

# CSCE 551/MATH 562, Homework 5

## due Monday 4/1/2024

For the following, you can assume that all devices use the binary alphabet  $\Sigma := \{0, 1\}$  for input/output/printing. For these problems, you may assume that every string can be interpreted as the encoding of a Turing machine.

1. Let

$$L := \{\langle M \rangle \mid 00 \in L(M) \text{ and } 11 \notin L(M)\} .$$

Show that  $A_{\text{TM}} \leq_m L$  and that  $\overline{A_{\text{TM}}} \leq_m L$ . Describe the two m-reductions directly, without appealing to Rice's theorem.

2. Let

$$L := \{\langle M \rangle \mid 00 \in L(M) \text{ or } 11 \notin L(M)\} .$$

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3. Recall that a string  $w$  is a *palindrome* if  $w = w^{\mathcal{R}}$ . Let  $PALINDROMES := \{w \mid w \text{ is a palindrome}\}$ , and let

$$L := \{\langle M \rangle \mid L(M) = PALINDROMES\} .$$

Show that  $A_{\text{TM}} \leq_m L$  and that  $\overline{A_{\text{TM}}} \leq_m L$ . Describe the two m-reductions directly, without appealing to Rice's theorem.

4. Let

$$L := \{\langle M \rangle \mid M \text{ is a DFA and } 0^*1^* \subseteq L(M)\} .$$

Show that  $L \in \mathbf{P}$  by giving a polynomial-time decision procedure for  $L$ . [Hint: DFA minimization, the product and complement constructions, and deciding whether two given DFAs are isomorphic (i.e., the same DFA up to state re-labeling) can all be done in polynomial time.]