

Def: A Turing Machine (TM) is a tuple

$$M = \langle Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej} \rangle$$

where

- Q is a finite set (state set)
- Σ is an alphabet (the input alphabet)
- Γ is an alphabet (the tape alphabet)
- q_0, q_{acc}, q_{rej} are states (elements of Q)
- q_0 - start state

the halting states $\begin{cases} q_{acc} - \text{accept state} \\ q_{rej} - \text{reject state} \end{cases}$

and

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

except, $\delta(q_{acc}, -)$ and $\delta(q_{rej}, -)$ are undefined

Furthermore

- $q_{acc} \neq q_{rej}$
- $\Sigma \subseteq \Gamma, \perp \in \Gamma - \Sigma$ (blank symbol)
- $Q \cap \Gamma = \emptyset$

cell

0	1	2	...

comp opes in discrete time steps $0, 1, 2, \dots$

At each time step,

- every cell of the tape contains a unique symbol
- the TM is in a unique state
- the TM's head is scanning a unique cell.

$0 | 1 | x | 3 | 2 | w | \dots$

↑

q

Initially: on input $w \in \Sigma^*$

- the symbols of w are on the leftmost portion of the tape (cells $0, \dots, n-1$ if $n=|w|$)
- cell 0 (leftmost cell) is scanned
- state is q_0 (start state)

At any time: suppose

- M is in state $q \in Q$
- symbol being scanned is $a \in \Gamma$, then

in the next time step:

If $\delta(q, a) = (r, b, R)$

then: a is replaced by b in that cell, state becomes r , and head moves one cell to the right.

If $\delta(q, a) = (r, b, L)$

then same except head moves one cell left.

(exception: if head is scanning cell 0, it does not move).

When or if state is a halting state, computation stops (accepts if q_{acc} and rejects if q_{rej}).

Def: $M = \langle Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej} \rangle$ be a TM. A configuration of M (aka an instantaneous description (ID)) is of the form

$$\alpha q \beta \in (Q \cup \Gamma)^*$$

where $\alpha, \beta \in \Gamma^*$ and $q \in Q$.

$\alpha q \beta$ gives a complete snapshot of the state of the computation on any given time (so that the next step is uniquely determined).

$\alpha q \beta$ means: M is in state q , $\alpha \beta$ is the contents of the tape, starting at cell 0 and all subsequent cells.