y<0)
        disp(' x positive or 0 and y negative');
    else
        disp(' x and  y positive or 0');
    end
end

% Note the indentation increases as more statements are nested.

% Nested FOR / WHILE

A=[]

for i=1:3
    for j=1:3
        [i j]
        A(i,j)=i+j;
    end % end for columns loop
end % end for rows loop

A


% Variable trace / debugging
```

```
% When you have an error, the error may be apparent or hidden.
% An apparent error may cause the program to stop and report

% the offending line.  The error may be a syntax error or a
% coding logic error (array index problem, divide by zero, etc)
% Hidden errors cause the code to run in unintended ways.
% Wither way, you may need to "trace" variable values to make
% sure the program is performing as expected.  In MATLAB, you

% can print variable values just by using the name of the
% variable without a semicolon.

A

% or use whos to get information on a variables

whos

whos A

whos n*


% SUBROUTINES

% User defined scripts
% In MATLAB, you can save a string of commands in a textfile

% with a .m extension.  Typing the name of the command at the

% prompt will cause the commands to be executed as if you
% were typing commands at the prompt.

t=[-1:.01:5];
u=t>=0;
y=exp(-t).*u;
subplot(2,1,1)
plot(t,u)
title('My name is')
subplot(2,1,2)
plot(t,y)
```

Print out the plot to turn in.

```
% User defined function
```

```
% If you continually are doing the same procedure, you can
% generalize the procedure to make your own function.  Some
% functions are built in (sin, exp, length, etc).
% The function takes input arguments, performs some
% operations, and returns output values.  In MATLAB, you put

% your function in a text file with a .m extension.
```

```
function mean = stat(x)
     %STAT Interesting statistics.
      n = length(x);
      mean = sum(x) / n;
```

```
% You can have more interesting functions that return
% multiple outputs:
```

```
function [mean,stdev] = stat(x)
     %STAT Interesting statistics.
      n = length(x);
      mean = sum(x) / n;
      stdev = sqrt(sum((x - mean).^2)/n);
```

```
% Scope of variables
% Variable scope is important!
% Inside functions, you may use new variables.  These are
% often called local variables.  In the previous example, n is

% a local variable.  n takes a value when the function is
% called.  If variable n had a value outside of the function,

% it would not be changed.
```

```
n=5
stat([1 2 3 4 5 5 6 7 8])
n
```

```
% Variables can be defined as global.  This means that they
% can be changed inside a subroutine, assuming the subroutine

% knows it is a global variable.
```

```
clear n
global n
```

```
n=5
stat([1 2 3 4 5 5 6 7 8])
n

% Recursive functions


function out=fact(x)

if (isreal(x))

   if (x==1)
      out=1
   else
      out=x*fact(x-1)
   end
end


% Everything to this point has been with respect to procedural

% programming.  CS often discuss object-oriented programming.

% This is a methodology that considers all data as objects.
% These objects all have a class.  One object may be a
% subclass of another object.  For example, you may have a
% class student.  There may be a subclass undergraduate and a

% subclass graduate.  All students should have a name, but
% undergraduates would have class standing and graduate
% students would have advisors.  Procedures can be written for

% each class.  MATLAB is not easy to use for object oriented

% programming, I suggest Java.
```

Chapter 15

TLAs of PSE

Three Letter Acronyms of Process Systems Engineering

There are many acronyms used in engineering and technical fields. This may serve as an initial foray into

Note that many of the terms are not actually acronyms, but rather initialisms. To my understanding, an acronym is usually pronounced as a word rather than individual letters.

From http://www.randomhouse.com/wotd/index.pperl?date=19980825

> In technical use among linguists and lexicographers, there are two main terms. An acronym is used for a word formed from the initial letters of the words (or main words) in a series of words, when the resulting word is pronounced as a word. Thus, OPEC, from Organization of Petroleum Exporting Countries, is considered an acronym, because it is pronounced "OH-peck," not as "Oh-pee-ee-see."

Additionally, some terms listed here were not borrowed from other sources. The appropriate acronym of initialism was included for completeness of some topics. Terms that were not borrowed from other sources will denoted by an asterict. Example: MUT*Made Up Terms.

15.1 VMM Various Modeling Methods

FPM* Fundamental Process Model

A FPM is based on fundamental principles derived from physics. These fundamental principles may be partially erroneous due to assumptions made during the model derivation. Some examples of simplifying assumptions include assumption of a well mixed reactor or the assumption of no axial dispersion in a plug flow reactor. As long as the assumptions hold and the fundamental principle is true, the model should be accurate. One may extrapolate using a FPM with some limited degree of confidence. Data may be required to fit the unknown model parameters.

EPM* Empirical Process Model

An EPM is based on process data and limited physical insight. The coeficients of the model are derived from the data once the model form is established. For example, given data for draining of water from a tank, an EPM may be assumed to be exponenental decay, and the height as a function of time could be established to follow the function $h(t) = 3e^{-2t}$. A FPM could be derived from the solution to a dynamic mass balance equation for the tank, assuming flow out of the tank is proportional to the square root of the height of water in the tank

$$V\frac{dh}{dt}(t) = 0 - k\sqrt{h(t)}$$

In the FPM, the relationship should hold for any tank system given known values for V and k. Empirically derived models typically have difficulty extrapolating to new operating conditions different from those where the model was established.

MEB* Mass and Energy Balance

This is really the basis for a large amount of chemical engineering. Given a system with a fixed boundary and mass or energy entering or leaving the system, the general form of the MEB appears as:

$$accumulation = in - out + created - destroyed$$

PME* Process Modeling Environment

Many PME's are available for computational modeling of chemical systems

LPS* Lumped Parameter System

IDS Infinite Dimensional System

SSS Steady State Simulation

ODE Ordinary Differential Equation

PDE Partial Differential Equation

FEM Finite Element Modeling

FVM Finite Volume Modeling

NSE Navier Stokes Equations

MDS Molecular Dynamics Simulation

MMS Molecular Modeling Simulation

CCS* Computational Chemistry Simulation

DFT Density Functional Theory

MCS Monte Carlo Simulation

KMC Kinetic Monte Carlo

DEM Discrete Element Modeling

DAE Differential Algebraic Equation

CIC *Consistent Initial Conditions

HDS Hybrid Dynamic System

MLD Mixed Logical Dynamic

ASS* Autonomous Switched System

PBE Population Balance Equations

PSD Particle Size Distribution

MWD Molecular Weight Distribution

PVM Process Video Microscopy

MDP* Multi Dimensional PBE

MSS Multi Scale System

MSM Multi Scale Modeling
ANN Artificial Neural Network
PNS Peri Net Simulation
HMM Hidden Markov Model
SBS* Stochastic Batch Simulation

15.2 ODEs

ODE Ordinary Differential Equation
NSS* Nonlinear State Space
LSS* Linear State Space
MIMO Multiple Input Multiple Output
SISO Single Input Single Output
LTI Linear Time Invariant
ONF* Observability Normal Form
CNF* Controllability Normal Form
LTV Linear Time Varying
TSE Taylor Series Expansion
LDS* Linearization of Dynamic System
AGI* Adams Gear Integration
RKI* Runge Kutta Integration
DTS Discrete Time Systems
SDS Sampled Data System
DTM Discrete Time Modeling

15.3 BLA Basic Linear Algebra

MRR* Matrix Row Reduction
MEP* Matrix Eigenvalue Problem
CEP Characteristic Equation Polynomial
LEV* Left Eigen Vector
REV* Right Eigen Vector
MPP Moore-Penrose Psuedo-Inverse
LPI* Left Pseudo Inverse
RPI* Right Pseudo Inverse
JNF* Jordan Normal Form
MES* Matrix Exponential Solution
STM State Transition Matrix
LUD* Lower Upper Decomposition
DMD Dumage Mendolson Decomposition
SVD Singular Value Decomposition
COA* Controllability and Observability Analysis
COG* Controllability and Observability Grammians

15.4 CNM Computational and Numerical Methods

FPA Floating Point Arithmetic
SMA Sparse Matrix Algebra
FDA Finite Difference Approximation
SNS Simple Newton Step
IVP Initial Value Problem
FVP Final Value Problem
NMS Newton's Method Solution
BMS Bisection Method Solution
GSC Grahm Schmitt Colocation
MIM Model Identification Methods
SSG Steady State Gain
OTC Open loop Time Constant
PLS Partial Least Squares
PCA Principal Component Analysis
PRBS Pseudo Random Binary Sequence
CLI Closed Loop Identification
LSE Least Squares Estimate
PFI Plant Friendly Input
MSA Multivariate Statistical Analysis
MOM Method Of Moments

15.5 LTM Laplace Transform Modeling

LOT Linear Operator Theory
OTF Open loop Transfer Function
DDF Dirac Delta Function
EDF Exponential Decay Function
HSF H Step Function
RPF Rectangular Pulse Function
SWF S Wave Function
IRF Ideal Ramp Function
FOS First Order System
TDS Time Delay System
FOTD First Order + Time Delay
SOS Second Order System
HOS High Order System
IRS Inverse Response System
LLS Lead Lag System
DFS Direct Feed System
OUS Open loop Unstable System
PZE Pole Zero Excess
SPS Strictly Proper System

BIBO Bounded Input Bounded Output
FVT Final Value Theorem
ITT Initial Value Theorem
LCF Linear Composite Function
PFE Partial Fraction Expansion
ILT Inverse Laplace Transform
FRA Frequency Response Analysis
RHP Right Half Plane
LHP Left Half Plane
HPF High Pass Filter
LPF Low Pass Filter
BPF Band Pass Filter
CTF Open loop Transfer Function

15.6 DTM Discrete Time Modeling

FIR Finite Impulse Response
FMM Fading Memory Model
ZOH Zero Order Hold
ARM Auto Regressive Model
MAM Moving Model
ARMA Auto Regressive Moving Average
DVM Discrete Volterra Model
VLP Volterra Laguere Polynomial

15.7 NSA Nonlinear System Analysis

SSL Steady State Locus
SSM Steady State Manifold
ASR Asymmetric Step Response
HNO Hard Nonlinear Operators
AS Actuator Saturation
AH Actuator Hysterisis
VS Valve Stiction
SLC Stable Limit Cycle
PFB Pitch Fork Bifurcation
SHB Sub (Super) Critical Hopf Bifucation
PPA Phase Plane Analysis
SF Stable Focus
UF Unstable Focus
SN Stable Node
UN Unstable Node
SP Saddle Point

PNM Process Nonlinearity Measures

15.8 BFC Basic Feedback Control

FCS Feedback Control System
STA Setpoint Tracking Analysis
DRA Disturbance Rejection Analysis
MMA Model Mismatch Analysis
DCS Distributed Control System
DAQ Data Acquisition
MNF Measurement Noise Filter
SSC Steady State Control
ECL Explicit Control Law
ICL Implicit Control Law
OLC Open Loop Control
NSC Nyquist Stability Criterion
CLS Closed-Loop Stability
FFC Feed Forward Control
PID Proportional Integral Derivative
ZNT Ziegler Nichols Tuning
CCT Cohen Coon Tuning
SAE Sum Absolute Error
SSE Sum Square Error
QDR Quarter Decay Ration
NST Ninety-five percent Settling Time
SSO Steady State Offset
SRC Step Response Curve

15.9 MNC Multivariable and Nonlinear Control

MTF Multivariable Transfer Function
STF State space Transfer Function
GSC Gain Scheduling Control
FLC Fuzzy Logic Control
NNC Neural Network Control
MMC Multi Model Control
IMC Internal Model Control
RGA Relative Gain Array
RLD Root Locus Diagram
MCM Multivariable Control Methods
MDC Multivariable Decoupling Control
MPC Model Predictive Control
DMC Dynamic Matrix Control

FSF Full State Feedback
MRC Multi Rate Control
LQR Linear Quadratic Regulator
LQG Linear Quadratic Gaussian
EKF Extended Kalman Filter
DGC Differential Geometric Control
IOL Input Output Linearization
SEL State space Exact Linerization
TDF Two Degree of Freedom
SSI SubSpace Identification
FFT Fast Fourier Transform
DSC Direct Synthesis Control
HIC H Infinity Control
MSC Mu Synthesis Control
ICM Inferential Control Methods
ACM Adaptive Control Methods
CCC CasCade Control
BBC Bang Bang Control
OOC Open loop Optimal Control
OCT Optimal Control Theory
LSC Lyapunov Stability Criterion
SGT Small Gain Theorem
LSC Loop Shaping Control
PMT Phase Margin Tuning
GMT Main Margin Tuning
NSC Nominal Stability Criterion
NPC Nominal Performance Criterion
RSC Robust Stability Criterion
RPC Robust Performance Criterion
MDD M Delta Diagram
MLS Measurement Location Selection
RTO Real Time Optimization

15.10 SPC Statistical Process Control

PDF Probability Density Function
GDF Gaussian Distribution Function
NDF Normal Distribution Function
WDF Weibel Distribution Function
BDF Binomial Distribution Function
STT Student T Test
ARE Algebraic Riccatti Equation

15.11 FDE Fault Diagnosis and Estimation

FDI Fault Detection and Isolation
QTA Qualitative Trend Analysis
ANN Artificial Neural Networks
RBF Radial Basis Function
BNN Butterfly Neural Network
HPL Hidden Perceptron Layer
SEM State Estimation Methods
PEM Parameter Estimation Methods
LOE Luenberger Observer Estimation
KFE Kalman Filter Estimation
WSS Wide Sense Stationary
ACF Auto Correlation Function
GDW Gaussian Distributed White noise
MHE Moving Horizon Estimation
RDA Residual Direction Analysis
FGM Fault Gain Matrix
EIV Error In Variables
MLM Maximum Likelihood Methods
ERS Expert Rule System
FTA Fault Tree Analysis
SDG Sign Directed Graph
DEA Disturbance Estimation and Analysis
OOP Object Oriented Programming
BBN Bayesian Belief Network
HMI Human Machine Interface
AAM Alarm Analysis Methods
AFH Alarm Flooding Handling

15.12 NOM Numerical Optimization Methods

NLP NonLinear Programming
KKT Karush Kuhn Tucker
LP Linear Programming
QP Quadratic Programming
IPM Interior Point Methods
GA Genetic Algorithm
SA Simulated Annealing
MINLP Mixed Integer Nonlinear Programming
GBD Generalized Benders Decomposition
OA Outer Approximation
DO Dynamic Optimization
MIDO Mixed Integer Dynamic Optimization

NCV NonConVex
CVF ConVex Function
CVS ConVex Set
UBD Upper BounD
LBD Lower BounD
CA Convexity Analysis
PSD Positive Semi Definite
IA Interval Analysis
DR Directed Rounding
CR Convex Relaxation
CH Convex Hull
PF Perturbation Function
NP Non-deterministic Polynomial
ABB Alpha Brach-and-Bound
B&B Branch and Bound
B&R Branch and Reduce
POS Pareto Optimal Surface
OUU Optimization Under Uncertainty
MLO Multi Level Optimization
BMC Big M Constraint
CP Constraint Programming
MAO Multi Agent Optimization
DPT Disjunctive Programming Techniques
GDP Generalized Disjunctive Programming
DP Dynamic Programming
PL Propositional Logic
TE Total Enumeration
RS Random Search
IP Integer Programming
MIP Mixed Integer Programming
BDM Business Decision Maker
ROI Return On Investment
OFC Objective Function Cut
LM Lagrange Multiplier
ACS Active Constraint Set
DGO Deterministic Global Optimization
SO Stochastic Optimization
DS Degenerate Solution
BFGS
BF Barrier Function
SQP Sequential Quadratic Programming
RGM Reduced Gradient Methods
CCA Computational Complexity Analysis
GC Gantt Chart
OR Operations Research

NO Network Optimization
BNA Bottle Neck Analysis
LSS Large Scale Scheduling
DSS Decision Support Systems
LMI Linear Matrix Inequality
BMI Bilinear Matrix Inequality
PTC Polynomial Time Complexity
ROI Return On Investment
TSP Traveling Salesman Problem
SMP Set Matching Problem
SCP Set Covering Problem
KSP KnapSack Problem
RAP Resource Allocation Problems
CPD Chemical Process Design

15.13 VAI Various Applications and Industries

MPA Metabolic Pathway Analysis
MCA Metabolic Control Analysis
DDS Drug Delivery Systems
HGP Human Genome Project
DCC Distillation Column Control
CRC Chemical Reactor Control
PWC Plant Wide Control
HCS Hierarchical Control System
BRC Bio Reactor Control
IDC Interaction of Design and Control
HS Hybrid Systems
MEM Micro Electro Mechanical
PS Particulate Systems
PPS Portable Power Systems
PEM Polymer Electrolyte Membrane
WGS Water Gas Shif
ATR AutoThermal Reforming
PROX PReferential OXidation
CPO Catalytic Partial Oxidation
PEM Proton Exchange Membrane
GDL Gas Diffusion Layer
CSA Cell Stack Assembly
FPS Fuel Processing System
TMS Thermal Management System
PCS Power Conditioning System
PCI PetroChemical Industries
SCI Specialty Chemical Industries

WWT Waste Water Treatment
ECM Environmentally Conscious Manufacturing
PPI Pulp and Paper Industries
CCI Commodity Chemicals Industries
PI Pharmaceutical Industries
BMI Bio Medical Industries
AI Automotive Industries
FAI Food and Agricultural Industries
PGS Power Generation Systems
PPS Portable Power Systems
PEM Polymer Electrolyte Membrane
AES Alternative Energy Systems
TI Textile Industries
MD Materials Development
CC Combinatorial Chemistry
DIA Defense Industry Applications
BS Batch Systems
MCS Micro Chemical Systems
IT Information Technology
MPI Mineral Processing Industries
CPI Consumer Products Industries
SI Steel Industries
AAI Airline and Aircraft Industries
EI Entertainment Industries
BFI Banking and Financial Industries
TTI Transportation and Trade Industries
PHC pH Control

15.14 GFA Governmental Funding Agencies

NSF National Science Foundation
NIH National Institute of Health
DOE Department of Energy
NASA National Space Administration
DOD Department of Defense
ARL Army Research Laboratory
ONR Office of Naval Research
NSA National Security Agency
NRO Naval Reconnassance Office
DEA Drug Enforcement Agancy
FBI Federal Buruea of Invesitation
CIA Central Intelligence Agency
NEA National Endowment for the Arts
USDA US Department of Agriculture

FDA Food and Drug Administration
DHS Department of Homeland Security
TSA Travel Security Agency

15.15 CPD Chemical Process Details

PFID Process Flow and Instrumentation Diagram
PV Process Variable
MV Manipulated Variable
CV Control Variable
DV Disturbance Variable
MCV Manual Control Valve
ACV Automatic Control Valve
RPS Remote Pressure Sensor
TTS Thermowell Temperature Sensor
OCM Online Concentration Measurement
MFM Magnetic Flow Meter
SFM Steam Flow Meter
LLI Liquid Level Indicator
LSM Laboratory Sample Measurement
BUO Basic Unit Operations
PFD Process Flow Diagram
PFR Plug Flow Reactor
CSTR Continuous Stirred Tank Reactor
FBR Fluidized Bed Reactor
HXN Heat eXchanger Network
BDC Binary Distillation Column
SSS Side Stream Splitter
SGP Steam Generation Plant
CWU Cold Water Utilities
HWU Hot Water Utilities
SSC Steam Stripping Column
ESP Electro Static Precipitator
EHP Environmental Holding Ponds
DEE Double Effect Evaporators
STH Shell and Tube Heat Exchanger
DIW De Ionized Water

15.16 MID Measurement and Instrumentation Devices

GC Gas Chromatograph
MS Mass Spectroscop
GCMS Gas Chromatograph / Mass Spectroscope

IR
FTIR
ATR
RS
SEM
TEM
AFM
XFM
XPS
XRD
TGA
BOD
COD

15.17 BCP Basic Computer Programming

DTM Deterministic Turing Machine
PPL Procedural Programming Language
OOP Object Oriented Programming
BEA Binary Executable Application
IPL Interpreted Programming Language
OOO Order Of Operations
PEMDAS Please Excuse My Dear Aunt Sally
BDS Basic Data Structures
CDS Complex Data Structures
DMA Dynamic Memory Allocation
IVV Initial Variable Value
VAS Variable Assignment Statement
VNC Variable Name Collision
FCS Flow Control Syntax
IWL Infinite While Loop
FCC Finite Convergence Criteria
BLC Boolean Logic Condition
PCE Psuedo Code Example
PBD Procedural Block Diagram
DIO Data Input / Output
REH Robust Error Handling
UVT Unknown Variable Trace
NIS Nested If Statement
RFC Recursive Function Call
LVS Local Variable Scope
GVS Global Variable Scope
SNAFU Situation Normal, All Fouled Up
TANSTAFEL There is No Such Thing As A FreE Lunch

GIGO Garbage In, Garbage Out
VSI Various Software Issues
OS Operating System
OSS Open Source Software
GNU GNU is Not Unix
GCC GNU C Compiler
GPL GNU Public License
YACC Yet Another Compiler Compiler
MPI Message Passing Interface
SMP Shared Memory Processing
G77 GNU Fortran 77
JVM Java Virtual Machine
C++ C Plus Plus
NFS Network File System
AFS Andrew File System
SMB SaMBa Network File System
SSH Secure Shell
SCP Secure CoPy
SFTP Secure File Transfer Protocol
FTP File Transfer Protocol
SMTP Simple Mail Transfer Protocol
POP
IMAP
HTML Hyper Text Markup Language
SGML Standard Generalized Markup Language
IT Information Technology
WWW World Wide Web
TCP / IP Transmission Control Protocol / Internet Protocol
RTS Real Time System
RTOS Real Time Operating System
VCP Various Computer Parts
CPU Central Processing Unit
FPU Floating Point Unit
GPU Game Processing Unit
MMU Memory Management Unit
L1C Level 1 Cache
L2C Level 2 Cache
RISC Reduced Instruction Set Commands
LCD Liquid Crystal Display
CRT Cathode Ray Tube
FDD Flash (or Floppy) Disk Drive
HDD Hard Disk Drive
USB Universal Serial Bus
SIMM Single Inline Memory Module
UPS Uninterruptible Power Supply

PNP Plug aNd Play
PCI Personal Computer Interface
VGA Video Graphics Adapter
SVGA Super Video Graphics Adapter
XGA
WAN Wireless Area Network
SAN Storage Area Network
LAN Local Area Network
NIC Network Interface Card