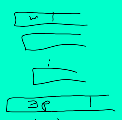
1) TQBF is NPSPACE-complete
2) TQBF is in PSPACE
Cor: PSPACE = NPSPACE

TM:



guess tape
1-way read only
Not counted in the space bound

Def
TM accepts an input w, iff there exist some contents of the guess tape that makes the TM accept.
per the standard def of accepting

High level algo can use the "guess x" primitive — copy symbol(s) from the guess tape to a work tape, call it x (x is a string).

Ex:
$EQ_{REX} := \{\langle R, S\rangle : R$ & S are regexes that are equivalent (denote the same language ($R = S$)$\}$

(eg. $a^* = aa^* \cup \varepsilon$)

EQ_{REX} is decidable:
(regex → NFA → DFA ...)

Prop: $\overline{EQ_{REX}} \in NPSPACE$:

"On input $\langle R, S\rangle$ where R, S are regexes over the same alphabet Σ:

1. Convert R to N_R (equiv NFA)
2. Convert S to N_S (equiv NFA)
3. Initialize sets of states for N_R & N_S on input ε
4. Repeat
   a) Guess a symbol $a \in \Sigma$
   b) Update set of states for each of N_R and N_S by reading a

   until one of N_R and N_S accepts & the other doesn't, or "too many guesses"
5. If too many guess, then reject; else accept."

Cor: $\overline{EQ_{REX}} \in PSPACE$

Proof: NPSPACE = PSPACE

Cor: $EQ_{REX} \in PSPACE$

Pf: PSPACE is closed under complementation. //

Review: 3 sections

1) Automata, regexes, reg langs
2) TMs, decidability, undecidability, mapping reductions, T-recognizability
3) Resource-bounded computation
   polynomial time, P, NP
   polynomial space, PSPACE (= NPSPACE)
   polynomial reducibility ($\leq_p$)
   NP-completeness
   NP-hardness
   PSPACE-completeness

$EQ_{REX^2} := \{\langle R, S\rangle : R, S$ regexes with squaring, and $R = S\}$

Squaring means: can use r^2 as shorthand for rr

EQ_{REX^2} is EXPSPACE-complete,
& EXPSPACE ≠ PSPACE
(space hierarchy theorem)

∴ $EQ_{REX^2} \notin PSPACE$

Final is cummulative, but more emphasis on topics not covered in the quizzes

Ex: space-bd comp
$TQBF \leq_p GG$
QBF input: is it true?

NP-completeness
General P, NP question

No: DFA minimization
Yes: state elim method for NFA → regex
Maybe: set of states for NFA → DFA

Prob: Show a lang is decidable

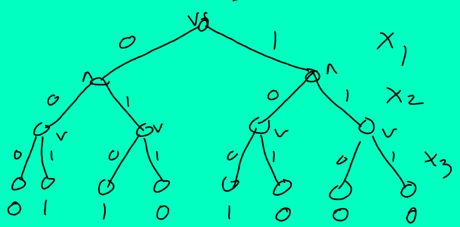
Yes: m-reduction from A_{TM} and/or $\overline{A_{TM}}$ to show undecidability and or non-T-rec.

Maybe: Prove that a function is not computable.

No: Simulate a TM formally on an input.

Ex:

$$\varphi := (\overset{\vee}{\exists} x_1)(\overset{\wedge}{\forall} x_2)(\overset{\vee}{\exists} x_3)\Big[(x_1 \vee x_2 \vee x_3) \wedge (\overline{x}_1 \vee \overline{x}_2 \vee x_3) \wedge (\overline{x}_2 \vee \overline{x}_3) \wedge (\overline{x}_1 \vee \overline{x}_3)\Big]$$



Yes: pumping lemma

Ex:

$L = \{ x \in \{0,1\}^* :$ x has a 0 somewhere in its first half (excl. middle if $|x|$ is odd)$\}$

Prop: L is not pumpable.

Proof:

$\forall p > 0$, let $s := 1^p 0 0^{p+1}$.

$s \in L$ & $|s| \geq p$.

Given x, y, z such that

1) $s = xyz$
2) $|xy| \leq p$
3) $|y| > 0$,

let $i := 2$. Then $xy^2z \notin L$ because:

$y = 1^k$ for some $k > 0$,

thus $xy^2z = 1^{p+k} 0 0^{p+1} = 1^{p+k} 0^{p+2}$

has no 0's in its 1st half,

$\therefore xy^2z \notin L$.

$\therefore$ L is not pumpable //

No: Myhill-Nerode Theorem.

TQBF $\leq_p$ TQBF-CNF