CSCE 551/MATH 562, Homework 6
due Wednesday 4/10/2024

For the following, you can assume that all devices use the binary alphabet $\Sigma := \{0, 1\}$ for input/output/printing. For these problems, you may assume that every string can be interpreted as the encoding of a Turing machine.

Textbook Exercise 7.5: Is the following formula satisfiable?

$$(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$$

Textbook Exercise 7.6 Show that P is closed under union, concatenation, and complement.

Textbook Exercise 7.7 Show that NP is closed under union and concatenation.

Textbook Problem 7.22: Let

$$\mathit{DOUBLE\text{-}SAT} = \{\langle \phi \rangle \mid \phi \text{ has a least two satisfying assignments}\}\ .$$

Show that *DOUBLE-SAT* is NP-complete.

Textbook Problem 7.35: A subset of nodes of a graph G is a ***dominating set*** if every other node of G is adjacent to some node in the subset. Let

$$\mathit{DOMINATING\text{-}SET} = \{\langle G, k \rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}\ .$$

Show that it is NP-complete by given a reduction from *VERTEX-COVER*.

Not in the textbook 1: Suppose $\mathsf{P} = \mathsf{NP}$. Show that there exists a polynomial-time algorithm A that on input $\langle G \rangle$ outputs the maximum size of any clique in G. [Note that if $\mathsf{P} = \mathsf{NP}$, then there exists a polynomial-time decider for the CLIQUE decision problem.]

Not in the textbook 2: Suppose $\mathsf{P} = \mathsf{NP}$. Show that there exists a polynomial-time algorithm B that on input $\langle G, k \rangle$ outputs a clique of G with k many vertices, if there is one; otherwise B outputs "none."

Not in the textbook 3: Suppose $\mathsf{P} = \mathsf{NP}$. Show that there exists a polynomial-time algorithm C that on input $\langle G \rangle$ outputs a maximum-size clique in G. [Hint: Combine the algorithms of the last problem or the last two problems.]